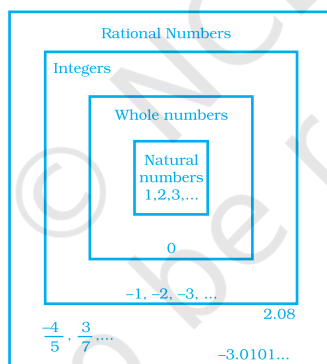


UNIT 1

RATIONAL NUMBERS

(A) Main Concepts and Results

- A number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a **rational number**.
- **Lowest form of a rational number** – A rational number $\frac{p}{q}$ is said to be in the lowest form or simplest form if p and q have no common factor other than 1 and $q \neq 0$.



Addition, subtraction, multiplication and division of rational numbers are done in the same way as we do for fractions.

- Rational numbers are closed under the operations of addition, subtraction and multiplication.
- The operations of addition and multiplication for rational numbers are
 - (i) commutative,
 - (ii) associative
- The rational number 0 is the additive identity for rational numbers.
- The rational number 1 is the multiplicative identity for rational numbers.

- The additive inverse of the rational number $\frac{a}{b}$ is $\frac{-a}{b}$ and vice-versa.
- The reciprocal or multiplicative inverse of the rational number $\frac{a}{b}$ is $\frac{b}{a}$ if $\frac{a}{b} \times \frac{b}{a} = 1$.
- **Distributivity of rational numbers** – For all rational numbers a , b and c

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$
- Rational numbers can be represented on a number line.
- Between any two given rational numbers there are infinitely many rational numbers. The idea of **mean** helps us to find rational numbers between two given rational numbers.

(B) Solved Examples

In examples 1 to 3, there are four options out of which one is correct. Choose the correct answer.

Example 1 : Which of the following is not true?

(a) $\frac{2}{3} + \frac{5}{4} = \frac{5}{4} + \frac{2}{3}$

(b) $\frac{2}{3} - \frac{5}{4} = \frac{5}{4} - \frac{2}{3}$

(c) $\frac{2}{3} \times \frac{5}{4} = \frac{5}{4} \times \frac{2}{3}$

(d) $\frac{2}{3} \div \frac{5}{4} = \frac{2}{3} \times \frac{4}{5}$

Solution : The correct answer is (b).

Example 2 : Multiplicative inverse of $\frac{0}{1}$ is

(a) 1

(b) -1

(c) 0

(d) not defined

Solution : The correct answer is (d).

Example 3 : Three rational numbers lying between $\frac{-3}{4}$ and $\frac{1}{2}$ are

(a) $-\frac{1}{2}, 0, \frac{3}{4}$

(b) $\frac{-1}{4}, \frac{1}{4}, \frac{3}{4}$

(c) $-\frac{1}{4}, 0, \frac{1}{4}$

(d) $-\frac{5}{4}, 0, \frac{1}{4}$

Solution : The correct answer is (c).

In examples 4 and 5, fill in the blanks to make the statements true.

Example 4 : The product of a non-zero rational number and its reciprocal is _____.

Solution : 1

Example 5 : If $x = \frac{1}{3}$ and $y = \frac{6}{7}$ then $xy - \frac{y}{x} =$ _____.

Solution : $-\frac{16}{7}$

In examples 6 and 7, state whether the given statements are true or false.

Example 6 : Every rational number has a reciprocal.

Solution : False

Example 7 : $-\frac{4}{5}$ is larger than $-\frac{5}{4}$.

Solution : True

Example 8 : Find $\frac{4}{7} \times \frac{14}{3} \div \frac{2}{3}$.

Solution : $\frac{4}{7} \times \frac{14}{3} \div \frac{2}{3} = \frac{4}{7} \times \left(\frac{14}{3} \times \frac{3}{2} \right)$
 $= \frac{4}{7} \times 7 = 4$

Example 9 : Using appropriate properties, find $\frac{2}{3} \times \frac{-5}{7} + \frac{7}{3} + \frac{2}{3} \times \frac{-2}{7}$.

Solution : $\frac{2}{3} \times \left(\frac{-5}{7} \right) + \frac{7}{3} + \frac{2}{3} \times \left(\frac{-2}{7} \right)$
 $= \frac{-5}{7} \times \frac{2}{3} - \frac{2}{7} \times \frac{2}{3} + \frac{7}{3}$

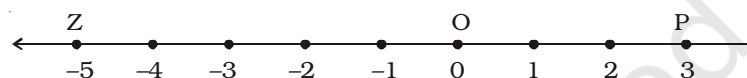
$$= \left(\frac{-5}{7} - \frac{2}{7} \right) \times \frac{2}{3} + \frac{7}{3}$$

$$= -\frac{2}{3} + \frac{7}{3} = \frac{5}{3}$$

Example 10 : Let O, P and Z represent the numbers 0, 3 and -5 respectively on the number line. Points Q, R and S are between O and P such that $OQ = QR = RS = SP$.

What are the rational numbers represented by the points Q, R and S. Next choose a point T between Z and O so that $ZT = TO$. Which rational number does T represent?

Solution :



As $OQ = QR = RS = SP$

and $OQ + QR + RS + SP = OP$

therefore Q, R and S divide OP into four equal parts.

So, R is the mid-point of OP, i.e. $R = \frac{0+3}{2} = \frac{3}{2}$

Q is the mid-point of OR, i.e. $Q = \frac{1}{2} \left(0 + \frac{3}{2} \right) = \frac{3}{4}$

and S is the mid-point of RP, i.e. $S = \frac{1}{2} \left(\frac{3}{2} + 3 \right) = \frac{9}{4}$

therefore, $Q = \frac{3}{4}$, $R = \frac{3}{2}$ and $S = \frac{9}{4}$

Also $ZT = TO$

So, T is the mid-point of OZ, i.e. $T = \frac{0+(-5)}{2} = \frac{-5}{2}$

Think and Discuss

- 1. Explain** the first step in solving an addition equation with fractions having *like* denominators.
- 2. Explain** the first step in solving an addition equation with fractions having *unlike* denominators.

Example 11 : A farmer has a field of area $49\frac{4}{5}$ ha. He wants to divide it equally among his one son and two daughters. Find the area of each one's share.
(ha means hectare; 1 hectare = 10,000 m²)

Solution : $49\frac{4}{5}$ ha = $\frac{249}{5}$ ha

$$\text{Each share} = \frac{1}{3} \times \frac{249}{5} \text{ ha} = \frac{83}{5} \text{ ha} = 16\frac{3}{5} \text{ ha}$$

Example 12 : Let a, b, c be the three rational numbers where $a = \frac{2}{3}, b = \frac{4}{5}$

$$\text{and } c = -\frac{5}{6}$$

Verify:

(i) $a + (b + c) = (a + b) + c$ (Associative property of addition)

(ii) $a \times (b \times c) = (a \times b) \times c$ (Associative property of multiplication)

Solution : (i) L.H.S = $a + (b + c)$

$$= \frac{2}{3} + \left[\frac{4}{5} + \left(-\frac{5}{6} \right) \right]$$

$$= \frac{2}{3} + \left[\frac{24 - 25}{30} \right]$$

$$= \frac{2}{3} + \left(-\frac{1}{30} \right)$$

$$= \frac{20 - 1}{30} = \frac{19}{30}$$

R.H.S. of (i) = $(a + b) + c$

$$= \left(\frac{2}{3} + \frac{4}{5} \right) + \left(-\frac{5}{6} \right)$$

$$= \left(\frac{10 + 12}{15} \right) + \left(-\frac{5}{6} \right)$$

$$= \frac{22}{15} - \frac{5}{6} = \frac{44 - 25}{30} = \frac{19}{30}$$

$$\text{So, } \frac{2}{3} + \left[\frac{4}{5} + \left(-\frac{5}{6} \right) \right] = \left(\frac{2}{3} + \frac{4}{5} \right) + \left(-\frac{5}{6} \right)$$

Hence verified.

$$\begin{aligned}
 \text{(ii) L.H.S} &= a \times (b \times c) \\
 &= \frac{2}{3} \times \left[\frac{4}{5} \times \left(\frac{-5}{6} \right) \right] \\
 &= \frac{2}{3} \times \left(\frac{-20}{30} \right) = \frac{2}{3} \times \left(\frac{-2}{3} \right) \\
 &= \frac{2 \times (-2)}{3 \times 3} = \frac{-4}{9}
 \end{aligned}$$

$$\begin{aligned}
 \text{R.H.S.} &= (a \times b) \times c \\
 &= \left(\frac{2}{3} \times \frac{4}{5} \right) \times \left(\frac{-5}{6} \right) \\
 &= \frac{2 \times 4}{3 \times 5} \times \frac{-5}{6} \\
 &= \frac{8}{15} \times \left(\frac{-5}{6} \right) \\
 &= \frac{8 \times (-5)}{15 \times 6} = \frac{-40}{90} = \frac{-4}{9}
 \end{aligned}$$

$$\text{So, } \frac{2}{3} \times \left[\frac{4}{5} \times \left(\frac{-5}{6} \right) \right] = \left[\frac{2}{3} \times \frac{4}{5} \right] \times \left(\frac{-5}{6} \right)$$

Example 13 : Solve the following questions and write your observations.

$$\text{(i) } \frac{5}{3} + 0 = ?$$

$$\text{(ii) } \frac{-2}{5} + 0 = ?$$

$$\text{(iii) } \frac{3}{7} + 0 = ?$$

$$\text{(iv) } \frac{2}{3} \times 1 = ?$$

$$\text{(v) } \frac{-6}{7} \times 1 = ?$$

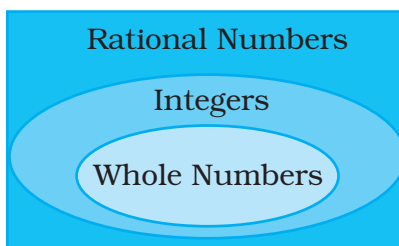
$$\text{(vi) } \frac{9}{8} \times 1 = ?$$

Solution

$$\text{: (i) } \frac{5}{3} + 0 = \frac{5}{3}$$

$$\text{(ii) } \frac{-2}{5} + 0 = \frac{-2}{5}$$

$$\text{(iii) } \frac{3}{7} + 0 = \frac{3}{7}$$



$$(iv) \frac{2}{3} \times 1 = \frac{2}{3} \quad (v) \frac{-6}{7} \times 1 = \frac{-6}{7} \quad (vi) \frac{9}{8} \times 1 = \frac{9}{8}$$

Observation

From (i) to (iii), we observe that: (i) When we add 0 to a rational number we get the same rational number.

From (iv) to (vi), we observe that: (ii) When we multiply a rational number by 1 we get the same rational number.

(iii) Therefore, 0 is the **additive identity** of rational numbers and 1 is the '**multiplicative identity**' of rational numbers.

Example 14 : Write any 5 rational numbers between $\frac{-5}{6}$ and $\frac{7}{8}$.

Solution : $\frac{-5}{6} = \frac{-5 \times 4}{6 \times 4} = \frac{-20}{24}$

and $\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$

Thus, rational numbers $\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, \dots, \frac{20}{24}$ lie

between $\frac{-5}{6}$ and $\frac{7}{8}$.

Example 15 : Identify the rational number which is different from the other three : $\frac{2}{3}, \frac{-4}{5}, \frac{1}{2}, \frac{1}{3}$. Explain your reasoning.

Solution : $\frac{-4}{5}$ is the rational number which is different from the other three, as it lies on the left side of zero while others lie on the right side of zero on the number line.

Example 16 : Problem Solving Strategies

Problem : The product of two rational numbers is -7 . If one of the number is -10 , find the other.

Solution : Understand and explore

- What information is given in the question?
One of the two rational numbers
Product of two rational numbers
- What are you finding?
The other rational number

Plan a strategy

- Let the unknown rational number be x . Form an equation with the conditions given. Then solve the equation.

Solve

Let the other rational number be x

$$-10 \times x = -7$$

$$x = \frac{-7}{-10}, x = \frac{7}{10}$$

Check

$$-10 \times \frac{7}{10} = -7. \text{ Hence, the result is correct.}$$

Think and Discuss

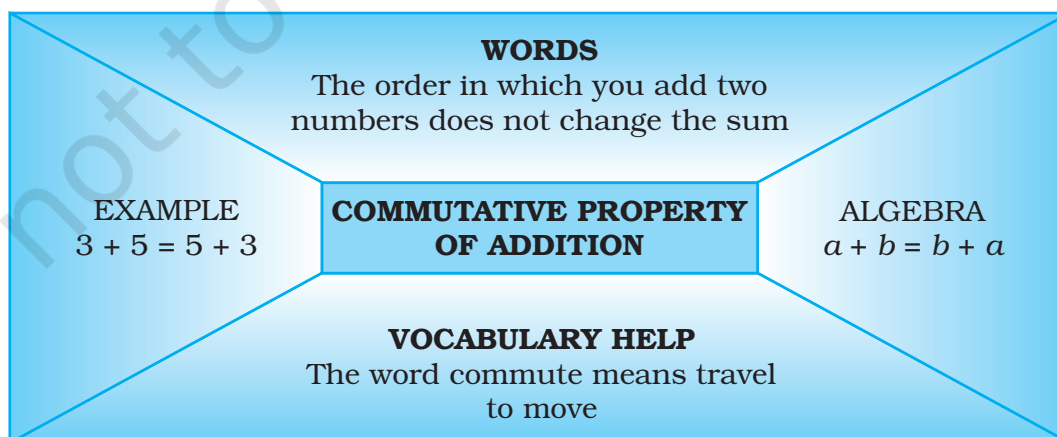
Some other easier ways to find the answer.

Is the product greater than both the rational number or less than both the rational numbers?

Note taking Skills

Focus on Graphic Organisers

You can use an information frame to organize information about a mathematical concept or property, such as the commutative property of addition.



Make an information frame for the distributive property.

(C) Exercise

In questions 1 to 25, there are four options out of which one is correct. Choose the correct answer.

- A number which can be expressed as $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is
 (a) natural number. (b) whole number.
 (c) integer. (d) rational number.
- A number of the form $\frac{p}{q}$ is said to be a rational number if
 (a) p and q are integers.
 (b) p and q are integers and $q \neq 0$
 (c) p and q are integers and $p \neq 0$
 (d) p and q are integers and $p \neq 0$ also $q \neq 0$.
- The numerical expression $\frac{3}{8} + \frac{(-5)}{7} = \frac{-19}{56}$ shows that
 (a) rational numbers are closed under addition.
 (b) rational numbers are not closed under addition.
 (c) rational numbers are closed under multiplication.
 (d) addition of rational numbers is not commutative.
- Which of the following is not true?
 (a) rational numbers are closed under addition.
 (b) rational numbers are closed under subtraction.
 (c) rational numbers are closed under multiplication.
 (d) rational numbers are closed under division.
- $-\frac{3}{8} + \frac{1}{7} = \frac{1}{7} + \left(\frac{-3}{8}\right)$ is an example to show that
 (a) addition of rational numbers is commutative.
 (b) rational numbers are closed under addition.
 (c) addition of rational number is associative.
 (d) rational numbers are distributive under addition.
- Which of the following expressions shows that rational numbers are associative under multiplication.
 (a) $\frac{2}{3} \times \left(\frac{-6}{7} \times \frac{3}{5}\right) = \left(\frac{2}{3} \times \frac{-6}{7}\right) \times \frac{3}{5}$

$$(b) \frac{2}{3} \times \left(\frac{-6}{7} \times \frac{3}{5} \right) = \frac{2}{3} \times \left(\frac{3}{5} \times \frac{-6}{7} \right)$$

$$(c) \frac{2}{3} \times \left(\frac{-6}{7} \times \frac{3}{5} \right) = \left(\frac{3}{5} \times \frac{2}{3} \right) \times \frac{-6}{7}$$

$$(d) \left(\frac{2}{3} \times \frac{-6}{7} \right) \times \frac{3}{5} = \left(\frac{-6}{7} \times \frac{2}{3} \right) \times \frac{3}{5}$$

7. Zero (0) is

- (a) the identity for addition of rational numbers.
- (b) the identity for subtraction of rational numbers.
- (c) the identity for multiplication of rational numbers.
- (d) the identity for division of rational numbers.

8. One (1) is

- (a) the identity for addition of rational numbers.
- (b) the identity for subtraction of rational numbers.
- (c) the identity for multiplication of rational numbers.
- (d) the identity for division of rational numbers.

9. The additive inverse of $\frac{-7}{19}$ is

- (a) $\frac{-7}{19}$
- (b) $\frac{7}{19}$
- (c) $\frac{19}{7}$
- (d) $\frac{-19}{7}$

10. Multiplicative inverse of a negative rational number is

- (a) a positive rational number.
- (b) a negative rational number.
- (c) 0
- (d) 1

11. If $x + 0 = 0 + x = x$, which is rational number, then 0 is called

- (a) identity for addition of rational numbers.
- (b) additive inverse of x .
- (c) multiplicative inverse of x .
- (d) reciprocal of x .

12. To get the product 1, we should multiply $\frac{8}{21}$ by

- (a) $\frac{8}{21}$
- (b) $\frac{-8}{21}$
- (c) $\frac{21}{8}$
- (d) $\frac{-21}{8}$

13. $-(-x)$ is same as

- (a) $-x$ (b) x (c) $\frac{1}{x}$ (d) $\frac{-1}{x}$

14. The multiplicative inverse of $-1\frac{1}{7}$ is

- (a) $\frac{8}{7}$ (b) $\frac{-8}{7}$ (c) $\frac{7}{8}$ (d) $\frac{7}{-8}$

15. If x be any rational number then $x + 0$ is equal to

- (a) x (b) 0 (c) $-x$ (d) Not defined

16. The reciprocal of 1 is

- (a) 1 (b) -1 (c) 0 (d) Not defined

17. The reciprocal of -1 is

- (a) 1 (b) -1 (c) 0 (d) Not defined

18. The reciprocal of 0 is

- (a) 1 (b) -1 (c) 0 (d) Not defined

19. The reciprocal of any rational number $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

- (a) $\frac{p}{q}$ (b) 1 (c) 0 (d) $\frac{q}{p}$

20. If y be the reciprocal of rational number x , then the reciprocal of y will be

- (a) x (b) y (c) $\frac{x}{y}$ (d) $\frac{y}{x}$

21. The reciprocal of $\frac{-3}{8} \times \left(\frac{-7}{13}\right)$ is

- (a) $\frac{104}{21}$ (b) $\frac{-104}{21}$ (c) $\frac{21}{104}$ (d) $\frac{-21}{104}$

22. Which of the following is an example of distributive property of multiplication over addition for rational numbers.

(a) $-\frac{1}{4} \times \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} = \left[-\frac{1}{4} \times \frac{2}{3} \right] + \left[-\frac{1}{4} \times \left(\frac{-4}{7} \right) \right]$

(b) $-\frac{1}{4} \times \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} = \left[\frac{1}{4} \times \frac{2}{3} \right] - \left(\frac{-4}{7} \right)$

$$(c) \quad -\frac{1}{4} \times \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} = \frac{2}{3} + \left(-\frac{1}{4} \right) \times \frac{-4}{7}$$

$$(d) \quad -\frac{1}{4} \times \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} = \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} - \frac{1}{4}$$

23. Between two given rational numbers, we can find

- (a) one and only one rational number.
- (b) only two rational numbers.
- (c) only ten rational numbers.
- (d) infinitely many rational numbers.



Plan a strategy

- Some problems contain a lot of information. Read the entire problem carefully to be sure you understand all the facts. You may need to read it over several times, perhaps aloud so that you can hear yourself and understand it well.
- Then decide which information is most important (prioritise). Is there any information that is absolutely necessary to solve the problem? This information is most important.
- Finally, put the information in order (sequence). Use comparison words like *before*, *after*, *longer*, *shorter*, and so on to help you. Write down the sequence before you try to solve the problem.

Read the problem given below, and then answer the questions that follow

- Five friends are standing in line for the opening of a show. They are in line according to their arrival. Shreya arrived 3 minutes after Sachin. Roy took his place in line at 9:01 P.M. He was 1 minute behind Reena and 7 minutes ahead of Shreya. The first person arrived at 9:00 P.M. Babu showed up 6 minutes after the first person. List the time of each person's arrival.
- (a) Whose arrival information helped you determine each person's arrival time?
- (b) Can you determine the order without the time?
- (c) List the friends' order of arrival from the earliest to the last.

24. $\frac{x+y}{2}$ is a rational number.
- Between x and y
 - Less than x and y both.
 - Greater than x and y both.
 - Less than x but greater than y .
25. Which of the following statements is always true?
- $\frac{x-y}{2}$ is a rational number between x and y .
 - $\frac{x+y}{2}$ is a rational number between x and y .
 - $\frac{x \times y}{2}$ is a rational number between x and y .
 - $\frac{x \div y}{2}$ is a rational number between x and y .

In questions 26 to 47, fill in the blanks to make the statements true.

26. The equivalent of $\frac{5}{7}$, whose numerator is 45 is _____.
27. The equivalent rational number of $\frac{7}{9}$, whose denominator is 45 is _____.
28. Between the numbers $\frac{15}{20}$ and $\frac{35}{40}$, the greater number is _____.
29. The reciprocal of a positive rational number is _____.
30. The reciprocal of a negative rational number is _____.
31. Zero has _____ reciprocal.
32. The numbers _____ and _____ are their own reciprocal.
33. If y be the reciprocal of x , then the reciprocal of y^2 in terms of x will be _____.
34. The reciprocal of $\frac{2}{5} \times \left(\frac{-4}{9}\right)$ is _____.
35. $(213 \times 657)^{-1} = 213^{-1} \times$ _____.
36. The negative of 1 is _____.

Writing Strategy:

Write a Convincing Argument

Your ability to write a convincing argument proves that you have understanding of the concept. An effective argument should include the following four parts:

- (1) A goal
- (2) A response to the goal
- (3) Evidence to support the response
- (4) A summary statement

Step 1 : Identify the goal

For any two numbers, explain whether using the greater number as the base will generally result in a greater number or using it as the exponent. Find one exception.

Step 2 : Provide a response to the goal

Using the greater number as the exponent usually gives the greater number.

Step 3 : Provide evidence to support your response

For the number 10 and 2
Using the greater number, 10, as the exponent will result in a greater number.

$$10^2 = 100$$

$$2^{10} = 1024$$

$$100 < 1024$$

$$10^2 < 2^{10}$$

Exception for the numbers 2 and 3. Using the greater number, 3, as the exponent will not result in a greater number.

$$3^2 = 9$$

$$2^3 = 8$$

$$9 > 8$$

$$3^2 > 2^3$$

Step 4 : Summarise your argument

Generally, for any two numbers, using the greater number as the exponent instead of as the base will result in a greater number.

Write about it
Compare 10^2 and 2^{10} . For any two numbers, what usually gives the greater number, using the greater number as the base or as the exponent? Give at least one exception.

37. For rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ we have $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f} \right) = \text{_____} + \text{_____}$.
38. $\frac{-5}{7}$ is _____ than -3 .
39. There are _____ rational numbers between any two rational numbers.
40. The rational numbers $\frac{1}{3}$ and $\frac{-1}{3}$ are on the _____ sides of zero on the number line.
41. The negative of a negative rational number is always a _____ rational number.
42. Rational numbers can be added or multiplied in any _____.
43. The reciprocal of $\frac{-5}{7}$ is _____.
44. The multiplicative inverse of $\frac{4}{3}$ is _____.
45. The rational number 10.11 in the form $\frac{p}{q}$ is _____.
46. $\frac{1}{5} \times \left[\frac{2}{7} + \frac{3}{8} \right] = \left[\frac{1}{5} \times \frac{2}{7} \right] + \text{_____}$.
47. The two rational numbers lying between -2 and -5 with denominator as 1 are _____ and _____.

In each of the following, state whether the statements are true (T) or false (F).

48. If $\frac{x}{y}$ is a rational number, then y is always a whole number.
49. If $\frac{p}{q}$ is a rational number, then p cannot be equal to zero.
50. If $\frac{r}{s}$ is a rational number, then s cannot be equal to zero.
51. $\frac{5}{6}$ lies between $\frac{2}{3}$ and 1.

52. $\frac{5}{10}$ lies between $\frac{1}{2}$ and 1.
53. $\frac{-7}{2}$ lies between -3 and -4.
54. $\frac{9}{6}$ lies between 1 and 2.
55. If $a \neq 0$, the multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$.
56. The multiplicative inverse of $\frac{-3}{5}$ is $\frac{5}{3}$.
57. The additive inverse of $\frac{1}{2}$ is -2.
58. If $\frac{x}{y}$ is the additive inverse of $\frac{c}{d}$, then $\frac{x}{y} + \frac{c}{d} = 0$.
59. For every rational number x , $x + 1 = x$.
60. If $\frac{x}{y}$ is the additive inverse of $\frac{c}{d}$, then $\frac{x}{y} - \frac{c}{d} = 0$.
61. The reciprocal of a non-zero rational number $\frac{q}{p}$ is the rational number $\frac{p}{q}$.
62. If $x + y = 0$, then $-y$ is known as the negative of x , where x and y are rational numbers.
63. The negative of the negative of any rational number is the number itself.
64. The negative of 0 does not exist.
65. The negative of 1 is 1 itself.
66. For all rational numbers x and y , $x - y = y - x$.
67. For all rational numbers x and y , $x \times y = y \times x$.

68. For every rational number x , $x \times 0 = x$.
69. For every rational numbers x , y and z , $x + (y \times z) = (x + y) \times (x + z)$.
70. For all rational numbers a , b and c , $a(b + c) = ab + bc$.
71. 1 is the only number which is its own reciprocal.
72. -1 is not the reciprocal of any rational number.
73. For any rational number x , $x + (-1) = -x$.
74. For rational numbers x and y , if $x < y$ then $x - y$ is a positive rational number.
75. If x and y are negative rational numbers, then so is $x + y$.
76. Between any two rational numbers there are exactly ten rational numbers.
77. Rational numbers are closed under addition and multiplication but not under subtraction.
78. Subtraction of rational number is commutative.
79. $-\frac{3}{4}$ is smaller than -2 .
80. 0 is a rational number.
81. All positive rational numbers lie between 0 and 1000.
82. The population of India in 2004 - 05 is a rational number.
83. There are countless rational numbers between $\frac{5}{6}$ and $\frac{8}{9}$.
84. The reciprocal of x^{-1} is $\frac{1}{x}$.
85. The rational number $\frac{57}{23}$ lies to the left of zero on the number line.
86. The rational number $\frac{7}{-4}$ lies to the right of zero on the number line.
87. The rational number $\frac{-8}{-3}$ lies neither to the right nor to the left of zero on the number line.

- 88.** The rational numbers $\frac{1}{2}$ and -1 are on the opposite sides of zero on the number line.
- 89.** Every fraction is a rational number.
- 90.** Every integer is a rational number.
- 91.** The rational numbers can be represented on the number line.
- 92.** The negative of a negative rational number is a positive rational number.
- 93.** If x and y are two rational numbers such that $x > y$, then $x - y$ is always a positive rational number.
- 94.** 0 is the smallest rational number.
- 95.** Every whole number is an integer.
- 96.** Every whole number is a rational number.
- 97.** 0 is whole number but it is not a rational number.
- 98.** The rational numbers $\frac{1}{2}$ and $-\frac{5}{2}$ are on the opposite sides of 0 on the number line.
- 99.** Rational numbers can be added (or multiplied) in any order
- $$\frac{-4}{5} \times \frac{-6}{5} = \frac{-6}{5} \times \frac{-4}{5}$$
- 100.** Solve the following: Select the rational numbers from the list which are also the integers.
- $$\frac{9}{4}, \frac{8}{4}, \frac{7}{4}, \frac{6}{4}, \frac{9}{3}, \frac{8}{3}, \frac{7}{3}, \frac{6}{3}, \frac{5}{2}, \frac{4}{2}, \frac{3}{1}, \frac{3}{2}, \frac{1}{1}, \frac{0}{1}, \frac{-1}{1}, \frac{-2}{1}, \frac{-3}{2}, \frac{-4}{2}, \frac{-5}{2}, \frac{-6}{2},$$
- 101.** Select those which can be written as a rational number with denominator 4 in their lowest form:
- $$\frac{7}{8}, \frac{64}{16}, \frac{36}{-12}, \frac{-16}{17}, \frac{5}{-4}, \frac{140}{28}$$

102. Using suitable rearrangement and find the sum:

(a) $\frac{4}{7} + \left(\frac{-4}{9}\right) + \frac{3}{7} + \left(\frac{-13}{9}\right)$

(b) $-5 + \frac{7}{10} + \frac{3}{7} + (-3) + \frac{5}{14} + \frac{-4}{5}$

103. Verify $-(-x) = x$ for

(i) $x = \frac{3}{5}$

(ii) $x = \frac{-7}{9}$

(iii) $x = \frac{13}{-15}$

104. Give one example each to show that the rational numbers are closed under addition, subtraction and multiplication. Are rational numbers closed under division? Give two examples in support of your answer.

105. Verify the property $x + y = y + x$ of rational numbers by taking

(a) $x = \frac{1}{2}, y = \frac{1}{2}$

(b) $x = \frac{-2}{3}, y = \frac{-5}{6}$

(c) $x = \frac{-3}{7}, y = \frac{20}{21}$

(d) $x = \frac{-2}{5}, y = \frac{-9}{10}$

106. Simplify each of the following by using suitable property. Also name the property.

(a) $\left[\frac{1}{2} \times \frac{1}{4}\right] + \left[\frac{1}{2} \times 6\right]$

(b) $\left[\frac{1}{5} \times \frac{2}{15}\right] - \left[\frac{1}{5} \times \frac{2}{5}\right]$

(c) $\frac{-3}{5} \times \left\{\frac{3}{7} + \left(\frac{-5}{6}\right)\right\}$

107. Tell which property allows you to compute

$\frac{1}{5} \times \left[\frac{5}{6} \times \frac{7}{9}\right]$ as $\left[\frac{1}{5} \times \frac{5}{6}\right] \times \frac{7}{9}$

108. Verify the property $x \times y = y \times x$ of rational numbers by using

(a) $x = 7$ and $y = \frac{1}{2}$

(b) $x = \frac{2}{3}$ and $y = \frac{9}{4}$

(c) $x = \frac{-5}{7}$ and $y = \frac{14}{15}$

(d) $x = \frac{-3}{8}$ and $y = \frac{-4}{9}$

109. Verify the property $x \times (y \times z) = (x \times y) \times z$ of rational numbers by using

(a) $x = 1, y = \frac{-1}{2}$ and $z = \frac{1}{4}$

(b) $x = \frac{2}{3}, y = \frac{-3}{7}$ and $z = \frac{1}{2}$

$$(c) \ x = \frac{-2}{7}, \ y = \frac{-5}{6} \text{ and } z = \frac{1}{4} \quad (d) \ x = 0, \ y = \frac{1}{2}$$

and What is the name of this property?

- 110.** Verify the property $x \times (y + z) = x \times y + x \times z$ of rational numbers by taking.

$$(a) \ x = \frac{-1}{2}, \ y = \frac{3}{4}, \ z = \frac{1}{4}$$

$$(b) \ x = \frac{-1}{2}, \ y = \frac{2}{3}, \ z = \frac{3}{4}$$

$$(c) \ x = \frac{-2}{3}, \ y = \frac{-4}{6}, \ z = \frac{-7}{9}$$

$$(d) \ x = \frac{-1}{5}, \ y = \frac{2}{15}, \ z = \frac{-3}{10}$$

- 111.** Use the distributivity of multiplication of rational numbers over addition to simplify

$$(a) \ \frac{3}{5} \times \left[\frac{35}{24} + \frac{10}{1} \right]$$

$$(b) \ \frac{-5}{4} \times \left[\frac{8}{5} + \frac{16}{15} \right]$$

$$(c) \ \frac{2}{7} \times \left[\frac{7}{16} - \frac{21}{4} \right]$$

$$(d) \ \frac{3}{4} \times \left[\frac{8}{9} - 40 \right]$$

- 112.** Simplify

$$(a) \ \frac{32}{5} + \frac{23}{11} \times \frac{22}{15}$$

$$(b) \ \frac{3}{7} \times \frac{28}{15} \div \frac{14}{5}$$

$$(c) \ \frac{3}{7} + \frac{-2}{21} \times \frac{-5}{6}$$

$$(d) \ \frac{7}{8} + \frac{1}{16} - \frac{1}{12}$$

- 113.** Identify the rational number that does not belong with the other three. Explain your reasoning

$$\frac{-5}{11}, \frac{-1}{2}, \frac{-4}{9}, \frac{-7}{3}$$

- 114.** The cost of $\frac{19}{4}$ metres of wire is Rs. $\frac{171}{2}$. Find the cost of one metre of the wire.

- 115.** A train travels $\frac{1445}{2}$ km in $\frac{17}{2}$ hours. Find the speed of the train in km/h.

- 116.** If 16 shirts of equal size can be made out of 24m of cloth, how much cloth is needed for making one shirt?
- 117.** $\frac{7}{11}$ of all the money in Hamid's bank account is Rs. 77,000. How much money does Hamid have in his bank account?
- 118.** A $117\frac{1}{3}$ m long rope is cut into equal pieces measuring $7\frac{1}{3}$ m each. How many such small pieces are these?
- 119.** $\frac{1}{6}$ of the class students are above average, $\frac{1}{4}$ are average and rest are below average. If there are 48 students in all, how many students are below average in the class?
- 120.** $\frac{2}{5}$ of total number of students of a school come by car while $\frac{1}{4}$ of students come by bus to school. All the other students walk to school of which $\frac{1}{3}$ walk on their own and the rest are escorted by their parents. If 224 students come to school walking on their own, how many students study in that school?
- 121.** Huma, Hubna and Seema received a total of Rs. 2,016 as monthly allowance from their mother such that Seema gets $\frac{1}{2}$ of what Huma gets and Hubna gets $1\frac{2}{3}$ times Seema's share. How much money do the three sisters get individually?
- 122.** A mother and her two daughters got a room constructed for Rs. 62,000. The elder daughter contributes $\frac{3}{8}$ of her mother's contribution while the younger daughter contributes $\frac{1}{2}$ of her mother's share. How much do the three contribute individually?
- 123.** Tell which property allows you to compare $\frac{2}{3} \times \left[\frac{3}{4} \times \frac{5}{7} \right]$ and $\left[\frac{2}{3} \times \frac{5}{7} \right] \times \frac{3}{4}$

124. Name the property used in each of the following.

$$(i) \quad -\frac{7}{11} \times \frac{-3}{5} = \frac{-3}{5} \times \frac{-7}{11}$$

$$(ii) \quad -\frac{2}{3} \times \left[\frac{3}{4} + \frac{-1}{2} \right] = \left[\frac{-2}{3} \times \frac{3}{4} \right] + \left[\frac{-2}{3} \times \frac{-1}{2} \right]$$

$$(iii) \quad \frac{1}{3} + \left[\frac{4}{9} + \left(\frac{-4}{3} \right) \right] = \left[\frac{1}{3} + \frac{4}{9} \right] + \left[\frac{-4}{3} \right]$$

$$(iv) \quad \frac{-2}{7} + 0 = 0 + \frac{-2}{7} = -\frac{2}{7}$$

$$(v) \quad \frac{3}{8} \times 1 = 1 \times \frac{3}{8} = \frac{3}{8}$$

125. Find the multiplicative inverse of

$$(i) \quad -1\frac{1}{8}$$

$$(ii) \quad 3\frac{1}{3}$$

126. Arrange the numbers $\frac{1}{4}, \frac{13}{16}, \frac{5}{8}$ in the descending order.

127. The product of two rational numbers is $\frac{-14}{27}$. If one of the numbers be $\frac{7}{9}$, find the other.

128. By what numbers should we multiply $\frac{-15}{20}$ so that the product may be $\frac{-5}{7}$?

129. By what number should we multiply $\frac{-8}{13}$ so that the product may be 24?

130. The product of two rational numbers is -7 . If one of the number is -5 , find the other?

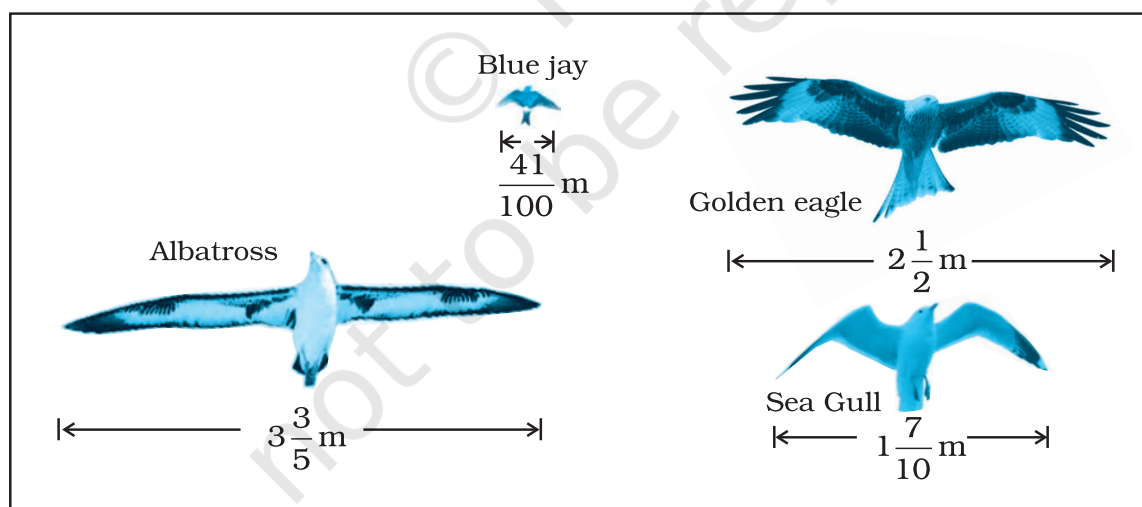
131. Can you find a rational number whose multiplicative inverse is -1 ?

132. Find five rational numbers between 0 and 1.

133. Find two rational numbers whose absolute value is $\frac{1}{5}$.

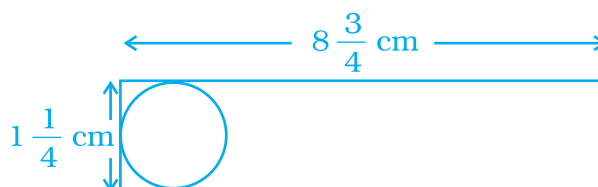
- 134.** From a rope 40 metres long, pieces of equal size are cut. If the length of one piece is $\frac{10}{3}$ metre, find the number of such pieces.
- 135.** $5\frac{1}{2}$ metres long rope is cut into 12 equal pieces. What is the length of each piece?
- 136.** Write the following rational numbers in the descending order.
 $\frac{8}{7}, \frac{-9}{8}, \frac{-3}{2}, 0, \frac{2}{5}$
- 137.** Find (i) $0 \div \frac{2}{3}$ (ii) $\frac{1}{3} \times \frac{-5}{7} \times \frac{-21}{10}$
- 138.** On a winter day the temperature at a place in Himachal Pradesh was -16°C . Convert it in degree Fahrenheit ($^{\circ}\text{F}$) by using the formula.

$$\frac{C}{5} = \frac{F - 32}{9}$$
- 139.** Find the sum of additive inverse and multiplicative inverse of 7.
- 140.** Find the product of additive inverse and multiplicative inverse of $-\frac{1}{3}$.
- 141.** The diagram shows the wingspans of different species of birds. Use the diagram to answer the question given below:



- (a) How much longer is the wingspan of an Albatross than the wingspan of a Sea gull?
- (b) How much longer is the wingspan of a Golden eagle than the wingspan of a Blue jay?

- 142.** Shalini has to cut out circles of diameter $1\frac{1}{4}$ cm from an aluminium strip of dimensions $8\frac{3}{4}$ cm by $1\frac{1}{4}$ cm. How many full circles can Shalini cut? Also calculate the wastage of the aluminium strip.



- 143.** One fruit salad recipe requires $\frac{1}{2}$ cup of sugar. Another recipe for the same fruit salad requires 2 tablespoons of sugar. If 1 tablespoon is equivalent to $\frac{1}{16}$ cup, how much more sugar does the first recipe require?
- 144.** Four friends had a competition to see how far could they hop on one foot. The table given shows the distance covered by each.

Name	Distance covered (km)
Seema	$\frac{1}{25}$
Nancy	$\frac{1}{32}$
Megha	$\frac{1}{40}$
Soni	$\frac{1}{20}$

- (a) How farther did Soni hop than Nancy?
- (b) What is the total distance covered by Seema and Megha?
- (c) Who walked farther, Nancy or Megha?
- 145.** The table given below shows the distances, in kilometres, between four villages of a state. To find the distance between two villages,

locate the square where the row for one village and the column for the other village intersect.

	Sonapur	Ramgarh	Himgaon	Rawalpur
Sonapur		$40\frac{2}{3}$	$100\frac{5}{6}$	$16\frac{1}{2}$
Ramgarh	$40\frac{2}{3}$		$210\frac{3}{8}$	$16\frac{2}{3}$
Himgaon	$100\frac{5}{6}$	$210\frac{3}{8}$		$98\frac{3}{4}$
Rawalpur	$16\frac{1}{2}$	$30\frac{2}{3}$	$98\frac{3}{4}$	

- (a) Compare the distance between Himgaon and Rawalpur to Sonapur and Ramgarh?
- (b) If you drove from Himgaon to Sonapur and then from Sonapur to Rawalpur, how far would you drive?

146. The table shows the portion of some common materials that are recycled.

Material	Recycled
Paper	$\frac{5}{11}$
Aluminium cans	$\frac{5}{8}$
Glass	$\frac{2}{5}$
Scrap	$\frac{3}{4}$

- (a) Is the rational number expressing the amount of paper recycled more than $\frac{1}{2}$ or less than $\frac{1}{2}$?

- (b) Which items have a recycled amount less than $\frac{1}{2}$?
- (c) Is the quantity of aluminium cans recycled more (or less) than half of the quantity of aluminium cans?
- (d) Arrange the rate of recycling the materials from the greatest to the smallest.

147. The overall width in cm of several wide-screen televisions are 97.28 cm, $98\frac{4}{9}$ cm, $98\frac{1}{25}$ cm and 97.94 cm. Express these numbers as rational numbers in the form $\frac{p}{q}$ and arrange the widths in ascending order.

148. Roller Coaster at an amusement park is $\frac{2}{3}$ m high. If a new roller coaster is built that is $\frac{3}{5}$ times the height of the existing coaster, what will be the height of the new roller coaster?

149. Here is a table which gives the information about the total rainfall for several months compared to the average monthly rains of a town. Write each decimal in the form of rational number $\frac{p}{q}$.

Month	Above/Below normal (in cm)
May	2.6924
June	0.6096
July	- 6.9088
August	- 8.636

150. The average life expectancies of males for several states are shown in the table. Express each decimal in the form $\frac{p}{q}$ and arrange the states from the least to the greatest male life expectancy.

State-wise data are included below; more indicators can be found in the "FACTFILE" section on the homepage for each state.

State	Male	$\frac{p}{q}$ form	Lowest terms
Andhra Pradesh	61.6		
Assam	57.1		
Bihar	60.7		
Gujarat	61.9		
Haryana	64.1		
Himachal Pradesh	65.1		
Karnataka	62.4		
Kerala	70.6		
Madhya Pradesh	56.5		
Maharashtra	64.5		
Orissa	57.6		
Punjab	66.9		
Rajasthan	59.8		
Tamil Nadu	63.7		
Uttar Pradesh	58.9		
West Bengal	62.8		
India	60.8		

Source: Registrar General of India (2003) SRS Based Abridged Life Tables. SRS Analytical Studies, Report No. 3 of 2003, New Delhi: Registrar General of India. The data are for the 1995-99 period; states subsequently divided are therefore included in their pre-partition states (Chhatisgarh in MP, Uttaranchal in UP and Jharkhand in Bihar)

- 151.** A skirt that is $35\frac{7}{8}$ cm long has a hem of $3\frac{1}{8}$ cm. How long will the skirt be if the hem is let down?
- 152.** Manavi and Kuber each receives an equal allowance. The table shows the fraction of their allowance each deposits into his/her saving account and the fraction each spends at the mall. If allowance of each is Rs. 1260 find the amount left with each.

Where money goes	Fraction of allowance	
	Manavi	Kuber
Saving Account	$\frac{1}{2}$	$\frac{1}{3}$
Spend at mall	$\frac{1}{4}$	$\frac{3}{5}$
Left over	?	?

(D) Games and Puzzles

1. Given below is a magic square. Place the numbers $\frac{70}{95}$, $\frac{-21}{-133}$, $\frac{25}{95}$, $\frac{24}{38}$ in the appropriate squares so that sum in each row, column and diagonal is equal.

$\frac{32}{38}$	$\frac{18}{38}$	$\frac{4}{38}$	$\frac{-14}{-38}$
$\frac{-18}{-57}$	?	?	$\frac{104}{152}$
$\frac{22}{38}$	?	?	$\frac{-20}{-95}$
$\frac{1}{19}$	$\frac{-16}{-38}$	$\frac{45}{57}$	$\frac{60}{114}$

Hint: (Rewrite each rational number in its lowest term.)

2. Solve the given crossword filling up the given boxes. Clues are given below for across as well as downward filling. Also, for across and down clues, clue number is written at the corner of the boxes. Answers of clues have to be filled in their respective boxes.

Down 1: $\frac{2}{3}$ and $\frac{-5}{4}$ are _____ numbers.

Down 2: The _____ inverse of $\frac{a}{b}$ is $\frac{-a}{b}$.

Down 3: The addition and multiplication of whole number integers and rational numbers is _____.

Down 4: Since $\frac{1}{0}$ doesn't exist hence 0 has no _____.

Down 5: The number line extends _____ on both the sides.

Down 6: The _____ of two integers may not lead to the formation of another integer.

Down 7: The multiplication of a number by its reciprocal gives _____.

Down 8: Rational numbers can be represented on a _____ line.

Across 1: There are _____ rational numbers between two integers.

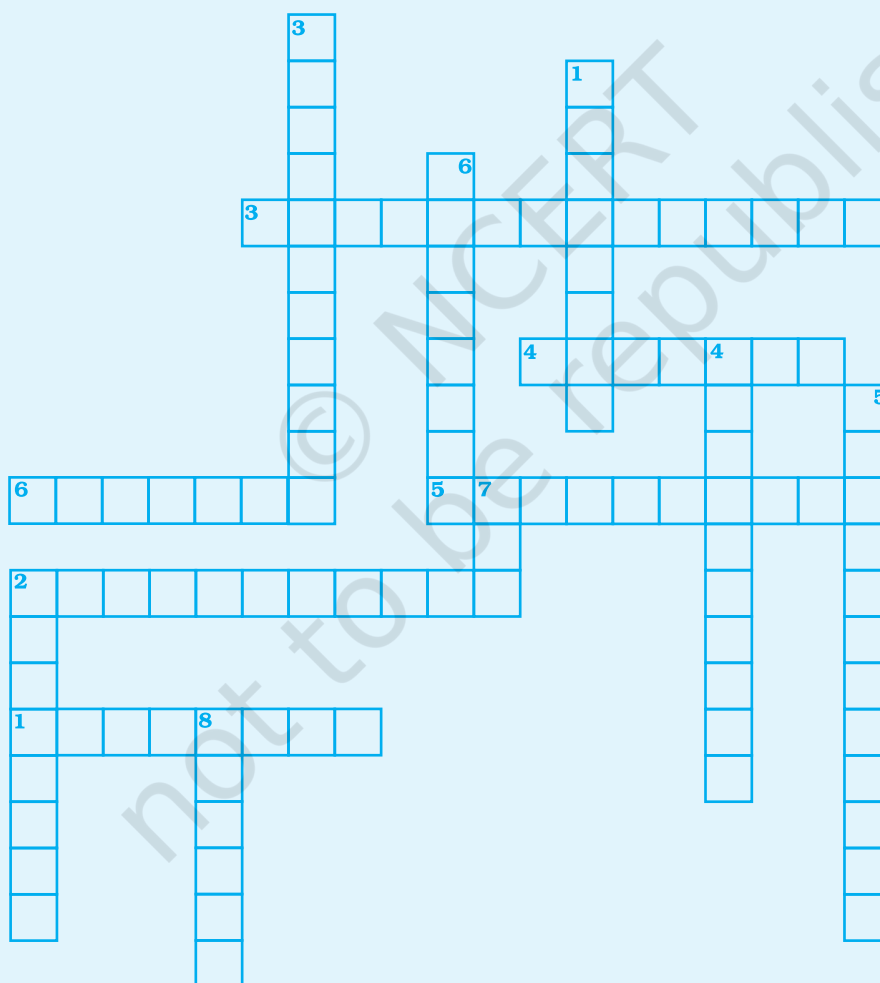
Across 2: The multiplication of rational numbers is commutative and _____.

Across 3: The addition and _____ of two rational numbers lead to the formation of another rational number.

Across 4: All the positive integers excluding 0 are known as _____ numbers.

Across 5: For any rational a ; $a \div 0$ is _____.

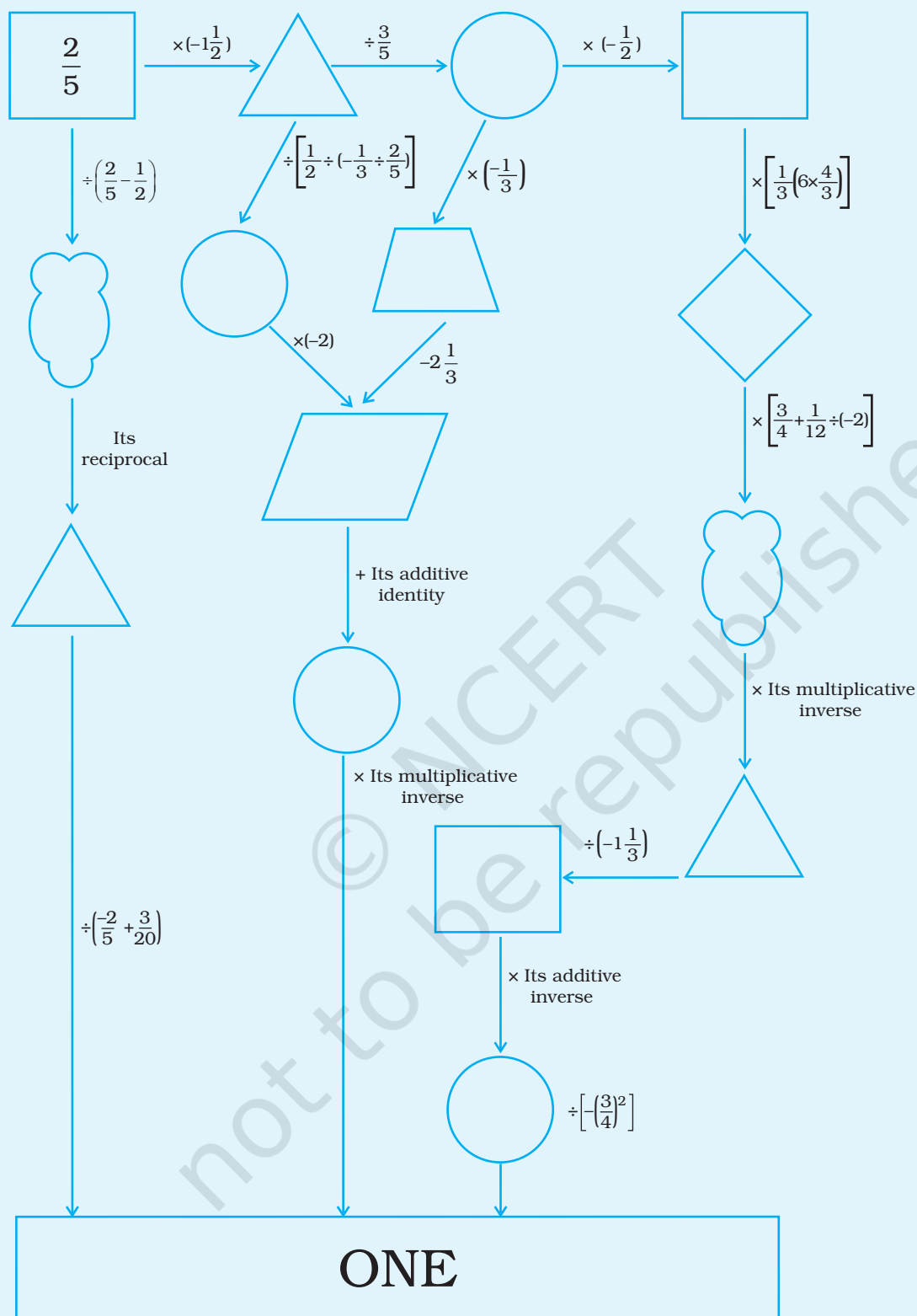
Across 6: Reciprocal is also known as the multiplicative _____.



3. Break the Code

Solve this riddle by reducing each rational number to its lowest term. The magnitude of the numerator of rational number so obtained gives you the letter you have to encircle in the word following it. Use the encircled letters to fill in the blanks given below:

S.No.	Rational Number	Lowest Term	Word						
(1)	$\frac{-12}{30}$		SPIN						
(2)	$\frac{-24}{-36}$		TYPE						
(3)	$\frac{39}{52}$		WITH						
(4)	$\frac{-48}{144}$		HOST						
(5)	$\frac{27}{90}$		SHARP						
(6)	$\frac{-34}{-170}$		GAIN						
(7)	$\frac{76}{95}$		PROOF						
(8)	$\frac{46}{-92}$		RAIN						
(9)	$\frac{29}{116}$		AWAY						
(10)	$\frac{14}{-42}$		SWEET						
1	2	3	4	5	6	7	8	9	10



Rough Work

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UNIT 2

DATA HANDLING

(A) Main Concepts and Results

- The information collected in term of numbers is called **data**.
- Data are represented graphically to have a quick glance on them.
- Data available in an unorganised form are called **raw data**.
- The number of times a particular observation occurs in a given data is called its **frequency**.
- When the data are large, they can be arranged in groups and each group is known as **Class Interval or Class**.
- A table showing the frequencies of various observations or class intervals of a given data is called a **Frequency Distribution table**.
- The *upper value* of a class interval is called its **Upper Class Limit** and the *lower value* of the class interval is called its **Lower Class Limit**.
- The difference between the upper class limit and lower class limit of a class is called the **Width or Size** of the class.
- The difference between the lowest and the highest observation in a given data is called its **Range**.
- Grouped data can be represented by a **histogram**.
- Histogram is a type of bar diagram, where the class intervals are shown on the horizontal axis and the heights of the bars (rectangles) show the frequency of the class interval, but there is no gap between the bars as there is no gap between the class intervals.
- Data can also be represented using a **pie chart** (circle graph). It shows the relationship between a whole and its parts.

- There are certain experiments whose outcomes have an equal chance of occurring. Such outcomes are said to be **equally likely**.
- Probability of an event =

$$\frac{\text{Number of outcomes that make an event}}{\text{Total number of outcomes of the experiment}},$$

when the outcomes are equally likely.

(B) Solved Examples

In examples 1 to 6, there are four options given out of which one is correct. Choose the correct answer.

Example 1 : The range of the data– 9, 8, 4, 3, 2, 1, 6, 4, 8, 10, 12, 15, 4, 3 is

- (a) 15 (b) 14 (c) 12 (d) 10

Solution : The correct answer is (b).

Example 2 : The following data : 2, 5, 15, 25, 20, 12, 8, 7, 6, 16, 21, 17, 30, 32, 23, 40, 51, 15, 2, 9, 57, 19, 25 is grouped in the classes 0 –5, 5 –10, 10 –15 etc. Find the frequency of the class 20 –25.

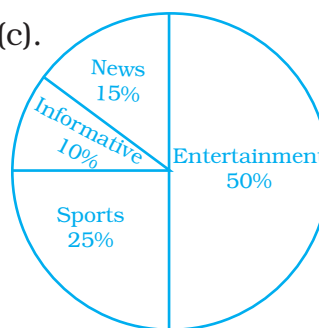
- (a) 5 (b) 4 (c) 3 (d) 2

Solution : The correct answer is (c).

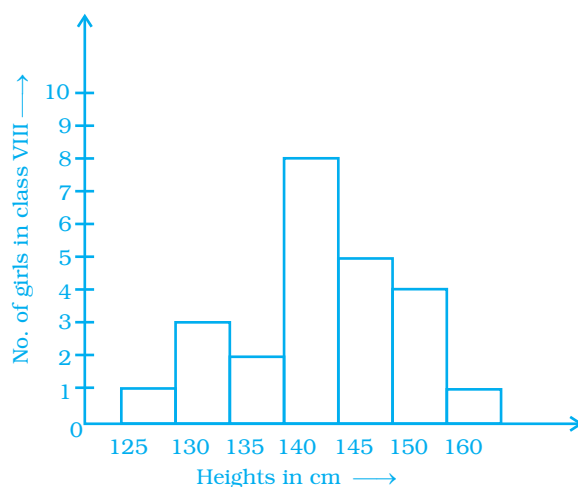
Example 3 : The pie chart depicts the information of viewers watching different type of channels on TV. Which type of programmes are viewed the most?

- (a) News (b) Sports (c) Entertainment
(d) Informative.

Solution : The correct answer is (c).



Example 4 :



Observe the histogram given above. The number of girls having height 145 cm and above is

- (a) 5 (b) 10 (c) 17 (d) 19

Solution : The correct answer is (b).

Example 5 : A dice is thrown two times and sum of the numbers appearing on the dice are noted. The number of possible outcomes is

- (a) 6 (b) 11 (c) 18 (d) 36

Solution : The correct answer is (b).

[Possible sums are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

Example 6 : The probability of getting a multiple of 2 when a dice is rolled is

- (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

Solution : The correct answer is (c).

In examples 7 to 9 fill in the blanks to make statements true.

Example 7 : The fourth class interval for a grouped data whose first and second class intervals are 10 –15 and 15 –20 respectively is _____.

Solution : 25 – 30

Example 8 : In the class interval 250 – 275, 250 is known as the _____.

Solution : Lower class limit.

Example 9 : The number of times a particular observation occurs in the given data is called its _____.

Solution : Frequency.

In examples 10 to 12, state whether the statements are true (T) or false (F).

Example 10 : The central angle of the sectors in a pie chart will be a fraction of 360° .

Solution : True.

Example 11 : On throwing a dice, the probability of occurrence of an odd number is $\frac{1}{2}$.

Solution : True.

Example 12 : A pie chart is also called a pictograph.

Solution : False .

Example 13 : The weekly wages (in Rs.) of 30 workers in a factory are 830, 835, 890, 810, 835, 836, 869, 845, 898, 890, 820, 860, 832, 833, 855, 845, 804, 808, 812, 840, 885, 835, 835, 836, 878, 840, 868, 890, 806, 840.

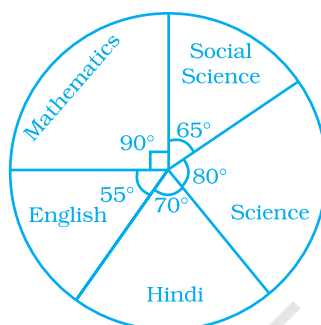
Using tally marks, make a frequency distribution table with class intervals 800 – 810, 810 – 820 and so on.

Solution :

Class interval	Tally marks	Frequency
800 – 810		3
810 – 820		2
820 – 830		1
830 – 840	<del> </del>	9
840 – 850	<del> </del>	5
850 – 860		1
860 – 870		3
870 – 880		1
880 – 890		1
890 – 900		4
	Total	30

Example 14 : The pie chart gives the marks scored in an examination by a student in different subjects. If the total marks obtained were 540, answer the following questions–

- In which subject did the student score 105 marks?
- How many more marks were obtained by the student in Mathematics than in Hindi?



Solution : (i) For 540 marks, central angle = 360°

$$\text{For 1 mark, central angle} = \frac{360^\circ}{540}$$

$$\begin{aligned} \text{For 105 marks, central angle} &= \frac{360^\circ}{540} \times 105 \\ &= 70^\circ \end{aligned}$$

Hence the student scored 105 marks in Hindi.

(ii) Central angle = 360° for 540 marks,

$$\text{For 1 mark, central angle} = \frac{360^\circ}{540}$$

$$\begin{aligned} \text{For 90 marks, central angle} &= \frac{540}{360} \times 90 \text{ marks} \\ &= 135 \text{ marks.} \end{aligned}$$

Thus, the student gets 135 marks in Mathematics.

From part (i) we get that the student gets 105 marks in Hindi.

$$\text{Difference in marks} = 135 - 105 = 30$$

Hence, the student gets 30 more marks in Mathematics than in Hindi.

Example 15 : Draw a pie chart for the given data.

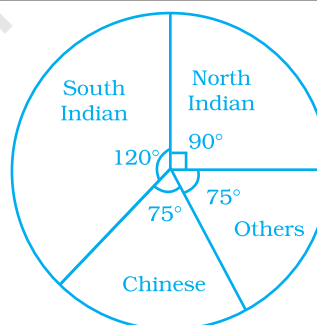
Favourite food	Number of people
North Indian	30
South Indian	40
Chinese	25
Others	25

Solution : Total number of people = 120

We find the central angle for each sector.

Favourite food	Number of people	In fraction	Central angle
North Indian	30	$\frac{30}{120} = \frac{1}{4}$	$\frac{1}{4} \times 360^\circ = 90^\circ$
South Indian	40	$\frac{40}{120} = \frac{1}{3}$	$\frac{1}{3} \times 360^\circ = 120^\circ$
Chinese	25	$\frac{25}{120} = \frac{5}{24}$	$\frac{5}{24} \times 360^\circ = 75^\circ$
Others	25	$\frac{25}{120} = \frac{5}{24}$	$\frac{5}{24} \times 360^\circ = 75^\circ$

The pie chart is drawn adjacently.



Try to write a formula for the probability of finding a paper.

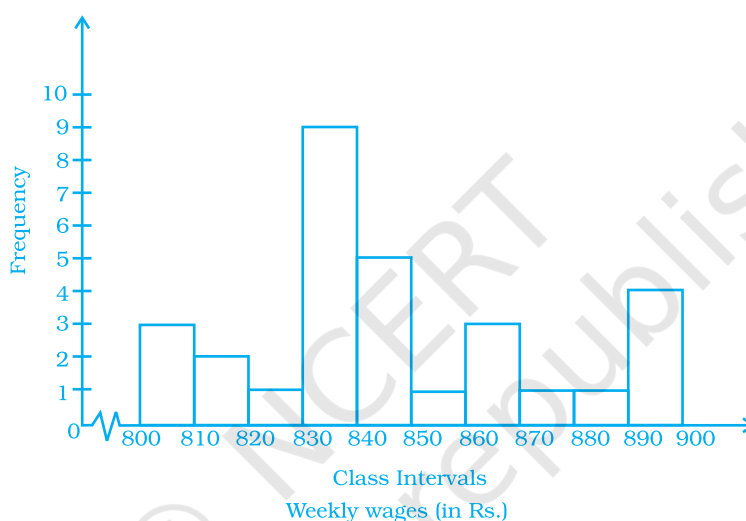
Data can be represented in several different ways, depending on the type of data and the message to be conveyed.

Type of Graph	
Line graph	Shows change in data over time.
Bar graph	Shows relationship or comparisons between groups.
Circle graph	Compares parts to a whole.
Histogram	Shows the frequency of data divided into equal groups.

Example 16 : Draw a histogram for the frequency distribution table given in Example 13 and answer the following questions.

- Which class interval has the maximum number of workers?
- How many workers earn Rs. 850 and more?
- How many workers earn less than Rs. 850?
- How many workers earn Rs. 820 or more but less than Rs. 880?

Solution :



- 830 – 840
- 10
- 20
- 20

Example 17 : Read the frequency distribution table given below and answer the questions that follow:

Class Interval	Frequency
25 – 35	1
35 – 45	5
45 – 55	5
55 – 65	4
65 – 75	0
75 – 85	8
85 – 95	2
Total	25

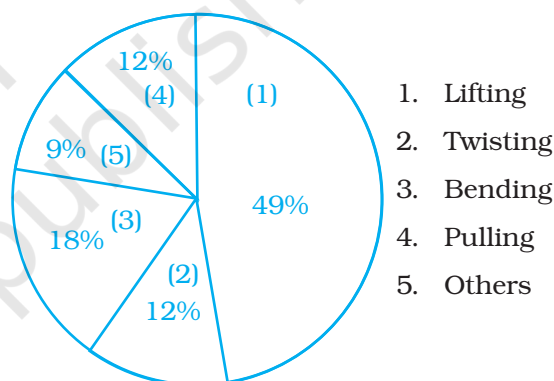
- (i) Class interval which has the lowest frequency.
- (ii) Class interval which has the highest frequency.
- (iii) What is the class size of the intervals?
- (iv) What is the upper limit of the fifth class?
- (v) What is the lower limit of the last class?

Solution : (i) 65 – 75
 (ii) 75 – 85
 (iii) 10
 (iv) 75
 (v) 85

Example 18 : Application on problem solving strategy

Given below is a pie chart depicting the reason given by people who had injured their lower back.

Study the pie chart and find the number of people who injured their back while either bending and lifting. A total of 600 people were surveyed.



Solution : **Understand and explore the problem**

- What information is given in the question?
 The percentages of the most common reasons given by 600 people.
- What are you trying to find?
 The total number of people out of 600 who have injured their back while bending and lifting.
- Is there any information that is not needed?
 The percentages of other reasons except bending and lifting are not required.

Plan a strategy

- You have learnt to solve questions dealing with percentages in earlier classes. Use the same method to solve this question.

- 18% have injured their back while bending and 49% have injured their back while lifting so–

To find

The total number of people can be found by finding $(18\% + 49\%) = 67\%$ of 600.

Solve

Total percentage of people who injured their back while bending and lifting = $18\% + 49\% = 67\%$.

Number of people who injured their back while bending and lifting = 67% of 600

$$= \frac{67}{100} \times 600$$

$$= 402$$

Hence, 402 people injured their back while bending and lifting.

Revise

You can check the answer by finding the total number of people who injured their backs for reasons other than bending and lifting and then subtracting the answer from the total number of people.

Total percentage of people who injured their backs for reasons other than bending and lifting = $12\% + 12\% + 9\%$
 $= 33\%$

$$\begin{aligned} \text{Number of such people} &= 33\% \text{ of } 600 = \frac{33}{100} \times 600 \\ &= 198 \end{aligned}$$

So, $600 - 198$ should give us the answer for our original question, and $600 - 198 = 402$ which is same as our answer. Hence our answer is correct.

Think and Discuss

- If the total angle covered by all sectors is 360° , find the angle covered by the sector representing the people who injured their back by pulling only.
- If the number of people surveyed is doubled, would the number of people who injured their back by bending and lifting also be doubled?

(C) Exercise

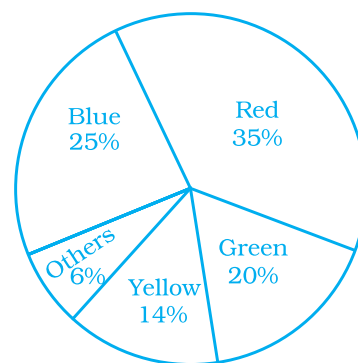
In questions 1 to 35 there are four options given, out of which one is correct. Choose the correct answer.

- The height of a rectangle in a histogram shows the
 (a) Width of the class (b) Upper limit of the class
 (c) Lower limit of the class (d) Frequency of the class
- A geometric representation showing the relationship between a whole and its parts is a
 (a) Pie chart (b) Histogram (c) Bar graph (d) Pictograph
- In a pie chart, the total angle at the centre of the circle is
 (a) 180° (b) 360° (c) 270° (d) 90°
- The range of the data 30, 61, 55, 56, 60, 20, 26, 46, 28, 56 is
 (a) 26 (b) 30 (c) 41 (d) 61
- Which of the following is not a random experiment?
 (a) Tossing a coin (b) Rolling a dice
 (c) Choosing a card from a deck of 52 cards
 (d) Throwing a stone from a roof of a building
- What is the probability of choosing a vowel from the alphabets?
 (a) $\frac{21}{26}$ (b) $\frac{5}{26}$ (c) $\frac{1}{26}$ (d) $\frac{3}{26}$
- In a school only, 3 out of 5 students can participate in a competition. What is the probability of the students who do not make it to the competition?
 (a) 0.65 (b) 0.4 (c) 0.45 (d) 0.6

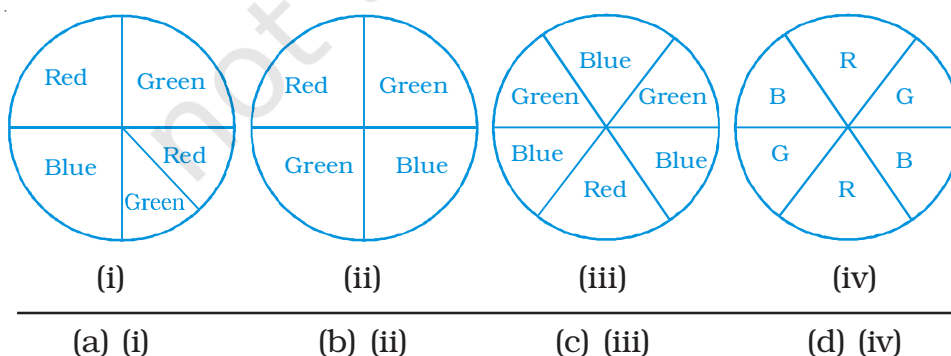
Students of a class voted for their favourite colour and a pie chart was prepared based on the data collected.

Observe the pie chart given below and answer questions 8 – 10 based on it.

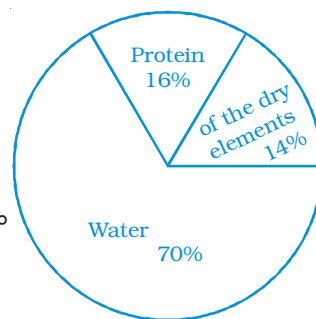
- Which colour received $\frac{1}{5}$ of the votes?
 (a) Red (b) Blue (c) Green
 (d) Yellow



9. If 400 students voted in all, then how many did vote 'Others' colour as their favourite?
 (a) 6 (b) 20 (c) 24 (d) 40
10. Which of the following is a reasonable conclusion for the given data?
 (a) $\frac{1}{20}$ th student voted for blue colour
 (b) Green is the least popular colour
 (c) The number of students who voted for red colour is two times the number of students who voted for yellow colour
 (d) Number of students liking together yellow and green colour is approximately the same as those for red colour.
11. Listed below are the temperature in °C for 10 days.
 -6, -8, 0, 3, 2, 0, 1, 5, 4, 4
 What is the range of the data?
 (a) 8 (b) 13°C (c) 10°C (d) 12°C
12. Ram put some buttons on the table. There were 4 blue, 7 red, 3 black and 6 white buttons in all. All of a sudden, a cat jumped on the table and knocked out one button on the floor. What is the probability that the button on the floor is blue?
 (a) $\frac{7}{20}$ (b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{4}$
13. Rahul, Varun and Yash are playing a game of spinning a coloured wheel. Rahul wins if spinner lands on red. Varun wins if spinner lands on blue and Yash wins if it lands on green. Which of the following spinner should be used to make the game fair?

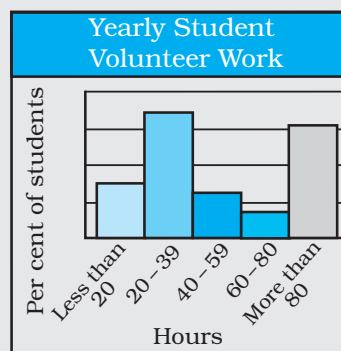
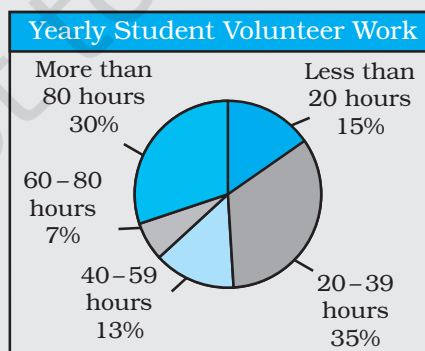


- 14.** In a frequency distribution with classes 0 –10, 10 –20 etc., the size of the class intervals is 10. The lower limit of fourth class is
 (a) 40 (b) 50 (c) 20 (d) 30
- 15.** A coin is tossed 200 times and head appeared 120 times. The probability of getting a head in this experiment is
 (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$
- 16.** Data collected in a survey shows that 40% of the buyers are interested in buying a particular brand of toothpaste. The central angle of the sector of the pie chart representing this information is
 (a) 120° (b) 150° (c) 144° (d) 40°
- 17.** Monthly salary of a person is Rs. 15000. The central angle of the sector representing his expenses on food and house rent on a pie chart is 60° . The amount he spends on food and house rent is
 (a) Rs. 5000 (b) Rs. 2500 (c) Rs. 6000 (d) Rs. 9000
- 18.** The following pie chart gives the distribution of constituents in the human body. The central angle of the sector showing the distribution of protein and other constituents is
 (a) 108° (b) 54° (c) 30° (d) 216°



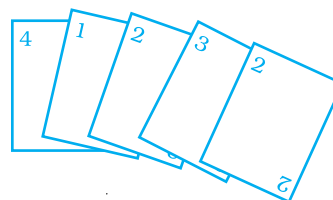
Selecting a Data Display

Which graph is a better display of the data on students volunteering for some work?

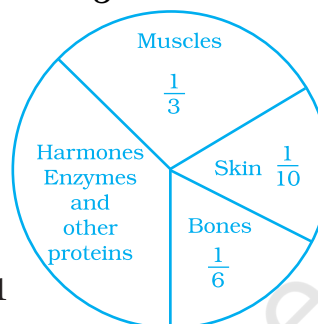


The data shows how groups of people who responded to the survey compare to the whole. The circle graph is the better representation.

19. Rohan and Shalu are playing with 5 cards as shown in the figure. What is the probability of Rohan picking a card without seeing, that has the number 2 on it?



- (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$
20. The following pie chart represents the distribution of proteins in parts of a human body. What is the ratio of distribution of proteins in the muscles to that of proteins in the bones?



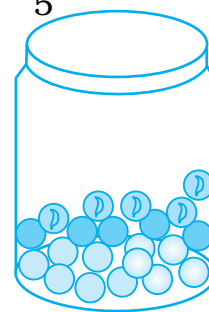
- (a) 3 : 1 (b) 1 : 2 (c) 1 : 3 (d) 2 : 1
21. What is the central angle of the sector (in the above pie chart) representing skin and bones together?
- (a) 36° (b) 60° (c) 90° (d) 96°
22. What is the central angle of the sector (in the above pie chart) representing hormones enzymes and other proteins.
- (a) 120° (b) 144° (c) 156° (d) 176°
23. A coin is tossed 12 times and the outcomes are observed as shown below:



The chance of occurrence of Head is

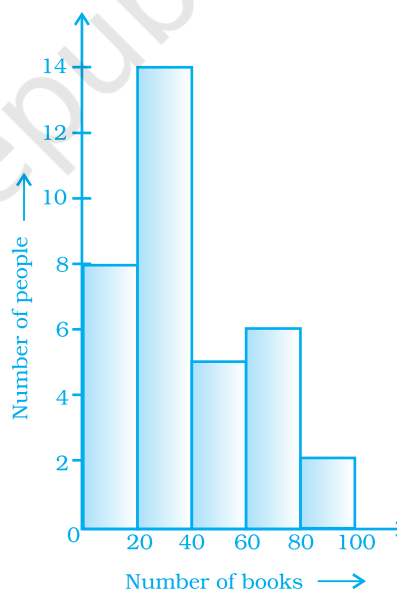
- (a) $\frac{1}{2}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{5}{7}$
24. Total number of outcomes, when a ball is drawn from a bag which contains 3 red, 5 black and 4 blue balls is
- (a) 8 (b) 7 (c) 9 (d) 12
25. A graph showing two sets of data simultaneously is known as
- (a) Pictograph (b) Histogram (c) Pie chart (d) Double bar graph

- 26.** Size of the class 150 –175 is
 (a) 150 (b) 175 (c) 25 (d) –25
- 27.** In a throw of a dice, the probability of getting the number 7 is
 (a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) 1 (d) 0
- 28.** Data represented using circles is known as
 (a) Bar graph (b) Histogram (c) Pictograph (d) Pie chart
- 29.** Tally marks are used to find
 (a) Class intervals (b) Range
 (c) Frequency (d) Upper limit
- 30.** Upper limit of class interval 75 –85 is
 (a) 10 (b) –10 (c) 75 (d) 85
- 31.** Numbers 1 to 5 are written on separate slips, i.e one number on one slip and put in a box. Wahida pick a slip from the box without looking at it. What is the probability that the slip bears an odd number?
 (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$
- 32.** A glass jar contains 6 red, 5 green, 4 blue and 5 yellow marbles of same size. Hari takes out a marble from the jar at random. What is the probability that the chosen marble is of red colour?
 (a) $\frac{7}{10}$ (b) $\frac{3}{10}$ (c) $\frac{4}{5}$ (d) $\frac{2}{5}$
- 33.** A coin is tossed two times. The number of possible outcomes is
 (a) 1 (b) 2 (c) 3 (d) 4
- 34.** A coin is tossed three times. The number of possible outcomes is
 (a) 3 (b) 4 (c) 6 (d) 8
- 35.** A die is tossed two times. The number of possible outcomes is
 (a) 12 (b) 24 (c) 36 (d) 30



In questions 36 to 58, fill in the blanks to make the statements true.

36. Data available in an unorganised form is called _____ data.
37. In the class interval 20 – 30, the lower class limit is _____.
38. In the class interval 26 – 33, 33 is known as _____.
39. The range of the data 6, 8, 16, 22, 8, 20, 7, 25 is _____.
40. A pie chart is used to compare _____ to a whole.
41. In the experiment of tossing a coin one time, the outcome is either _____ or _____.
42. When a dice is rolled, the six possible outcomes are _____.
43. Each outcome or a collection of outcomes in an experiment makes an _____.
44. An experiment whose outcomes cannot be predicted exactly in advance is called a _____ experiment.
45. The difference between the upper and lower limit of a class interval is called the _____ of the class interval.
46. The sixth class interval for a grouped data whose first two class intervals are 10 – 15 and 15 – 20 is _____.
Histogram given on the right shows the number of people owning the different number of books. Answer 47 to 50 based on it.
47. The total number of people surveyed is _____.
48. The number of people owning books more than 60 is _____.
49. The number of people owning books less than 40 is _____.
50. The number of people having books more than 20 and less than 40 is _____.
51. The number of times a particular observation occurs in a given data is called its _____.



- 52.** When the number of observations is large, the observations are usually organised in groups of equal width called _____.
- 53.** The total number of outcomes when a coin is tossed is _____.
- 54.** The class size of the interval 80 – 85 is _____.
- 55.** In a histogram _____ are drawn with width equal to a class interval without leaving any gap in between.
- 56.** When a dice is thrown, outcomes 1, 2, 3, 4, 5, 6 are equally _____.
- 57.** In a histogram, class intervals and frequencies are taken along _____ axis and _____ axis.
- 58.** In the class intervals 10 –20, 20 –30, etc., respectively, 20 lies in the class _____.

In questions 59 to 81, state whether the statements are true (T) or false (F).

- 59.** In a pie chart a whole circle is divided into sectors.
- 60.** The central angle of a sector in a pie chart cannot be more than 180° .
- 61.** Sum of all the central angles in a pie chart is 360° .
- 62.** In a pie chart two central angles can be of 180° .
- 63.** In a pie chart two or more central angles can be equal.
- 64.** Getting a prime number on throwing a die is an event.

Using the following frequency table, answer question 65-68

Marks (obtained out of 10)	4	5	7	8	9	10
Frequency	5	10	8	6	12	9

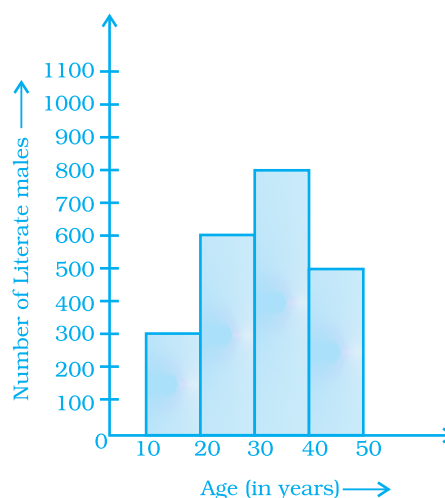
- 65.** 9 students got full marks.
- 66.** The frequency of less than 8 marks is 29.
- 67.** The frequency of more than 8 marks is 21.
- 68.** 10 marks the highest frequency.
- 69.** If the fifth class interval is 60 – 65, fourth class interval is 55 – 60, then the first class interval is 45 –50.

70. From the histogram given on the right, we can say that 1500 males above the age of 20 are literate.

71. The class size of the class interval 60 – 68 is 8.

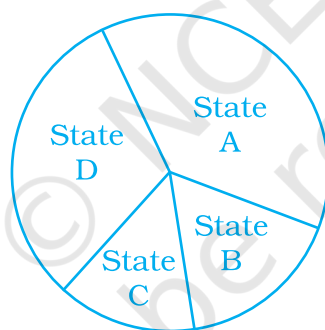
72. If a pair of coins is tossed, then the number of outcomes are 2.

73. On throwing a dice once, the probability of occurrence of an even number is $\frac{1}{2}$.



74. On throwing a dice once, the probability of occurrence of a composite number is $\frac{1}{2}$.

75. From the given pie chart, we can infer that production of Manganese is least in state B.



Production of Manganese
in 4 different states

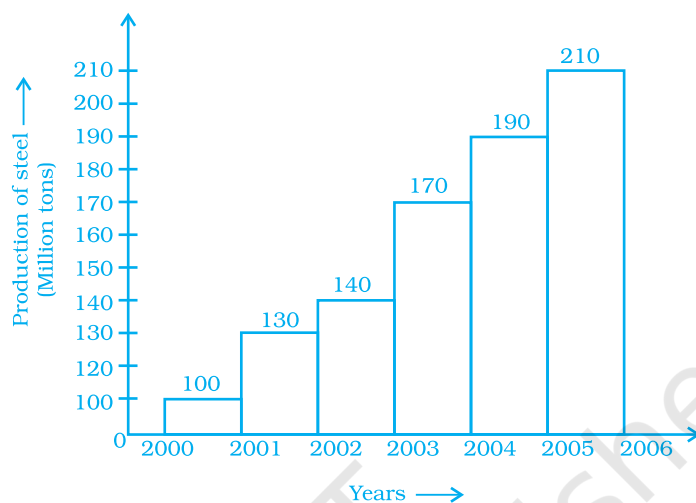
76. One or more outcomes of an experiment make an event.

77. The probability of getting number 6 in a throw of a dice is $\frac{1}{6}$. Similarly the probability of getting a number 5 is $\frac{1}{5}$.

78. The probability of getting a prime number is the same as that of a composite number in a throw of a dice.

79. In a throw of a dice, the probability of getting an even number is the same as that of getting an odd number.

- 80.** To verify pythagoras theorem is a random experiment.
- 81.** The following pictorial representation of data is a histogram.



- 82.** Given below is a frequency distribution table. Read it and answer the questions that follow:

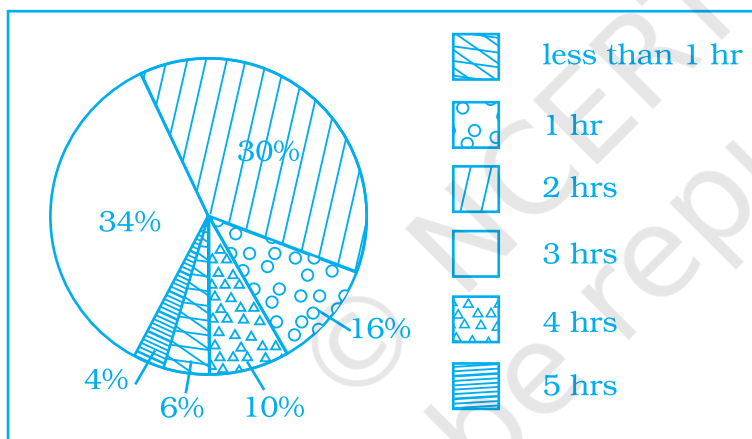
Class Interval	Frequency
10 – 20	5
20 – 30	10
30 – 40	4
40 – 50	15
50 – 60	12

- (a) What is the lower limit of the second class interval?
- (b) What is the upper limit of the last class interval?
- (c) What is the frequency of the third class?
- (d) Which interval has a frequency of 10?
- (e) Which interval has the lowest frequency?
- (f) What is the class size?

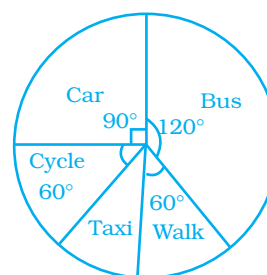
83. The top speeds of thirty different land animals have been organised into a frequency table. Draw a histogram for the given data.

Maximum Speed (km/h)	Frequency
10 – 20	5
20 – 30	5
30 – 40	10
40 – 50	8
50 – 60	0
60 – 70	2

84. Given below is a pie chart showing the time spend by a group of 350 children in different games. Observe it and answer the questions that follow.



- (a) How many children spend at least one hour in playing games?
- (b) How many children spend more than 2 hours in playing games?
- (c) How many children spend 3 or lesser hours in playing games?
- (d) Which is greater — number of children who spend 2 hours or more per day or number of children who play for less than one hour?
85. The pie chart on the right shows the result of a survey carried out to find the modes of travel used by the children to go to school. Study the pie chart and answer the questions that follow.



- (a) What is the most common mode of transport?
- (b) What fraction of children travel by car?
- (c) If 18 children travel by car, how many children took part in the survey?
- (d) How many children use taxi to travel to school?
- (e) By which two modes of transport are equal number of children travelling?

86. A dice is rolled once. What is the probability that the number on top will be

- (a) Odd
- (b) Greater than 5
- (c) A multiple of 3
- (d) Less than 1
- (e) A factor of 36
- (f) A factor of 6

87. Classify the following statements under appropriate headings.

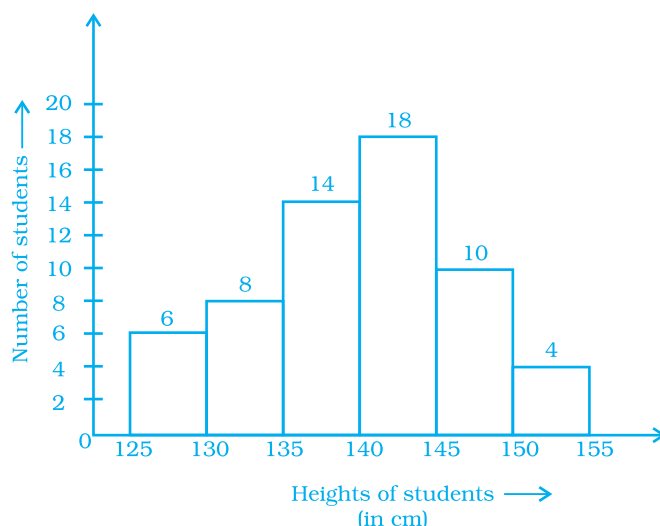
- (a) Getting the sum of angles of a triangle as 180° .
- (b) India winning a cricket match against Pakistan.
- (c) Sun setting in the evening.
- (d) Getting 7 when a die is thrown.
- (e) Sun rising from the west.
- (f) Winning a racing competition by you.

Certain to happen	Impossible to happen	May or may not happen

88. Study the pie chart given below depicting the marks scored by a student in an examination out of 540. Find the marks obtained by him in each subject.

89. Ritwik draws a ball from a bag that contains white and yellow balls. The probability of choosing a white ball is $\frac{2}{9}$. If the total number of balls in the bag is 36, find the number of yellow balls.

90. Look at the histogram below and answer the questions that follow.



- (a) How many students have height more than or equal to 135 cm but less than 150 cm?
- (b) Which class interval has the least number of students?
- (c) What is the class size?
- (d) How many students have height less than 140 cm?
91. Following are the number of members in 25 families of a village:
6, 8, 7, 7, 6, 5, 3, 2, 5, 6, 8, 7, 7, 4, 3, 6, 6, 6, 7, 5, 4, 3, 3, 2, 5.
Prepare a frequency distribution table for the data using class intervals 0–2, 2–4, etc.
92. Draw a histogram to represent the frequency distribution in question 91.
93. The marks obtained (out of 20) by 30 students of a class in a test are as follows:
14, 16, 15, 11, 15, 14, 13, 16, 8, 10, 7, 11, 18, 15, 14, 19, 20, 7, 10, 13, 12, 14, 15, 13, 16, 17, 14, 11, 10, 20.
Prepare a frequency distribution table for the above data using class intervals of equal width in which one class interval is 4–8 (excluding 8 and including 4).
94. Prepare a histogram from the frequency distribution table obtained in question 93.

- 95.** The weights (in kg) of 30 students of a class are:

39, 38, 36, 38, 40, 42, 43, 44, 33, 33, 31, 45, 46, 38, 37, 31, 30, 39, 41, 41, 46, 36, 35, 34, 39, 43, 32, 37, 29, 26.

Prepare a frequency distribution table using one class interval as (30 – 35), 35 not included.

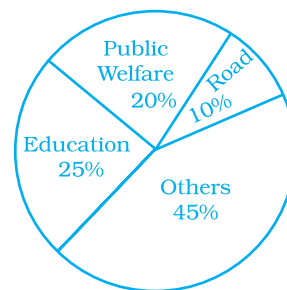
- Which class has the least frequency?
- Which class has the maximum frequency?

- 96.** Shoes of the following brands are sold in Nov. 2007 at a shoe store. Construct a pie chart for the data.

Brand	Number of pair of shoes sold
A	130
B	120
C	90
D	40
E	20

- 97.** The following pie chart depicts the expenditure of a state government under different heads.

- If the total spending is 10 crores, how much money was spent on roads?
- How many times is the amount of money spent on education compared to the amount spent on roads?
- What fraction of the total expenditure is spent on both roads and public welfare together?

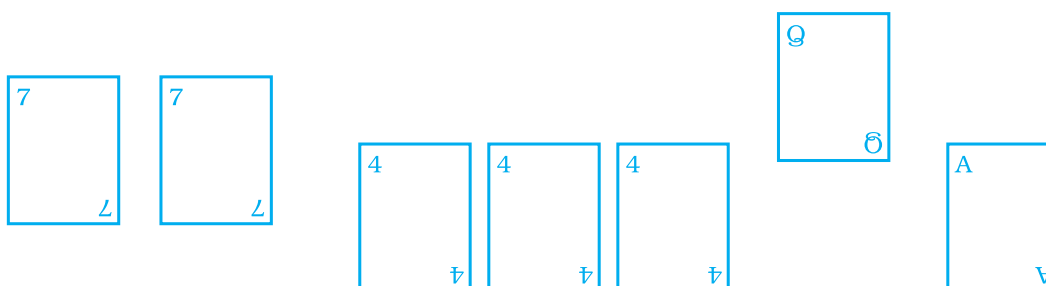


- 98.** The following data represents the different number of animals in a zoo. Prepare a pie chart for the given data.

Animals	Number of animals
Deer	42
Elephant	15
Giraffe	26
Reptiles	24
Tiger	13

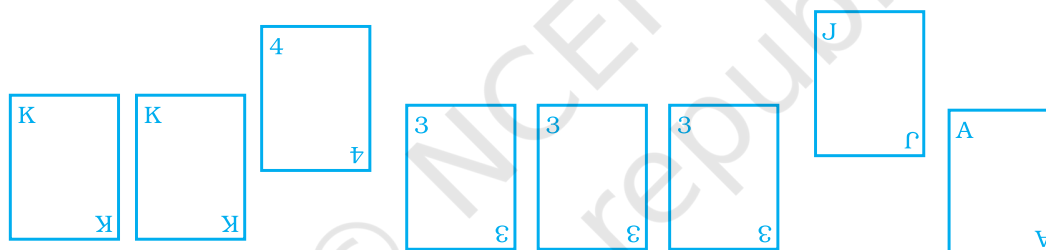
99. Playing cards

(a) From a pack of cards the following cards are kept face down:



Suhail wins if he picks up a face card. Find the probability of Suhail winning?

(b) Now the following cards are added to the above cards:



What is the probability of Suhail winning now? Reshma wins if she picks up a 4. What is the probability of Reshma winning?

[Queen, King and Jack cards are called face cards.]

100. Construct a frequency distribution table for the following weights (in grams) of 35 mangoes, using the equal class intervals, one of them is 40 – 45 (45 not included).

30, 40, 45, 32, 43, 50, 55, 62, 70, 70, 61, 62, 53, 52, 50, 42, 35, 37, 53, 55, 65, 70, 73, 74, 45, 46, 58, 59, 60, 62, 74, 34, 35, 70, 68.

(a) How many classes are there in the frequency distribution table?

(b) Which weight group has the highest frequency?

101. Complete the following table:

Weights (in kg.)	Tally Marks	Frequency (Number of persons)
40 – 50		
50 – 60		
60 – 70		
70 – 80		
80 – 90		

Find the total number of persons whose weights are given in the above table.

102. Draw a histogram for the following data.

Class interval	10–15	15–20	20–25	25–30	30–35	35–40
Frequency	30	98	80	58	29	50

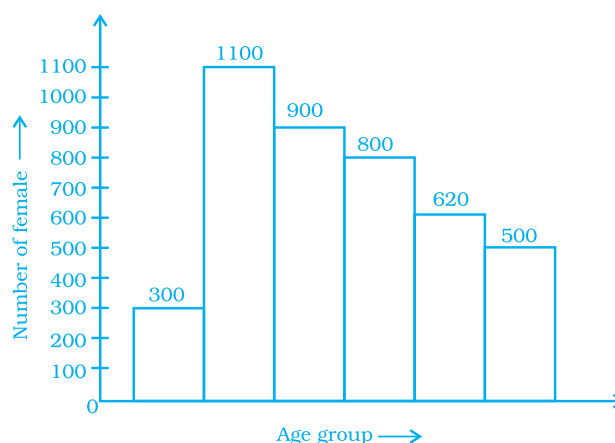
103. In a hypothetical sample of 20 people, the amount of money (in thousands of rupees) with each was found to be as follows:

114, 108, 100, 98, 101, 109, 117, 119, 126, 131, 136, 143, 156, 169, 182, 195, 207, 219, 235, 118.

Draw a histogram of the frequency distribution, taking one of the class intervals as 50–100.

104. The below histogram shows the number of literate females in the age group of 10 to 40 years in a town.

- Write the classes assuming all the classes are of equal width.
- What is the classes width?
- In which age group are literate females the least?
- In which age group is the number of literate females the highest?



- 105.** The following histogram shows the frequency distribution of teaching experiences of 30 teachers in various schools:
- What is the class width?
 - How many teachers are having the maximum teaching experience and how many have the least teaching experience?
 - How many teachers have teaching experience of 10 to 20 years?

- 106.** In a district, the number of branches of different banks is given below:

Bank	State Bank of India	Bank of Baroda	Punjab National Bank	Canara Bank
Number of Branches	30	17	15	10

Draw a pie chart for this data.

- 107.** For the development of basic infrastructure in a district, a project of Rs 108 crore approved by Development Bank is as follows:

Item Head	Road	Electricity	Drinking water	Sewerage
Amount in crore (Rs.)	43.2	16.2	27.00	21.6

Draw a pie chart for this data.

- 108.** In the time table of a school, periods allotted per week to different teaching subjects are given below:

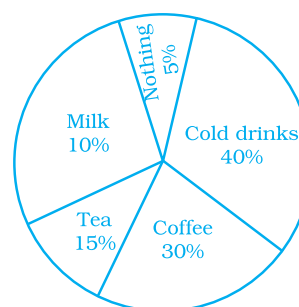
Subject	Hindi	English	Maths	Science	Social Science	Computer	Sanskrit
Periods Allotted	7	8	8	8	7	4	3

Draw a pie chart for this data.

- 109.** A survey was carried out to find the favourite beverage preferred by a certain group of young people. The following pie chart shows the findings of this survey.

From this pie chart answer the following:

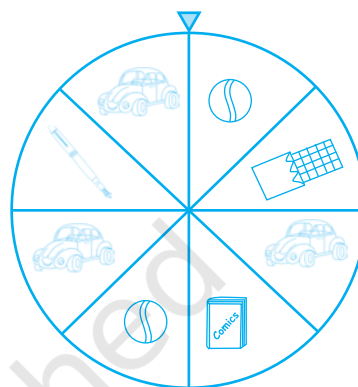
- Which type of beverage is liked by the maximum number of people?
- If 45 people like tea, how many people were surveyed?



- 110.** The following data represents the approximate percentage of water in various oceans. Prepare a pie chart for the given data.

Pacific	40%
Atlantic	30%
Indian	20%
Others	10%

- 111.** At a Birthday Party, the children spin a wheel to get a gift. Find the probability of
- getting a ball
 - getting a toy car
 - any toy except a chocolate

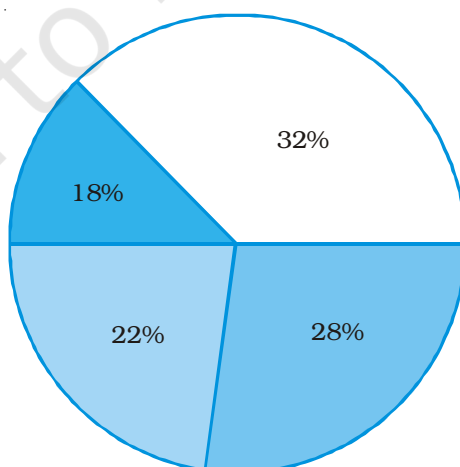


- 112.** Sonia picks up a card from the given cards.

R 1	Y 2	Y 3	R 4	B 5
B 6	G 7	Y 8	R 9	G 10

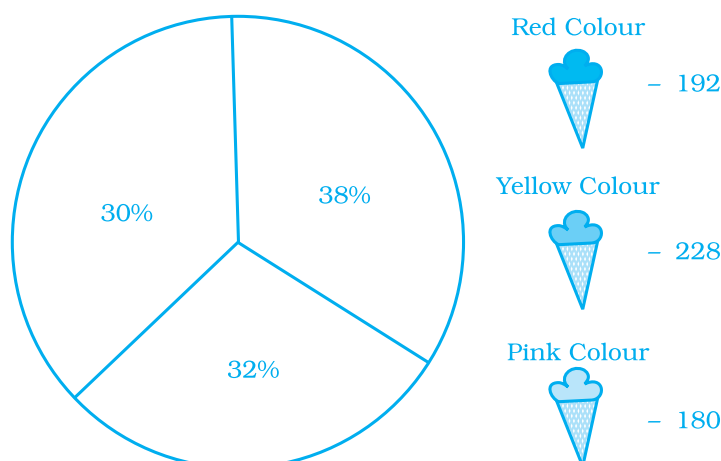
Calculate the probability of getting

- an odd number
 - a Y card
 - a G card
 - B card bearing number > 7
- 113.** Identify which symbol should appear in each sector in 113, 114.

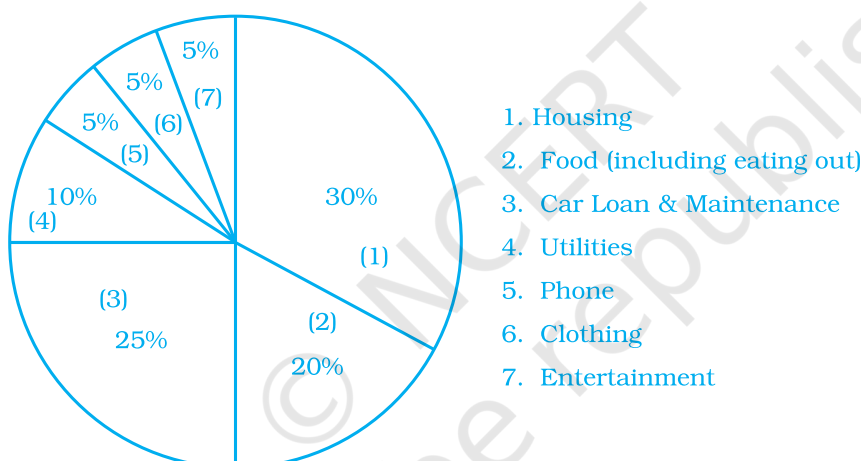


	- 800
	- 700
	- 550
	- 450

114.



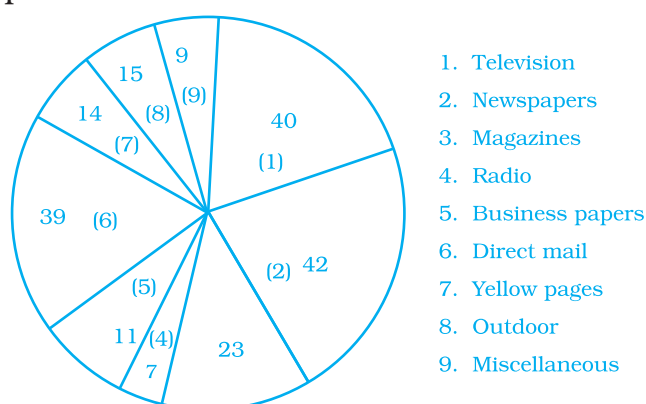
115. A financial counselor gave a client this pie chart describing how to budget his income. If the client brings home Rs. 50,000 each month, how much should he spend in each category?



116. Following is a pie chart showing the amount spent in rupees (in thousands) by a company on various modes of advertising for a product.

Now answer the following questions.

- Which type of media advertising is the greatest amount of the total?
- Which type of media advertising is the least amount of the total?



- What per cent of the total advertising amount is spent on direct mail campaigns?
- What per cent of the advertising amount is spent on newspaper and magazine advertisements?
- What media types do you think are included in miscellaneous? Why aren't those media types given their own category?

(D) Application, Games and Puzzles

1 Card Activity



Have you ever seen a pack of cards?

No didi.



Then, take a pack of cards and try to complete the table given below.

It will be fun didi.



	Face Cards			Number Cards										
	K	Q	J	10	9	8	7	6	5	4	3	2	A	Total
 Spade														
 Heart														
 Diamond														
 Club														

Total



Did you have a look at all the cards carefully?

Yes Didi. But what do the alphabets A, K, Q and J stand for?





A stands for Ace,
K stands for King,
Q stands for Queen
and
J stands for Jack.

Ok



Now, try to
answer some
questions.

I will
try didi.



1. How many colours can you observe?
2. How many cards are there in all?
3. How many cards of one type are there?
4. How many types of cards can you observe? Name them.
5. How many black cards are there in all?
6. How many red cards are there in all?
7. How many face cards of each type are there?
8. How many picture cards are there in all?



What is the fraction
of number of red
cards to total
number of cards?

Number of red cards
is 26.
Total cards are 52.
So fraction becomes

$$\frac{26}{52}$$



Do you know
this fraction
is also the probability
of getting a red
card out of the pack
of cards?

Really didi.
So now, I can
calculate the
probabilities also.



Let us see
if you can
answer these
questions?

9. From a pack of well-shuffled cards, what is the probability of getting
- (i) a black face card (ii) a red jack (iii) a 4 of spade
 - iv) a picture card (v) a red card of ace (vi) a black king
 - (vii) an ordinary card (viii) a picture card of heart
 - (ix) an ace of club (x) a king
 - (xi) a card of diamond (xii) a black ordinary card

2 Playing with dice

(a) Complete the table given below and answer the questions that follow:

A				B			
Dice 1	Dice 2	Out-comes	Sum	Dice 1	Dice 2	Out-comes	Sum
		(1,1)	2				
		(1,2)	3				
		(1,3)	4				
		(1,4)	5				
							
							
							
							
							
							
							
							
							
							
							
							
							
							

The Paper Chase

Pratibha's desk has 8 drawers. When she receives a paper, she usually chooses a drawer at random to put it in. However, 2 out of 10 times she forgets to put the paper away, and it gets lost.

The probability that a paper will get

lost is $\frac{2}{10}$, or $\frac{1}{5}$.

- What is the probability that a paper will be put into a drawer?
- If all drawers are equally likely to be chosen, what is the probability that a paper will be put in drawer 3?

When Pratibha needs a document, she looks first in drawer 1 and then checks each drawer in order until the paper is found or until she has looked in all the drawers.

1. If Pratibha checked drawer 1 and didn't find the paper she was looking for, what is the probability that the paper will be found in one of the remaining 7 drawers?
2. If Pratibha checked drawers 1, 2 and 3, and didn't find the paper she was looking for, what is the probability that the paper will be found in one of the remaining 5 drawers?
3. If Pratibha checked drawers 1–7 and didn't find the paper she was looking for, what is the probability that the paper will be found in the last drawer?



(b) Complete the table given below.

Sum of dots on both the dice	Tally marks	Number of outcomes	Probability
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Two dice are rolled together, using the above table find the probability of—

- sum of digits to be more than 6.
- sum of digits to be less than 3.
- sum of digits to be either 5 or 6.
- sum of digits to be 12.
- sum of digits to be less than 9 but more than 5.

3 DATA COLLECTION

Read the paragraph given below and complete the tables given.

All of us have some concept of statistics because magazines, newspapers, radio and TV advertisements are full of statistics or numerical data. Existence of the practice of collecting numerical data in ancient India is evident from the fact that during the reign of Chandragupta Maurya, there was a good system of collecting such data especially with regard to births and deaths. During Akbar's reign, Raja Todarmal, the Land and Revenue Minister, maintained good records of land and agricultural statistics. In Ain-i-Akbari written by Abul Fazal, a detailed account of the administrative and statistical surveys conducted during that period can be found.

From the paragraph given on the previous page prepare the frequency table of all the letters of the English alphabet and answer the questions that follow.

1. Frequency table for each letter of the alphabet.

Letter	Tally marks	Frequency
A		
B		
C		
-		
-		
-		
Z		

- (a) Which is the least frequently occurring letter?
 - (b) Which vowel is most commonly used?
 - (c) Which consonant is most commonly used?
 - (d) Find the ratio of vowels to that of consonants.
2. Frequency table for words with two or more letters.

Number of words with	Tally marks	Frequency
2 letters		
3 letters		
4 letters		
5 letters		
6 letters		
more than 6 letters		

- (a) How many two letter words are used in the paragraph?
 - (b) How many words are used in all?
 - (c) How many words have five letters or more?
 - (d) What is the ratio of three letter words and five letter words?
- 4 **Fun Activity**

Take a packet which has different colours of toffees/candies in it. Count the number of toffees of each colour and fill the data in the table given below. Also draw a pie chart to depict the data.

Colour of candies	Number	Fraction	Fraction of 360°
Red			
Green			

5 Conducting Survey

Conduct a class survey to know the favourite T.V. channels and note the responses in the following table.

Channels	Number of Votes	Fraction of Total Votes	Estimated per cent of Total Votes	Caculated per cent of Total Votes
News				
Movies				
History and Nature				
Cartoon				
Sports				

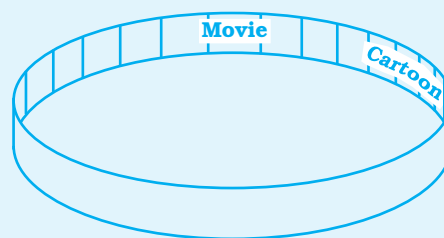
How accurate is your estimation?

Now, take a strip of thick chart paper, 1 cm wide and divide it into equal-sized rectangles – one for each student of your class. The entire strip represents your whole class, or 100% of the votes. On your strip, colour groups of rectangles according to the number of votes each choice received. Use a different colour for each choice. For example, if 5 students voted for movie, colour the first 5 rectangles blue. If 7 choose cartoon, colour the next 7 rectangle green. When you are finished, all the rectangles should be coloured.

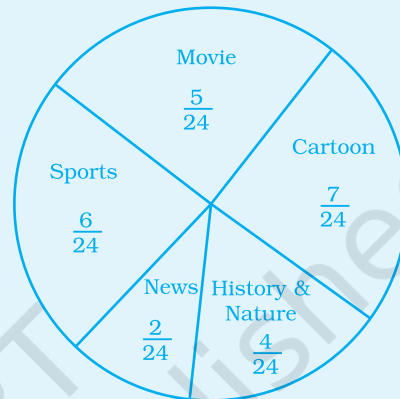
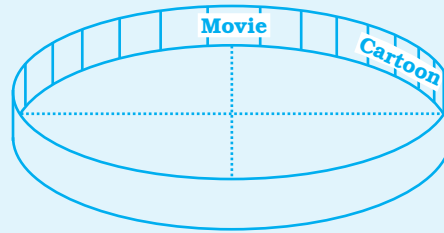


Now create a circle graph as shown below.

- Tape the ends of your strip together, with no overlap, to form a loop with the coloured rectangles inside.
- Tape four copies of the quarter-circle template together to form a circle.



- Place your above loop around the circle. On the edge of the circle, mark where each colour begins and ends.
- Remove the loop, and use a ruler to connect each mark you made to the centre of the circle.
- Colour the sections of your graph. Label each section with the channels name and the fraction of votes that channel received. For example, your circle graph known as pie chart might look like this.

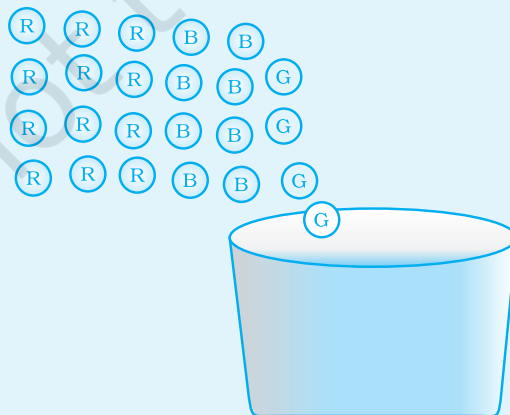


Circle graphs in books, magazines, and newspapers are often labeled with per cents. Add per cent labels to your pie chart.

6 Marble Game

Pramod is babysitting his little sister Monika and her two friends, Puja and Jyoti. Monika is wearing red, Puja is wearing blue and Jyoti is wearing green coloured clothes.

Pramod fills a bucket with 12 red (R) marbles, 8 blue (B) marbles and 4 green (G) marbles. He tells the girls that they will play a game. He will reach into the bucket and pull out a marble at random. The girl whose clothes match the colour of the marble scores 1 point.



- a. What is the probability of each girl scoring 1 point on the first draw?

Monika :

Puja :

Jyoti :

- b. What is the probability of not drawing a green marble on the first draw?
- c. If two marbles of each colour are added to the bucket, do the probabilities in part (a) change? Explain your answer.
- d. If the number of each colour is doubled, do the probabilities in part (a) change? Explain why or why not.

7 Crossword Puzzle

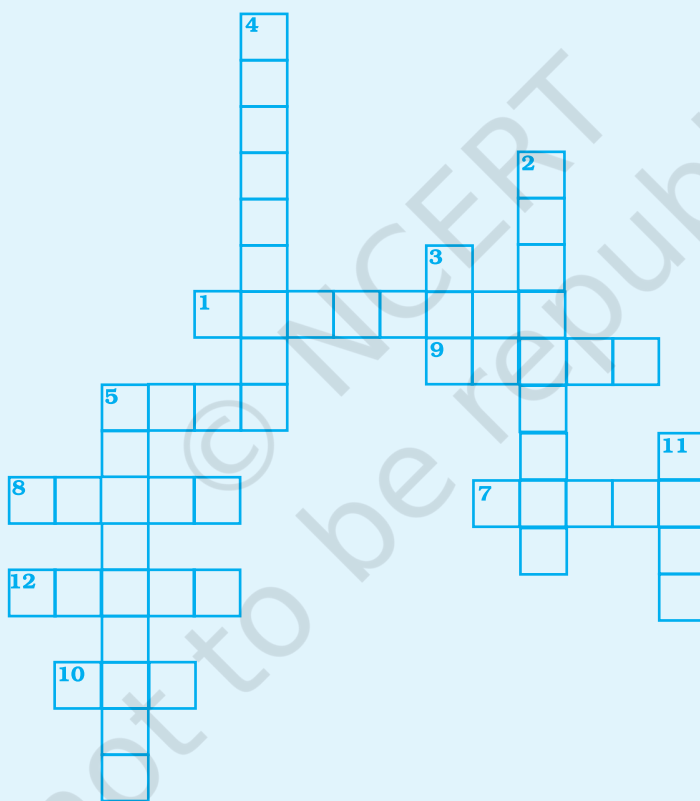
Solve the crossword (given on the next page) and then fill up the given boxes. Clues are given below for across as well as downward filling. Also, for across and down clues, due number is written at the corner of the boxes. Answer of clues have to be filled in their respective boxes.

Across

1. Another name for a circle graph is _____.
5. Class width of the interval 10-15 is _____.
7. Difference of highest and lowest observations in a given data is called _____.
8. Each outcome or a collection of outcomes in an experiment is known as _____.
9. Pie chart represents the comparison of parts to a _____.
10. Probability of sun rising in the east is _____.
12. Probability of getting a head or a tail on tossing a coin once is _____.

Down

2. Representation of grouped data graphically is called _____.
3. Unorganised and ungrouped data are called _____.
4. Difference between upper and lower class limit is known as _____.
6. The number of times a particular observation occurs in the given data is called _____.
11. If today is Saturday, then the probability of two days after tomorrow being a Monday is _____.



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UNIT 3

SQUARE-SQUARE ROOT AND CUBE-CUBE ROOT

(A) Main Concepts and Results

- A natural number is called a **perfect square** if it is the square of some natural number.
i.e., if $m = n^2$, then m is a perfect square where m and n are natural numbers.
- A natural number is called a **perfect cube** if it is the cube of some natural number.
i.e., if $m = n^3$, then m is a perfect cube where m and n are natural numbers.
- Number obtained when a number is multiplied by itself is called the square of the number.
- Number obtained when a number is multiplied by itself three times are called **cube number**.
- Squares and cubes of even numbers are even.
- Squares and cubes of odd numbers are odd.
- A perfect square can always be expressed as the product of pairs of prime factors.
- A perfect cube can always be expressed as the product of triplets of prime factors.

- The unit digit of a perfect square can be only 0, 1, 4, 5, 6 or 9.
- The square of a number having:
 - 1 or 9 at the units place ends in 1.
 - 2 or 8 at the units place ends in 4.
 - 3 or 7 at the units place ends in 9.
 - 4 or 6 at the units place ends in 6.
 - 5 at the units place ends in 5.
- There are $2n$ natural numbers between the squares of numbers n and $n+1$.
- A number ending in odd numbers of zeroes is not a perfect square.
- The sum of first n odd natural numbers is given by n^2 .
- Three natural numbers a, b, c are said to form a pythagorean triplet if $a^2 + b^2 = c^2$.
- For every natural number $m > 1$, $2m$, m^2-1 and $m^2 + 1$ form a pythagorean triplet.
- The square root of a number x is the number whose square is x . Positive square root of a number x is denoted by $\sqrt{x}$.
- The cube root of a number x is the number whose cube is x . It is denoted by $\sqrt[3]{x}$.
- Square root and cube root are the inverse operations of squares and cubes respectively.
- If a perfect square is of n digits, then its square root will have $\frac{n}{2}$ digit if n is even or $\left(\frac{n+1}{2}\right)$ digit if n is odd.
- Cubes of the numbers ending with the digits 0, 1, 4, 5, 6 and 9 end with digits 0, 1, 4, 5, 6 and 9 respectively.

Think and Discuss

1. **Describe** what is meant by a perfect square. Give an example.
2. **Explain** how many square roots a positive number can have. How are these square roots different?

Key Concept**To be Noted****SQUARE ROOTS**

- Words** A square root of a number n is a number m which, when multiplied by itself, equals n .
- Numbers** The square roots of 16 are 4 and -4 because $4^2 = 16$ and $(-4)^2 = 16$.
- Algebra** If $m^2 = n$, then m is a square root of n .

Think and Discuss

- Which type of number has an exact square root?
- Which type of number has an approximate square root?
- How can we use perfect squares to estimate a square root, such as $\sqrt{8}$?

- Cube of the number ending in 2 ends in 8 and cube root of the number ending in 8 ends in 2.
- Cube of the number ending in 3 ends in 7 and cube root of the number ending in 7 ends in 3.

(B) Solved Examples

In examples 1 to 7, out of given four choices only one is correct. Write the correct answer.

Example 1 : Which of the following is the square of an odd number?

- (a) 256 (b) 361 (c) 144 (d) 400

Solution : Correct answer is (b).

Example 2 : Which of the following will have 1 at its units place?

- (a) 19^2 (b) 17^2 (c) 18^2 (d) 16^2

Solution : Correct answer is (a).

Example 3 : How many natural numbers lie between 18^2 and 19^2 ?

- (a) 30 (b) 37 (c) 35 (d) 36

Solution : Correct answer is (d).

Example 4 : Which of the following is not a perfect square?

- (a) 361 (b) 1156 (c) 1128 (d) 1681

Solution : Correct answer is (c).

Example 5 : A perfect square can never have the following digit at ones place.

- (a) 1 (b) 6 (c) 5 (d) 3

Solution : Correct answer is (d).

Example 6 : The value of $\sqrt{176} + \sqrt{2401}$ is

- (a) 14 (b) 15 (c) 16 (d) 17

Solution : Correct answer is (b).

$$\left(\sqrt{176} + \sqrt{2401} = \sqrt{176 + 49} = \sqrt{225} = 15 \right)$$

Example 7 : Given that $\sqrt{5625} = 75$, the value of $\sqrt{0.5625} + \sqrt{56.25}$ is:

- (a) 82.5 (b) 0.75 (c) 8.25 (d) 75.05

Solution : Correct answer is (c).

If ($\sqrt{5625} = 75$, then $\sqrt{0.5625} = 0.75$ and $\sqrt{56.25} = 7.5$)

In examples 8 to 14, fill in the blanks to make the statements true.

Example 8 : There are _____ perfect squares between 1 and 50.

Solution : 6

Example 9 : The cube of 100 will have _____ zeroes.

Solution : 6

Example 10 : The square of 6.1 is _____.

Solution : 37.21

1. Squaring a number and taking a square root are inverse operations. What other inverse operations do you know?
2. When the factors of a perfect square are written in order from the least to greatest, what do you notice?
3. Why do you think numbers such as 4, 9, 16, ... are called perfect squares?
4. Suppose you list the factors of a perfect square. Why is one factor square root and not the other factors?

Example 11 : The cube of 0.3 is _____.

Solution : 0.027

Connect

here are some ways to tell whether a number is a square number.

- If we can find a division sentence for a number so that the quotient is equal to the divisor, the number is a square number.

For example, $16 \div 4 = 4$, so 16 is a square number.

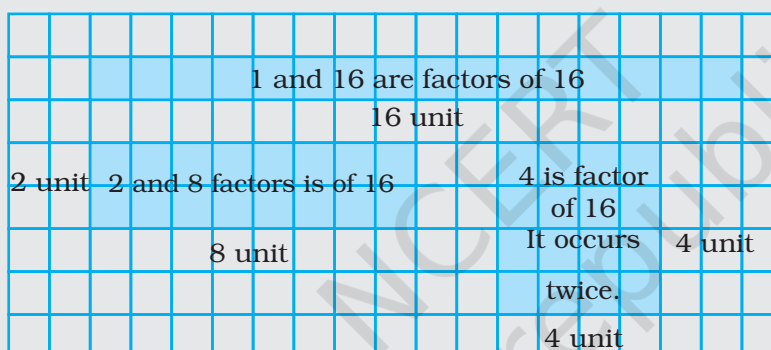


dividend divisor quotient

- We can also use factoring.

Factors of a number occur in pairs.

These are the dimensions of a rectangle.



Sixteen has 5 factors: 1, 2, 4, 8, 16

Since there is an odd number of factors, one rectangle is a square.

The square has side length of 4 units.

We say that 4 is a square root of 16.

We write $4 = \sqrt{16}$

When a number has an odd number of factors, it is a square number.

A factor that occurs twice is only written once in the list of factors.

Think and Discuss

1. **Discuss** whether 9.5 is a good first guess for $\sqrt{75}$.
2. **Determine** which square root or roots would have 7.5 as a good first guess.

Example 12 : 68^2 will have _____ at the units place.

Solution : 4

Example 13 : The positive square root of a number x is denoted by _____.

Solution : $\sqrt{x}$

Example 14 : The least number to be multiplied with 9 to make it a perfect cube is _____.

Solution : 3

In examples 15 to 19, state whether the statements are true (T) or false (F)

Example 15 : The square of 0.4 is 0.16.

Solution : True

Example 16 : The cube root of 729 is 8.

Solution : False

Example 17 : There are 21 natural numbers between 10^2 and 11^2 .

Solution: False

Example 18 : The sum of first 7 odd natural numbers is 49.

Solution : True

Example 19 : The square root of a perfect square of n digits will have $\frac{n}{2}$ digits if n is even.

Solution : True

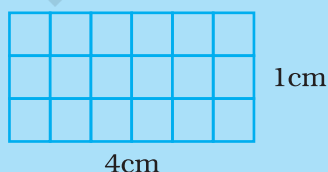
Example 20 : Express 36 as a sum of successive odd natural numbers.

Solution : $1+3+5+7+9+11 = 36$

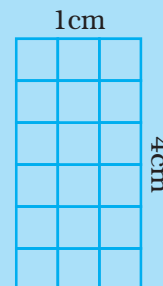
A rectangle is a quadrilateral with 4 right angles.

A square also has 4 right angles.

A rectangle with base 4 cm and height 1cm is the same as a rectangle with base 1cm and height 4 cm.

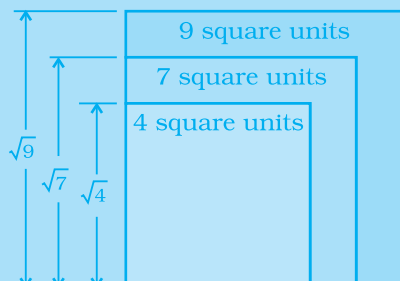


These two rectangles are congruent.
Is every square a rectangle?
Is every rectangle a square?



Copy this diagram on grid paper.

Then estimate the value of $\sqrt{7}$ to one decimal place.



Example 21 : Check whether 90 is a perfect square or not by using prime factorisation.

Solution : Prime factorisation of 90 is

2	90
3	45
3	15
5	5
	1

$$90 = 2 \times 3 \times 3 \times 5$$

The prime factors 2 and 5 do not occur in pairs. Therefore, 90 is not a perfect square.

Example 22 : Check whether 1728 is a perfect cube by using prime factorisation.

Solution : Prime factorisation of 1728 is

$$1728 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$$

Since all prime factors can be grouped in triplets. Therefore, 1728 is a perfect cube.

Apply

Use square tiles. Make as many different rectangles as you can with area 28 square units.

Draw your rectangles on grid paper.

Is 28 a perfect square? Justify your answer.

Example 23 : Using distributive law, find the square of 43.

Solution : $43 = 40 + 3$

$$\begin{aligned}\text{So } 43^2 &= (40 + 3)^2 = (40 + 3)(40 + 3) = 40(40 + 3) + 3(40 + 3) \\ &= 40 \times 40 + 40 \times 3 + 3 \times 40 + 3 \times 3 \\ &= 1600 + 240 + 9 \\ &= 1849\end{aligned}$$

$$\text{So, } 43^2 = 1849$$

Example 24 : Write a pythagorean triplet whose smallest number is 6.

Solution : Smallest number is 6

$$2m = 6 \text{ or } m = 3$$

$$m^2 + 1 = 3^2 + 1 = 9 + 1 = 10$$

$$m^2 - 1 = 3^2 - 1 = 9 - 1 = 8$$

So, the pythagorean triplet is 6, 8, 10.

Connect

Here is one way to estimate the value of $\sqrt{20}$:

- 25 is the square number closest to 20, but greater than 20.

On grid paper, draw a square with area 25.

Its side length: $\sqrt{25} = 5$

- 16 is the square number closest to 20, but less than 20.

Draw a square with area 16

Its side length: $\sqrt{16} = 4$

Draw the squares so that they overlap.

A square with area 20 lies between these two squares.

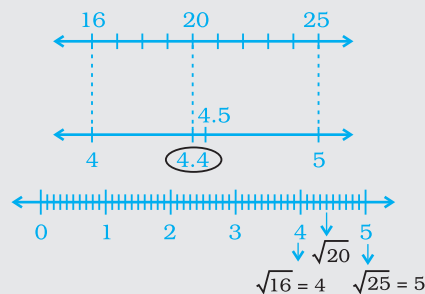
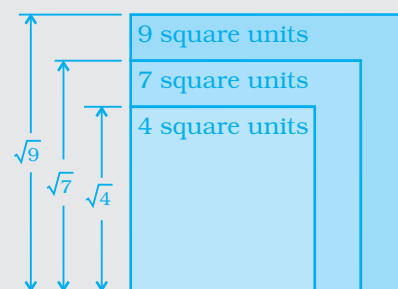
Its side length $\sqrt{20}$.

20 is between 16 and 25, but closer to 16.

So, $\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$, but closer to $\sqrt{16}$.

So, $\sqrt{20}$ is between 4 and 5, but closer to 4.

An estimate of $\sqrt{20}$ is 4.4 to one decimal place.





Application on Problem Solving Strategy

A couple wants to install a square glass window that has an area of 500 square cm. Calculate the length of each side and the length of trim needed to the nearest tenth of cm.



Understand the problem

First find the length of a side. Then you can use the length of the side to find the perimeter – the length of the trim around the window.



Make a Plan

The length of a side, in cm, is the number that you multiply by itself to get 500. Find this number to the nearest tenth.

Use guess and check to find $\sqrt{500}$.



Solve

Because 500 is between 22^2 (484) and 23^2 (529), the square root of 500 is between 22 and 23.

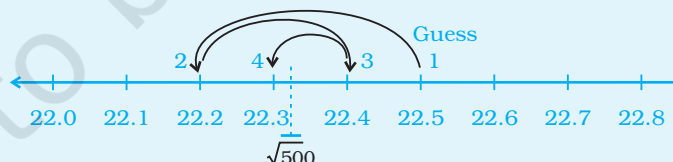
The square root is between 22.3 and 22.4. To round to the nearest tenth, consider 22.35.

$$22.35^2 = 499.5225 \quad \text{low}$$

Guess 22.5	Guess 22.2	Guess 22.4	Guess 22.3
22.5^2 506.25	22.2^2 492.84	22.4^2 501.76	22.3^2 497.29
high	low	high	low
Square root is between 22 and 22.5	Square root is between 22.2 and 22.5	Square root is between 22.2 and 22.4	Square root is between 22.3 and 22.4

The square root must be greater than 22.35, so you can round up.

To the nearest tenth, $\sqrt{500}$ is about 22.4.



Now estimate the length around the window. The length of a side of the window to the nearest tenth of an inch is 22.4 inches.

$$4 \times 22.4 = 89.6 \quad (\text{Perimeter} = 4 \times \text{side})$$

The trim is about 89.6 cm long.



Look Back

The length 90 cm divided by 4 is 22.5 cm. A 22.5 cm square has an area of 506 square cm, which is close to 500, so the answers are reasonable.

Example 25 : Using prime factorisation, find the cube root of 5832.

Solution : The prime factorisation of 5832 is

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$\begin{aligned}\text{Therefore, } \sqrt[3]{5832} &= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3} \\ &= 2 \times 3 \times 3 \\ &= 18\end{aligned}$$

Take It Further

a) Find the square root of each palindromic number.

A palindromic number is a number that reads the same – forward and backward.

(i) $\sqrt{121}$

(ii) $\sqrt{12321}$

(iii) $\sqrt{1234321}$

(iv) $\sqrt{123454321}$

b) Continue the pattern.

Write the next 4 palindromic numbers in the pattern and their square roots.

Think and Discuss

1. Is 1 a square number? How can you tell?
2. Suppose you know the area of a square. How can you find its perimeter?
3. Suppose you know the perimeter of a square. How can you find its area?

Example 26 : Evaluate the square root of 22.09 by long division method.

Solution : 4.7

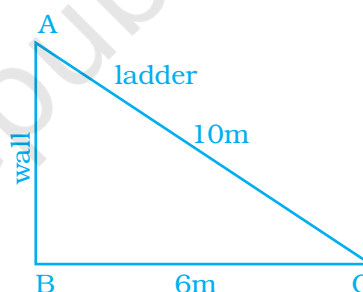
$$\begin{array}{r} 4 \overline{) 22.09} \\ \underline{16} \\ 609 \\ \underline{609} \\ 0 \end{array}$$

Therefore, $\sqrt{22.09} = 4.7$

Example 27 : Find the smallest perfect square divisible by 3, 4, 5 and 6.

Solution : The least number divisible by 3, 4, 5 and 6 is their LCM. The LCM of 3, 4, 5 and 6 is 60. Now, $60 = 2 \times 2 \times 5 \times 3$. We see that prime factors 5 and 3 are not in pairs. Therefore 60 is not a perfect square. So, 60 should be multiplied by $5 \times 3 = 15$ to get a perfect square. Thus, the required least square number $= 60 \times 15 = 900$.

Example 28 : A ladder 10m long rests against a vertical wall. If the foot of the ladder is 6m away from the wall and the ladder just reaches the top of the wall, how high is the wall?



Solution : Let AC be the ladder. Therefore, $AC = 10\text{m}$

Let BC be the distance between the foot of the ladder and the wall.

Therefore, $BC = 6\text{m}$

$\triangle ABC$ forms a right angled triangle, right angled at B.

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$10^2 = AB^2 + 6^2$$

$$\text{or} \quad AB^2 = 10^2 - 6^2 = 100 - 36 = 64$$

$$\text{or} \quad AB = \sqrt{64} = 8\text{m}$$

Hence, the wall is 8m high.

Example 29 : Find the length of a diagonal of a rectangle with dimensions 20m by 15m.

Solution : Using Pythagoras theorem, we have

Length of diagonal of the rectangle

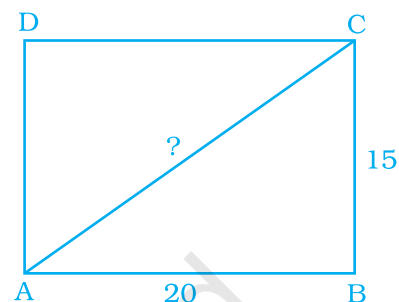
$$= \sqrt{(l^2 + b^2)} \text{ units}$$

$$= \sqrt{(20^2 + 15^2)} \text{ m}$$

$$= \sqrt{400 + 225} \text{ m}$$

$$= \sqrt{625} \text{ m}$$

$$= 25 \text{ m}$$



Hence, the length of diagonal is 25m.

Investigate

Work with a partner.

You will need grid paper and 20 square tiles.

Use the tiles to make as many different rectangles as you can with each area.

4 square units	12 square units
6 square units	16 square units
8 square units	20 square units
9 square units	

Draw the rectangles on grid paper.

- For how many areas given above were you able to make a square?
- What is the side length of each square you made?
- How is the side length of a square related to its area?

Think and Discuss

Compare your strategies and results with those of another pair of classmates.

Find two areas greater than 20 square units for which you could use tiles to make a square.

How do you know you could make a square for each of these areas?

Example 30 : The area of a rectangular field whose length is twice its breadth is 2450 m^2 . Find the perimeter of the field.

Solution : Let the breadth of the field be x metres. Then length of the field is $2x$ metres.

$$\begin{aligned}\text{Therefore, area of the rectangular field} &= \text{length} \times \text{breadth} \\ &= (2x)(x) = (2x^2) \text{ m}^2\end{aligned}$$

Given that area is 2450 m^2 .

$$\text{Therefore, } 2x^2 = 2450$$

$$x^2 = \frac{2450}{2}$$

$$x = \sqrt{1225} \text{ or } x = 35\text{m}$$

Hence, breadth = 35m and length $35 \times 2 = 70\text{m}$

$$\begin{aligned}\text{Perimeter of the field} &= 2(l + b) \\ &= 2(70+35)\text{m} = 2 \times 105\text{m} = 210\text{m}\end{aligned}$$

Example 31 : During a mass drill exercise, 6250 students of different schools are arranged in rows such that the number of students in each row is equal to the number of rows. In doing so, the instructor finds out that 9 children are left out. Find the number of children in each row of the square.

Solution : Total number of students = 6250

$$\begin{aligned}\text{Number of students forming a square} &= 6250 - 9 \\ &= 6241\end{aligned}$$

Thus, 6241 students form a big square which has number of rows equal to the number of students in each row.

Let the number of students in each row be x , then the number of rows = x

$$\text{Therefore, } x \times x = 6241$$

$$\text{or } x = \sqrt{6241} = 79$$

Hence, there are 79 students in each row of the square formed.

Example 32 : Find the least number that must be added to 1500 so as to get a perfect square. Also find the square root of the perfect square.

Solution :

	38
3	1500
	9
68	600
	544
	56

We observe that $38^2 < 1500 < 39^2$

$$\begin{aligned}\text{Hence the number to be added} &= 39^2 - 1500 \\ &= 1521 - 1500 \\ &= 21\end{aligned}$$

Therefore, the perfect square is $1500 + 21 = 1521$

$$\sqrt{1521} = 39$$

Hence the required number is 21 and the square root is 39.

Tsunamis, sometimes called tidal waves, move across deep oceans at high speeds with barely a ripple on the water surface. It is only when tsunamis hit shallow water that their energy moves them upward into a huge destructive force.

- The speed of a tsunami, in metre per second, can be found by the formula $r = \sqrt{9.7344d}$, where d is the water depth in metre. Suppose the water depth is 6400m. How fast is the tsunami moving?

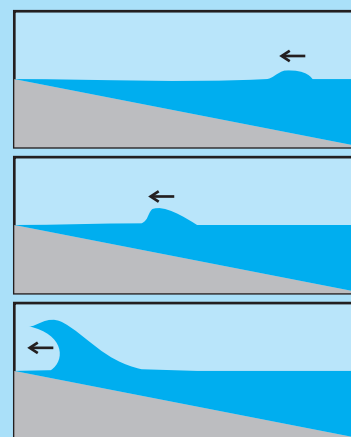
- The speed of a tsunami in km per hour can be found using $r = \sqrt{4.4944d}$ where d is the water depth in metre. Suppose the water depth is 8100 metre

- How fast is the tsunami moving in km per hour?
- How long would it take a tsunami to travel

3000 km if the water depth was a consistent 3000 m?



Tsunamis can be caused by earthquakes, volcanoes, landslides, or meteorites.



As the wave approaches the beach, it slows, builds in height and crashes on shore

Example 33 : Application of problem solving strategies

- Find the smallest number by which 1620 must be divided to get a perfect square.

Solution : Understand and Explore

- What information is given in the question? – A number which is not a perfect square.
- What are you trying to find? – The smallest number by which 1620 must be divided to get a perfect square.

Plan a strategy

- You have already learnt prime factorisation. Use it to find the product of prime factors of 1620.
- Pair the prime factors to see if any factor is left unpaired.
- This unpaired factor will be the smallest number that must be divided to get a perfect square.

Solve

Prime factorisation of 1620 is

2	1620
2	810
5	405
3	81
3	27
3	9
3	3
	1

The product of prime factors = $2 \times 2 \times 5 \times 3 \times 3 \times 3 \times 3$

Pair these prime factors = $2 \times 2 \times 5 \times 3 \times 3 \times 3 \times 3$

The factor 5 is left unpaired.

Hence, the required smallest number is 5.

Revise

Divide 1620 by 5 and check if it is a perfect square.

$$1620 \div 5 = 324$$

We see that 324 is a perfect square, hence our answer is verified.

Think and Discuss

1. Find the square root of the number obtained in step IV.
2. Can you find the smallest number that can be multiplied to 1620 to get a perfect square?
3. Find the square root.

(C) Exercise

In each of the questions, 1 to 24, write the correct answer from the given four options.

1. 196 is the square of
(a) 11 (b) 12 (c) 14 (d) 16
2. Which of the following is a square of an even number?
(a) 144 (b) 169 (c) 441 (d) 625
3. A number ending in 9 will have the units place of its square as
(a) 3 (b) 9 (c) 1 (d) 6

Magic Squares

A magic square is a square with numbers arranged so that the sum of the numbers in each row, column and diagonal is the same.

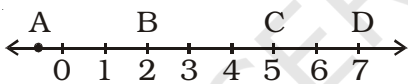
Complete each magic square below.

$\sqrt{36}$	$\square$	2^2
8^0	$\sqrt{9}$	$\square$
$\square$	$3^2 - 2$	$\square$

$\square$	$-(\sqrt{4} + 4)$	$-(9^2)$
$-(\sqrt{16})$	$\square$	0^3
$-(\sqrt{9})$	$2^0 + 1$	$\square$

Use the numbers $-4, -3, -2, -1, 0, 1, 2, 3$ and 4 to make a magic square with row, column and diagonal sums of 0.

2^3	3^1	2^2	$8 + 3 + 4 = 15$
8^0	5^1	3^2	$1 + 5 + 9 = 15$
6^1	7^1	2^1	$6 + 7 + 2 = 15$
$8 + 1 + 6 = 15$	$3 + 5 + 7 = 15$	$4 + 9 + 2 = 15$	
			$6 + 5 + 4 = 15$
			$8 + 5 + 2 = 15$

4. Which of the following will have 4 at the units place?
 (a) 14^2 (b) 62^2 (c) 27^2 (d) 35^2
5. How many natural numbers lie between 5^2 and 6^2 ?
 (a) 9 (b) 10 (c) 11 (d) 12
6. Which of the following cannot be a perfect square?
 (a) 841 (b) 529 (c) 198
 (d) All of the above
7. The one's digit of the cube of 23 is
 (a) 6 (b) 7 (c) 3 (d) 9
8. A square board has an area of 144 square units. How long is each side of the board?
 (a) 11 units (b) 12 units (c) 13 units (d) 14 units
9. Which letter best represents the location of $\sqrt{25}$ on a number line?
 (a) A (b) B (c) C (d) D
- 
10. If one member of a pythagorean triplet is $2m$, then the other two members are
 (a) m, m^2+1
 (b) m^2+1, m^2-1
 (c) m^2, m^2-1
 (d) $m^2, m+1$
11. The sum of successive odd numbers 1, 3, 5, 7, 9, 11, 13 and 15 is
 (a) 81 (b) 64 (c) 49 (d) 36
12. The sum of first n odd natural numbers is
 (a) $2n+1$ (b) n^2 (c) n^2-1 (d) n^2+1
13. Which of the following numbers is a perfect cube?
 (a) 243 (b) 216 (c) 392 (d) 8640
14. The hypotenuse of a right triangle with its legs of lengths $3x \times 4x$ is
 (a) $5x$ (b) $7x$ (c) $16x$ (d) $25x$
15. The next two numbers in the number pattern 1, 4, 9, 16, 25 ... are
 (a) 35, 48 (b) 36, 49 (c) 36, 48 (d) 35, 49

- 16.** Which among 43^2 , 67^2 , 52^2 , 59^2 would end with digit 1?
 (a) 43^2 (b) 67^2 (c) 52^2 (d) 59^2
- 17.** A perfect square can never have the following digit in its ones place.
 (a) 1 (b) 8 (c) 0 (d) 6
- 18.** Which of the following numbers is not a perfect cube?
 (a) 216 (b) 567 (c) 125 (d) 343
- 19.** $\sqrt[3]{1000}$ is equal to
 (a) 10 (b) 100 (c) 1
 (d) None of these
- 20.** If m is the square of a natural number n , then n is
 (a) the square of m
 (b) greater than m
 (c) equal to m
 (d) $\sqrt{m}$
- 21.** A perfect square number having n digits where n is even will have square root with
 (a) $n+1$ digit (b) $\frac{n}{2}$ digit (c) $\frac{n}{3}$ digit (d) $\frac{n+1}{2}$ digit
- 22.** If m is the cube root of n , then n is
 (a) m^3 (b) $\sqrt{m}$ (c) $\frac{m}{3}$ (d) $\sqrt[3]{m}$
- 23.** The value of $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$ is
 (a) 14 (b) 12 (c) 16 (d) 13
- 24.** Given that $\sqrt{4096} = 64$, the value of $\sqrt{4096} + \sqrt{40.96}$ is
 (a) 74 (b) 60.4 (c) 64.4 (d) 70.4

In questions 25 to 48, fill in the blanks to make the statements true.

- 25.** There are _____ perfect squares between 1 and 100.
- 26.** There are _____ perfect cubes between 1 and 1000.
- 27.** The units digit in the square of 1294 is _____.

28. The square of 500 will have _____ zeroes.
29. There are _____ natural numbers between n^2 and $(n + 1)^2$
30. The square root of 24025 will have _____ digits.
31. The square of 5.5 is _____.
32. The square root of 5.3×5.3 is _____.
33. The cube of 100 will have _____ zeroes.
34. $1\text{m}^2 =$ _____ cm^2 .
35. $1\text{m}^3 =$ _____ cm^3 .
36. Ones digit in the cube of 38 is _____.
37. The square of 0.7 is _____.
38. The sum of first six odd natural numbers is _____.
39. The digit at the ones place of 57^2 is _____.
40. The sides of a right triangle whose hypotenuse is 17cm are _____ and _____.
41. $\sqrt{1.96} =$ _____.
42. $(1.2)^3 =$ _____.
43. The cube of an odd number is always an _____ number.
44. The cube root of a number x is denoted by _____.
45. The least number by which 125 be multiplied to make it a perfect square is _____.
46. The least number by which 72 be multiplied to make it a perfect cube is _____.
47. The least number by which 72 be divided to make it a perfect cube is _____.
48. Cube of a number ending in 7 will end in the digit _____.

In questions 49 to 86, state whether the statements are true (T) or false (F).

49. The square of 86 will have 6 at the units place.
50. The sum of two perfect squares is a perfect square.
51. The product of two perfect squares is a perfect square.
52. There is no square number between 50 and 60.

53. The square root of 1521 is 31.
54. Each prime factor appears 3 times in its cube.
55. The square of 2.8 is 78.4.
56. The cube of 0.4 is 0.064.
57. The square root of 0.9 is 0.3.
58. The square of every natural number is always greater than the number itself.
59. The cube root of 8000 is 200.
60. There are five perfect cubes between 1 and 100.
61. There are 200 natural numbers between 100^2 and 101^2 .
62. The sum of first n odd natural numbers is n^2 .
63. 1000 is a perfect square.
64. A perfect square can have 8 as its units digit.
65. For every natural number m , $(2m-1, 2m^2-2m, 2m^2-2m+1)$ is a pythagorean triplet.
66. All numbers of a pythagorean triplet are odd.
67. For an integer a , a^3 is always greater than a^2 .
68. If x and y are integers such that $x^2 > y^2$, then $x^3 > y^3$.
69. Let x and y be natural numbers. If x divides y , then x^3 divides y^3 .
70. If a^2 ends in 5, then a^3 ends in 25.
71. If a^2 ends in 9, then a^3 ends in 7.
72. The square root of a perfect square of n digits will have $\left(\frac{n+1}{2}\right)$ digits, if n is odd.
73. Square root of a number x is denoted by $\sqrt{x}$.
74. A number having 7 at its ones place will have 3 at the units place of its square.

What's the Error? A student said that since the square roots of a certain number are 1.5 and -1.5, the number must be their product, -2.25. What error did the student make?

75. A number having 7 at its ones place will have 3 at the ones place of its cube.
76. The cube of a one digit number cannot be a two digit number.
77. Cube of an even number is odd.
78. Cube of an odd number is even.
79. Cube of an even number is even.
80. Cube of an odd number is odd.
81. 999 is a perfect cube.
82. 363×81 is a perfect cube.
83. Cube roots of 8 are +2 and -2.
84. $\sqrt[3]{8+27} = \sqrt[3]{8} + \sqrt[3]{27}$.
85. There is no cube root of a negative integer.
86. Square of a number is positive, so the cube of that number will also be positive.

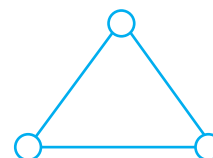
Solve the following questions.

87. Write the first five square numbers.
88. Write cubes of first three multiples of 3.
89. Show that 500 is not a perfect square.
90. Express 81 as the sum of first nine consecutive odd numbers.
91. Using prime factorisation, find which of the following are perfect squares.
 (a) 484 (b) 11250 (c) 841 (d) 729
92. Using prime factorisation, find which of the following are perfect cubes.
 (a) 128 (b) 343 (c) 729 (d) 1331
93. Using distributive law, find the squares of
 (a) 101 (b) 72
94. Can a right triangle with sides 6cm, 10cm and 8cm be formed? Give reason.
95. Write the Pythagorean triplet whose one of the numbers is 4.

- 96.** Using prime factorisation, find the square roots of
(a) 11025 (b) 4761
- 97.** Using prime factorisation, find the cube roots of
(a) 512 (b) 2197
- 98.** Is 176 a perfect square? If not, find the smallest number by which it should be multiplied to get a perfect square.
- 99.** Is 9720 a perfect cube? If not, find the smallest number by which it should be divided to get a perfect cube.
- 100.** Write two Pythagorean triplets each having one of the numbers as 5.
- 101.** By what smallest number should 216 be divided so that the quotient is a perfect square. Also find the square root of the quotient.
- 102.** By what smallest number should 3600 be multiplied so that the quotient is a perfect cube. Also find the cube root of the quotient.
- 103.** Find the square root of the following by long division method.
(a) 1369 (b) 5625
- 104.** Find the square root of the following by long division method.
(a) 27.04 (b) 1.44
- 105.** What is the least number that should be subtracted from 1385 to get a perfect square? Also find the square root of the perfect square.
- 106.** What is the least number that should be added to 6200 to make it a perfect square?
- 107.** Find the least number of four digits that is a perfect square.
- 108.** Find the greatest number of three digits that is a perfect square.
- 109.** Find the least square number which is exactly divisible by 3, 4, 5, 6 and 8.
- 110.** Find the length of the side of a square if the length of its diagonal is 10cm.
- 111.** A decimal number is multiplied by itself. If the product is 51.84, find the number.
- 112.** Find the decimal fraction which when multiplied by itself gives 84.64.

- 113.** A farmer wants to plough his square field of side 150m. How much area will he have to plough?
- 114.** What will be the number of unit squares on each side of a square graph paper if the total number of unit squares is 256?
- 115.** If one side of a cube is 15m in length, find its volume.
- 116.** The dimensions of a rectangular field are 80m and 18m. Find the length of its diagonal.
- 117.** Find the area of a square field if its perimeter is 96m.
- 118.** Find the length of each side of a cube if its volume is 512 cm^3 .
- 119.** Three numbers are in the ratio 1:2:3 and the sum of their cubes is 4500. Find the numbers.
- 120.** How many square metres of carpet will be required for a square room of side 6.5m to be carpeted.
- 121.** Find the side of a square whose area is equal to the area of a rectangle with sides 6.4m and 2.5m.
- 122.** Difference of two perfect cubes is 189. If the cube root of the smaller of the two numbers is 3, find the cube root of the larger number.
- 123.** Find the number of plants in each row if 1024 plants are arranged so that number of plants in a row is the same as the number of rows.
- 124.** A hall has a capacity of 2704 seats. If the number of rows is equal to the number of seats in each row, then find the number of seats in each row.
- 125.** A General wishes to draw up his 7500 soldiers in the form of a square. After arranging, he found out that some of them are left out. How many soldiers were left out?
- 126.** 8649 students were sitting in a lecture room in such a manner that there were as many students in the row as there were rows in the lecture room. How many students were there in each row of the lecture room?
- 127.** Rahul walks 12m north from his house and turns west to walk 35m to reach his friend's house. While returning, he walks diagonally from his friend's house to reach back to his house. What distance did he walk while returning?

- 128.** A 5.5m long ladder is leaned against a wall. The ladder reaches the wall to a height of 4.4m. Find the distance between the wall and the foot of the ladder.
- 129.** A king wanted to reward his advisor, a wise man of the kingdom. So he asked the wiseman to name his own reward. The wiseman thanked the king but said that he would ask only for some gold coins each day for a month. The coins were to be counted out in a pattern of one coin for the first day, 3 coins for the second day, 5 coins for the third day and so on for 30 days. Without making calculations, find how many coins will the advisor get in that month?
- 130.** Find three numbers in the ratio 2:3:5, the sum of whose squares is 608.
- 131.** Find the smallest square number divisible by each one of the numbers 8, 9 and 10.
- 132.** The area of a square plot is $101\frac{1}{400}$ m². Find the length of one side of the plot.
- 133.** Find the square root of 324 by the method of repeated subtraction.
- 134.** Three numbers are in the ratio 2:3:4. The sum of their cubes is 0.334125. Find the numbers.
- 135.** Evaluate : $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$
- 136.** $\left\{ \left(5^2 + (12^2)^{\frac{1}{2}} \right) \right\}^3$
- 137.** $\left\{ \left(6^2 + (8^2)^{\frac{1}{2}} \right) \right\}^3$
- 138.** A perfect square number has four digits, none of which is zero. The digits from left to right have values that are: even, even, odd, even. Find the number.
- 139.** Put three different numbers in the circles so that when you add the numbers at the end of each line you always get a perfect square.



- 140.** The perimeters of two squares are 40 and 96 metres respectively. Find the perimeter of another square equal in area to the sum of the first two squares.
- 141.** A three digit perfect square is such that if it is viewed upside down, the number seen is also a perfect square. What is the number?
(Hint: The digits 1, 0 and 8 stay the same when viewed upside down, whereas 9 becomes 6 and 6 becomes 9.)
- 142.** 13 and 31 is a strange pair of numbers such that their squares 169 and 961 are also mirror images of each other. Can you find two other such pairs?

(D) Applications, Games and Puzzles

1. Quick Tricks.



Let me teach you a trick, Karan.

Alright Geeta.



Pick any 2-digit number ending in 5.

45



Take the tens part of the number and multiply it with its successor.

Tens part of 45 is 4. Its successor is 5, so $4 \times 5 = 20$.



Now prefix this number to the square of 5.

Square of 5 is 25, so number obtained is 2025.





By actual multiplication check if the square of 45 is 2025.

Yes, it is! You are a genius Geeta.



Friends, you can also use the same trick to find the square of any 2-digit number ending in 5.

You all will definitely enjoy it. Now can you find the squares of 25, 75 or 95?



Geeta, can we also try it for any 3-digit number ending in 5?



I think we can if we consider the number at tens and hundreds place together.

Let us try it for 225.



Take the tens part of the number 225 and multiple by it with its successor.

The successor or of 22 is 23 so $22 \times 23 = 506$



Now prefix this number to the square of 5.

So, the number now becomes 50625.





Good. Check it by actual multiplication.

The square is correct.



Friends, you can also try this trick for 425 or 705 or any other 3-digit number ending in 5. You will definitely enjoy it.

Geeta, now let me teach you one trick.



Sure

This trick can help you to find the cube root $\frac{1}{n}$ of any 4, 5 or 6-digit perfect cube orally.



Alright. But what do we have to do for it?

Pick any 4, 5 or 6 $\frac{1}{n}$ digit perfect cube.



91125

From the right put a comma after 3 digits.



91, 125

See the digit at the units place and find the units place of its cube.





Digit at units place is 5 and digit at ones place of its cube is also 5.

Good! This number is the digit at units place of the cube root. Now see the digits before the comma.



91

Ascertain which number's cube is less than this number.



The cube of 4 is 64 which is less than 91.

Absolutely. This is the digit at the tens place of the cube root.



So it means that the cube root is 45.

You are right.



Fascinating!

Friends, you can also try the same for 13824, 2197, 50653 or any other perfect cube of 4, 5 or 6 digit.



Bye for now!



2. Cross Number Puzzle**Down**

1. Cube of 9.
2. Missing number to make 12, ____, 37, a pythagorean triplet.
4. Smallest number by which 248 be multiplied to make the resultant a perfect cube number.
5. Square of 75.
6. Smallest square number that is divisible by each of 5 and 11
9. Without adding, find the sum of $1 + 3 + 5 + 7 + 9 + 11$.
10. Smallest number which when added to 7669 makes the resultant a perfect square.

Across

2. Square of 19.
3. Look at the numbers given below and find the number which cannot be a perfect square.
81, 100, 144, 25000
7. Square root of 4489
8. Smallest natural number other than 1 which is a perfect square as well as a perfect cube number.
10. Cube root of 357911.
11. Smallest number which when subtracted from 374695 makes the resultant a perfect square number.

1	2			6
3	5			
4	7	10		
			9	
11			8	

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UNIT 4

LINEAR EQUATIONS IN ONE VARIABLE

(A) Main Concepts and Results

- An algebraic equation is an equality involving variables. It has an equality sign. The expression on the left of the equality sign is the Left Hand Side (LHS) and the expression on the right of the equality sign is the Right Hand Side (RHS).
- In an equation the values of the expressions on the LHS and RHS are equal for certain values of the variables. These values are the solutions of the equation.
- Equations where the expressions which form the equation contain only one variable and the highest power of the variable appearing in the equation is 1, are called linear equations in one variable.
- A linear equation may have linear expressions on both sides of the equality sign.
- To find the solution of an equation we perform the same mathematical operations on both sides of the equation, so that the balance between the LHS and RHS is not disturbed.
- A linear equation may have any rational number as its solution.
- In an equation, variables can be transposed from one side of the equation to the other.

(B) Solved Examples

In examples 1 and 2, there are four options given out of which one is correct. Choose the correct answer.

Example 1 : If $x = a$, then which of the following is not always true for an integer k .

(a) $kx = ak$

(b) $\frac{x}{k} = \frac{a}{k}$

(c) $x - k = a - k$

(d) $x + k = a + k$

Solution : Correct answer is (b).

Example 2 : If $3x - 4(64 - x) = 10$, then the value of x is

(a) -266

(b) 133

(c) 66.5

(d) 38

Solution: Correct answer is (d).

In examples 3 and 4, fill in the blanks to make the statements true.

Example 3 : Fifteen added to thrice a whole number gives 93. The number is _____.

Solution : Correct answer is 26.

Example 4 : If $\frac{1}{3} - x = -\frac{2}{3}$, then x is _____.

Solution : Correct answer is 1.

In examples 5 and 6, state whether the given statements are true (T) or false (F).

Example 5 : Three consecutive even numbers whose sum is 156 are 51, 52 and 53.

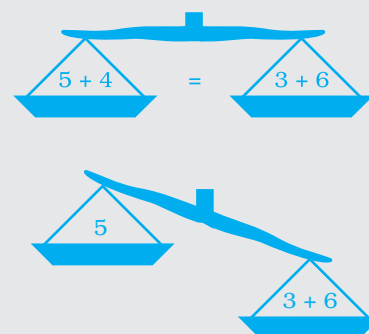
Solution : False.

You Have to Keep Equations Balanced

An equation is like a scale. The bit before the equals sign has the same value as the bit after the equals sign, so the scale is balanced.

When manipulating equations, you have to keep the scale balanced. You can't take 4 from one side and not from the other because then the two sides aren't equal.

The only way to keep the scale balanced is to always do the same thing to both sides.



Example 6 : $x = -12$ is the solution of the linear equation
 $5x - 3(2x + 1) = 21 + x$

Solution : True.

In examples 7 to 10 solve each of them.

Example 7 : Solve : $\frac{x}{2} + \frac{x}{4} + \frac{x}{5} + 10000 = x$

Solution : $\frac{x}{2} + \frac{x}{4} + \frac{x}{5} + 10000 = x$

$$\frac{x}{2} + \frac{x}{4} + \frac{x}{5} - x = -10000$$

$$\frac{10x + 5x + 4x - 20x}{20} = -10000$$

$$\frac{19x - 20x}{20} = -10000$$

$$\frac{-x}{20} = -10000$$

$$x = 200000$$

Example 8 : The present age of father is four times the age of his son.
 After 10 years, age of father will become three times the
 age of his son. Find their present ages.

Solution: Let the present age of son be x years

$\therefore$ the present age of father = $4x$ years

After 10 years

Age of son = $(x + 10)$ years

Age of father = $(4x + 10)$ years

According to the given condition

$$4x + 10 = 3(x + 10)$$

$$4x + 10 = 3x + 30$$

$$4x - 3x = 30 - 10$$

$$x = 20$$

$\therefore$ Present age of son = 20 years.

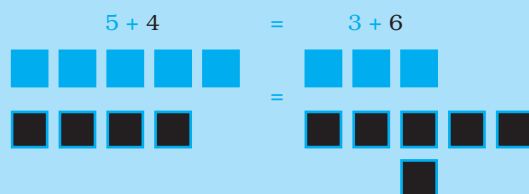
and present age of father = $4x = 4 \times 20 = 80$ years.

Here's another way to show the balance in the equation.

There are a total of 9 boxes on each side of the equals sign – so the equation is balanced.

If you take any 3 boxes from the left-hand side, you also need to take 3 boxes from the right-hand side to keep the equation balanced.

You still have to keep both sides of an equation balanced when there are variables involved.



Both sides still have the same total number of coloured boxes.

Example 9 : A steamer goes downstream from one point to another in 7 hours. It covers the same distance upstream in 8 hours. If the speed of stream be 2 km/hr, find the speed of the steamer in still water and the distance between the ports.

Solution : Let speed of steamer in still water = x km/hr

Speed of stream = 2 km/hr

Speed downstream = $(x + 2)$ km/hr

Speed upstream = $(x - 2)$ km/hr

Distance covered in 7 hours while downstream = $7(x + 2)$

Distance covered in 8 hours while upstream = $8(x - 2)$

According to the condition,

$$7(x + 2) = 8(x - 2)$$

$$7x + 14 = 8x - 16$$

$$x = 30 \text{ km/hr}$$

$$\text{Total Distance} = 7(x + 2) \text{ km}$$

$$= 7(30 + 2) \text{ km}$$

$$= 7 \times 32 \text{ km}$$

$$= 224 \text{ km}$$

Example 10 : Distance between two stations A and B is 690 km. Two cars start simultaneously from A and B towards each other, and the distance between them after 6 hours is 30 km. If the speed of one car is less than the other by 10 km/hr, find the speed of each car.

Solution : Let speed of faster car = x km/hr

Then speed of other = $(x - 10)$ km/hr

Let 1st one start from A and other from B.

M and N be their position after 6 hours.



$$AM = 6x, BN = 6(x - 10)$$

According to condition,

$$6x + 6x - 60 + 30 = 690$$

$$12x = 690 + 30$$

$$12x = 720$$

$$x = 60 \text{ km/hr}$$

Speed of other car = 50 km/hr.

Example 11 : Application on problem solving strategy

A home-owner is installing a fence around the square garden. The garden has a perimeter of 6480 cm. Write and solve the equation to find the garden's dimensions.

Solution : Understand and explore the problem

- What do you know?

Perimeter of square garden = 6480 cm

To find: Side of garden?

Plan a strategy

- To visualise that fencing around a garden means fencing its perimeter.
- Recall that a square has four equal sides, say s each.

Solve

Fence around square garden = Perimeter of square garden

$$s + s + s + s = 6480 \text{ cm}$$

$$4s = 6480 \text{ cm}$$

$$s = 1620 \text{ cm}$$

Thus, side of garden = 1620 cm

Check

Verify your answer by adopting some other plan.

e.g. Here in this problem instead of taking perimeter as sum of its sides, use the formula

$$4s = 6480 \text{ cm}$$

$$s = 1620 \text{ cm} \quad \text{Hence verified.}$$

Think and Discuss

- What other values will be needed if instead of square it is a rectangular or circular garden?
- What will happen if we have to level the grass inside it instead of fencing the garden?
- What will happen if there is a path running inside it?

(C) Exercise

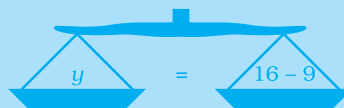
In questions 1 to 15 out of the four options only one is correct, write the correct answer.

- The solution of which of the following equations is neither a fraction nor an integer.
 - $3x + 2 = 5x + 2$
 - $4x - 18 = 2$
 - $4x + 7 = x + 2$
 - $5x - 8 = x + 4$
- The solution of the equation $ax + b = 0$ is
 - $x = \frac{a}{b}$
 - $x = -b$
 - $x = \frac{-b}{a}$
 - $x = \frac{b}{a}$

An equation like $y + 9 = 16$ is balanced just like one with only numbers. To find the value of y , you need to get the variable alone on one side of the equals sign.



If the variable has something added to it, use subtraction to get it on its own. In $y + 9 = 16$, subtract 9 from both sides to get y on its own.



You can do exactly the same without drawing the scales.

$$\begin{array}{l} y + 9 = 16 \\ y + 9 - 9 = 16 - 9 \\ y = 7 \end{array} \quad \begin{array}{l} +9 \text{ and } -9 \\ \text{cancel each other out.} \end{array}$$

You can check that $y = 7$ is the correct solution by substituting it back into the original equation.

$$7 + 9 = 16 \text{ — this is true, so } y = 7 \text{ is correct.}$$

3. If $8x - 3 = 25 + 17x$, then x is
 (a) a fraction (b) an integer
 (c) a rational number (d) cannot be solved
4. The shifting of a number from one side of an equation to other is called
 (a) Transposition (b) Distributivity
 (c) Commutativity (d) Associativity
5. If $\frac{5x}{3} - 4 = \frac{2x}{5}$, then the numerical value of $2x - 7$ is
 (a) $\frac{19}{13}$ (b) $-\frac{13}{19}$ (c) 0 (d) $\frac{13}{19}$
6. The value of x for which the expressions $3x - 4$ and $2x + 1$ become equal is
 (a) -3 (b) 0 (c) 5 (d) 1
7. If a and b are positive integers, then the solution of the equation $ax = b$ has to be always
 (a) positive (b) negative (c) one (d) zero
8. Linear equation in one variable has
 (a) only one variable with any power.
 (b) only one term with a variable.
 (c) only one variable with power 1.
 (d) only constant term.

If a statement is a proportion, the cross-products of the terms are equal.

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

To solve equations like $5t = -20$, you still need to get the variable t , on its own. The variable has been multiplied by a number, 5 — so you can get the variable on its own by dividing both sides by that numbers. In this case, you need to divide by 5.

$$\begin{aligned} 5t &= -20 \\ 5t \div 5 &= -20 \div 5 \\ t &= -20 \div 5 \\ t &= -4 \end{aligned}$$

Divide both sides by the same number by which the variable is multiplied.

9. Which of the following is a linear expression:
 (a) $x^2 + 1$ (b) $y + y^2$ (c) 4 (d) $1 + z$
10. A linear equation in one variable has
 (a) Only one solution
 (b) Two solutions
 (c) More than two solutions
 (d) No solution
11. Value of S in $\frac{1}{3} + S = \frac{2}{5}$
 (a) $\frac{4}{5}$ (b) $\frac{1}{15}$ (c) 10 (d) 0
12. $\frac{-4}{3}y = -\frac{3}{4}$, then $y =$
 (a) $-\left(\frac{3}{4}\right)^2$ (b) $-\left(\frac{4}{3}\right)^2$ (c) $\left(\frac{3}{4}\right)^2$ (d) $\left(\frac{4}{3}\right)^2$
13. The digit in the tens place of a two digit number is 3 more than the digit in the units place. Let the digit at units place be b . Then the number is
 (a) $11b + 30$ (b) $10b + 30$ (c) $11b + 3$ (d) $10b + 3$
14. Arpita's present age is thrice of Shilpa. If Shilpa's age three years ago was x . Then Arpita's present age is
 (a) $3(x - 3)$ (b) $3x + 3$
 (c) $3x - 9$ (d) $3(x + 3)$

A one-step equation is one that can be solved in one step by either adding, subtracting, multiplying or dividing by one thing.

There are four main methods. For example:

- (i) $a + 3 = 4.2$ — solve by **subtracting** 3 from both sides to get $a = 1.2$
- (ii) $s - 7 = 12$ — solve by **adding** 7 to both sides to get $s = 19$
- (iii) $9m = 27$ — solve by **dividing** both sides by 9 to get $m = 3$
- (iv) $d \div 8 = 2$ — solve by **multiplying** both sides by 8 to get $d = 16$

Before you can solve an equation, you must be able to spot what kind of equation you have.

15. The sum of three consecutive multiples of 7 is 357. Find the smallest multiple.

(a) 112

(b) 126

(c) 119

(d) 116

In questions 16 to 32, fill in the blanks to make each statement true.

16. In a linear equation, the _____ power of the variable appearing in the equation is one.

17. The solution of the equation $3x - 4 = 1 - 2x$ is _____.

18. The solution of the equation $2y = 5y - \frac{18}{5}$ is _____.

19. Any value of the variable which makes both sides of an equation equal is known as a _____ of the equation.

20. $9x - \underline{\hspace{2cm}} = -21$ has the solution (-2)

21. Three consecutive numbers whose sum is 12 are _____, _____ and _____.

22. The share of A when Rs 25 are divided between A and B so that A gets Rs. 8 more than B is _____.

23. A term of an equation can be transposed to the other side by changing its _____.

24. On subtracting 8 from x , the result is 2. The value of x is _____.

25. $\frac{x}{5} + 30 = 18$ has the solution as _____.

26. When a number is divided by 8, the result is -3 . The number is _____.

27. 9 is subtracted from the product of p and 4, the result is 11. The value of p is _____.

28. If $\frac{2}{5}x - 2 = 5 - \frac{3}{5}x$, then $x = \underline{\hspace{2cm}}$.

29. After 18 years, Swarnim will be 4 times as old as he is now. His present age is _____.

30. Convert the statement Adding 15 to 4 times x is 39 into an equation _____.



Application on Problem Solving Strategy

A family spent Rs. 52.00 for circus tickets. This cost included a Rs. 3.25 service fee for the order, with the cost of the circus tickets being Rs. 9.75 each. How many tickets did the family buy? Justify your answer.



Understand the problem

The answer is the number of tickets that family bought. List the important information– The service fee is Rs. 3.25 per order, the tickets cost Rs. 9.75 each, and the total cost is Rs. 52.

Let t represent the number of tickets bought.

$$\begin{array}{rclcl} \text{Total cost} & = & \text{Tickets} & + & \text{Service Fee} \\ 52.00 & = & 9.75t & + & 3.25 \end{array}$$



Make a Plan

Think: First the variable is multiplied by 9.75, and then 3.25 is added to the result. Work backward to solve the equation. Undo the operations in reverse order. First subtract 3.25 from both sides of the equation and then divide sides of the new equation by 9.75.



Solve

$$\begin{array}{rclcl} 52.00 & = & 9.75t + 3.25 & & \\ -3.25 & & -3.25 & \text{Subtract 3.25 from both sides.} & \\ 48.75 & = & 9.75t & & \\ \frac{48.75}{9.75} = \frac{9.75t}{9.75} & & & \text{Divide both sides by 9.75.} & \\ 5 = t & & & \text{The family bought 5 tickets.} & \end{array}$$



Look Back

You can use a table to decide whether your answer is reasonable. Five tickets is a reasonable answer.

Tickets	Cost of Tickets	Service charge	Total cost
1	Rs. 9.75	Rs. 3.25	Rs. 13.00
2	Rs. 19.50	Rs. 3.25	Rs. 22.75
3	Rs. 29.25	Rs. 3.25	Rs. 32.50
4	Rs. 39.00	Rs. 3.25	Rs. 42.25
5	Rs. 48.75	Rs. 3.25	Rs. 52.00

Sometimes, a two-step equation contains a term or an expression with a denominator. In these cases, it is often easier to first multiply both sides of the equation by the denominator in order to remove it, and then work to isolate the variable.

- 31.** The denominator of a rational number is greater than the numerator by 10. If the numerator is increased by 1 and the denominator is decreased by 1, then expression for new denominator is _____.
- 32.** The sum of two consecutive multiples of 10 is 210. The smaller multiple is _____.

In questions 33 to 48, state whether the statements are true (T) or false (F).

- 33.** 3 years ago, the age of a boy was y years. His age 2 years ago was $(y - 2)$ years.
- 34.** Shikha's present age is p years. Reemu's present age is 4 times the present age of Shikha. After 5 years Reemu's age will be $15p$ years.
- 35.** In a 2 digit number, the units place digit is x . If the sum of digits be 9, then the number is $(10x - 9)$.
- 36.** Sum of the ages of Anju and her mother is 65 years. If Anju's present age is y years then her mother's age before 5 years is $(60 - y)$ years.
- 37.** The number of boys and girls in a class are in the ratio 5:4. If the number of boys is 9 more than the number of girls, then number of boys is 9.
- 38.** A and B are together 90 years old. Five years ago A was thrice as old as B was. Hence, the ages of A and B five years back would be $(x - 5)$ years and $(85 - x)$ years respectively.
- 39.** Two different equations can never have the same answer.
- 40.** In the equation $3x - 3 = 9$, transposing -3 to RHS, we get $3x = 9$.
- 41.** In the equation $2x = 4 - x$, transposing $-x$ to LHS, we get $x = 4$.
- 42.** If $\frac{15}{8} - 7x = 9$, then $-7x = 9 + \frac{15}{8}$
- 43.** If $\frac{x}{3} + 1 = \frac{7}{15}$, then $\frac{x}{3} = \frac{6}{15}$
- 44.** If $6x = 18$, then $18x = 54$
- 45.** If $\frac{x}{11} = 15$, then $x = \frac{11}{15}$
- 46.** If x is an even number, then the next even number is $2(x + 1)$.

- 47.** If the sum of two consecutive numbers is 93 and one of them is x , then the other number is $93 - x$.
- 48.** Two numbers differ by 40, when each number is increased by 8, the bigger becomes thrice the lesser number. If one number is x , then the other number is $(40 - x)$.

Solve the following:

49. $\frac{3x-8}{2x} = 1$

50. $\frac{5x}{2x-1} = 2$

51. $\frac{2x-3}{4x+5} = \frac{1}{3}$

52. $\frac{8}{x} = \frac{5}{x-1}$

53. $\frac{5(1-x)+3(1+x)}{1-2x} = 8$

54. $\frac{0.2x+5}{3.5x-3} = \frac{2}{5}$

55. $\frac{y-(4-3y)}{2y-(3+4y)} = \frac{1}{5}$

56. $\frac{x}{5} = \frac{x-1}{6}$

57. $0.4(3x-1) = 0.5x + 1$

58. $8x - 7 - 3x = 6x - 2x - 3$

59. $10x - 5 - 7x = 5x + 15 - 8$

60. $4t - 3 - (3t + 1) = 5t - 4$

61. $5(x - 1) - 2(x + 8) = 0$

62. $\frac{x}{2} - \frac{1}{4}\left(x - \frac{1}{3}\right) = \frac{1}{6}(x + 1) + \frac{1}{12}$

$$63. \frac{1}{2}(x+1) + \frac{1}{3}(x-1) = \frac{5}{12}(x-2)$$

$$64. \frac{x+1}{4} = \frac{x-2}{3}$$

$$65. \frac{2x-1}{5} = \frac{3x+1}{3}$$

$$66. 1 - (x-2) - [(x-3) - (x-1)] = 0$$

$$67. 3x - \frac{x-2}{3} = 4 - \frac{x-1}{4}$$

$$68. \frac{3t+5}{4} - 1 = \frac{4t-3}{5}$$

$$69. \frac{2y-3}{4} - \frac{3y-5}{2} = y + \frac{3}{4}$$

$$70. 0.25(4x-5) = 0.75x + 8$$

$$71. \frac{9-3y}{1-9y} = \frac{8}{5}$$

$$72. \frac{3x+2}{2x-3} = -\frac{3}{4}$$

$$73. \frac{5x+1}{2x} = -\frac{1}{3}$$

$$74. \frac{3t-2}{3} + \frac{2t+3}{2} = t + \frac{7}{6}$$

$$75. m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$$

$$76. 4(3p+2) - 5(6p-1) = 2(p-8) - 6(7p-4)$$

$$77. 3(5x-7) + 2(9x-11) = 4(8x-7) - 111$$

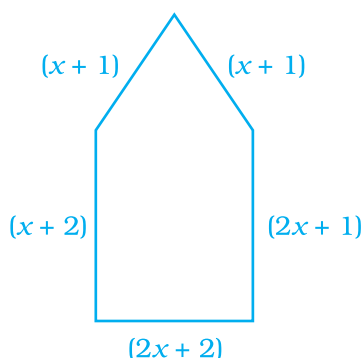
$$78. 0.16(5x-2) = 0.4x + 7$$

79. Radha takes some flowers in a basket and visits three temples one by one. At each temple, she offers one half of the flowers from the basket. If she is left with 3 flowers at the end, find the number of flowers she had in the beginning.

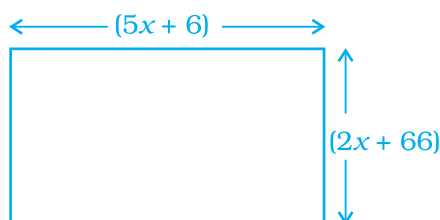
80. Rs. 13500 are to be distributed among Salma, Kiran and Jenifer in such a way that Salma gets Rs. 1000 more than Kiran and Jenifer gets Rs. 500 more than Kiran. Find the money received by Jenifer.
81. The volume of water in a tank is twice of that in the other. If we draw out 25 litres from the first and add it to the other, the volumes of the water in each tank will be the same. Find the volume of water in each tank.
82. Anushka and Aarushi are friends. They have equal amount of money in their pockets. Anushka gave $\frac{1}{3}$ of her money to Aarushi as her birthday gift. Then Aarushi gave a party at a restaurant and cleared the bill by paying half of the total money with her. If the remaining money in Aarushi's pocket is Rs.1600, find the sum gifted by Anushka.
83. Kaustubh had 60 flowers. He offered some flowers in a temple and found that the ratio of the number of remaining flowers to that of flowers in the beginning is 3:5. Find the number of flowers offered by him in the temple.
84. The sum of three consecutive even natural numbers is 48. Find the greatest of these numbers.
85. The sum of three consecutive odd natural numbers is 69. Find the prime number out of these numbers.
86. The sum of three consecutive numbers is 156. Find the number which is a multiple of 13 out of these numbers.
87. Find a number whose fifth part increased by 30 is equal to its fourth part decreased by 30.
88. Divide 54 into two parts such that one part is $\frac{2}{7}$ of the other.
89. Sum of the digits of a two-digit number is 11. The given number is less than the number obtained by interchanging the digits by 9. Find the number.
90. Two equal sides of a triangle are each 4m less than three times the third side. Find the dimensions of the triangle, if its perimeter is 55m.

91. After 12 years, Kanwar shall be 3 times as old as he was 4 years ago. Find his present age.
92. Anima left one-half of her property to her daughter, one-third to her son and donated the rest to an educational institute. If the donation was worth Rs. 1,00,000, how much money did Anima have?
93. If $\frac{1}{2}$ is subtracted from a number and the difference is multiplied by 4, the result is 5. What is the number?
94. The sum of four consecutive integers is 266. What are the integers?
95. Hamid has three boxes of different fruits. Box A weighs $2\frac{1}{2}$ kg more than Box B and Box C weighs $10\frac{1}{4}$ kg more than Box B. The total weight of the three boxes is $48\frac{3}{4}$ kg. How many kilograms (kg) does Box A weigh?
96. The perimeter of a rectangle is 240 cm. If its length is increased by 10% and its breadth is decreased by 20%, we get the same perimeter. Find the length and breadth of the rectangle.
97. The age of A is five years more than that of B. 5 years ago, the ratio of their ages was 3:2. Find their present ages.
98. If numerator is 2 less than denominator of a rational number and when 1 is subtracted from numerator and denominator both, the rational number in its simplest form is $\frac{1}{2}$. What is the rational number?
99. In a two digit number, digit in units place is twice the digit in tens place. If 27 is added to it, digits are reversed. Find the number.
100. A man was engaged as typist for the month of February in 2009. He was paid Rs. 500 per day but Rs. 100 per day were deducted for the days he remained absent. He received Rs. 9,100 as salary for the month. For how many days did he work?
101. A steamer goes downstream and covers the distance between two ports in 3 hours. It covers the same distance in 5 hours when it goes upstream. If the stream flows at 3 km/hr, then find what is the speed of the steamer upstream?

- 102.** A lady went to a bank with Rs. 1,00,000. She asked the cashier to give her Rs. 500 and Rs. 1,000 currency notes in return. She got 175 currency notes in all. Find the number of each kind of currency notes.
- 103.** There are 40 passengers in a bus, some with Rs. 3 tickets and remaining with Rs.10 tickets. The total collection from these passengers is Rs. 295. Find how many passengers have tickets worth Rs. 3?
- 104.** Denominator of a number is 4 less than its numerator. If 6 is added to the numerator it becomes thrice the denominator. Find the fraction.
- 105.** An employee works in a company on a contract of 30 days on the condition that he will receive Rs. 120 for each day he works and he will be fined Rs. 10 for each day he is absent. If he receives Rs. 2300 in all, for how many days did he remain absent?
- 106.** Kusum buys some chocolates at the rate of Rs. 10 per chocolate. She also buys an equal number of candies at the rate of Rs. 5 per candy. She makes a 20% profit on chocolates and 8% profit on candies. At the end of the day, all chocolates and candies are sold out and her profit is Rs. 240. Find the number of chocolates purchased.
- 107.** A steamer goes downstream and covers the distance between two ports in 5 hours while it covers the same distance upstream in 6 hours. If the speed of the stream is 1 km/hr, find the speed of the steamer in still water.
- 108.** Distance between two places A and B is 210 km. Two cars start simultaneously from A and B in opposite direction and distance between them after 3 hours is 54 km. If speed of one car is less than that of other by 8 km/hr, find the speed of each.
- 109.** A carpenter charged Rs. 2500 for making a bed. The cost of materials used is Rs. 1100 and the labour charges are Rs. 200/hr. For how many hours did the carpenter work?
- 110.** For what value of x is the perimeter of shape 77 cm?



111. For what value of x is the perimeter of shape 186 cm?



112. On dividing Rs. 200 between A and B such that twice of A's share is less than 3 times B's share by 200, B's share is?
113. Madhulika thought of a number, doubled it and added 20 to it. On dividing the resulting number by 25, she gets 4. What is the number?

(D) Applications, Games and Puzzles

1. Ranika wanted her friend Radhika's mobile number. But Radhika played a trick. She gave her the number as

9 X Y Z P 1 Q 2 R 3

and told her to decode it with the help of following equations :

(a) $16 - 35 = 7 - 8$

(b) $\frac{6Y - 7}{3Y + 9} = \frac{1}{3}$

(c) $\frac{Z^2 - 9}{5 + Z^2} = \frac{-5}{9}$

(d) $P + \frac{3}{10}P = \frac{13}{10}$

(e) $4(Q + 4) = 5(Q + 2)$

(f) $3(R + 10) + 200 = 236$

2. Determine the missing value in the puzzle below :

$$\begin{array}{rcl} \diamond & \star & = 8 \\ \diamond & \diamond & \star = 10 \\ \diamond & \star & \star & \star & \star = ? \end{array}$$

3. Game : Who will be the Lakhpati ???

Rohit and Saurabh are playing a game. The one who solves the following equations will be a winner. Find out if you were at their place would you have been be a winner. Till what money did you reach successfully?

Rules of the game

- (a) You can only move to the next problem if the previous answer is correct.
- (b) Winning amount slab

Question Number	Amount Won
1	Rs. 1,000
2	Rs. 2,000
3	Rs. 3,000
4	Rs. 4,000
5	Rs. 10,000
6	Rs. 12,000
7	Rs. 14,000
8	Rs. 20,000
9	Rs. 40,000
10	Rs. 1,00,000

- (c) Problems:

(i) $\frac{x+1}{2x+7} = \frac{3}{8}$

(ii) $\frac{1}{(x-1)} + \frac{2}{(x+1)} = 2$

(iii) $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$

(iv) $3m = 7m - \frac{8}{7}$

(v) $-x = \frac{-6}{5}(x-10)$

$$(vi) \quad 5x + \frac{7}{2} = \frac{3}{2}x - 14$$

$$(vii) \quad \frac{x}{3} + 1 = \frac{8}{15}$$

$$(viii) \quad \frac{x}{2} + \frac{3x}{4} - \frac{5x}{6} = 2$$

$$(ix) \quad \frac{50}{x} + 4 = 14$$

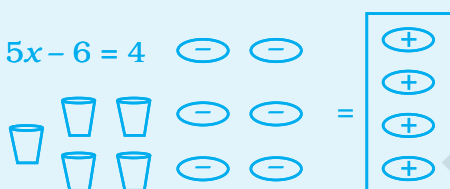
$$(x) \quad x + \frac{2}{3}x + \frac{x}{7} = 97 - \frac{x}{2}$$

4. Work with a partner.

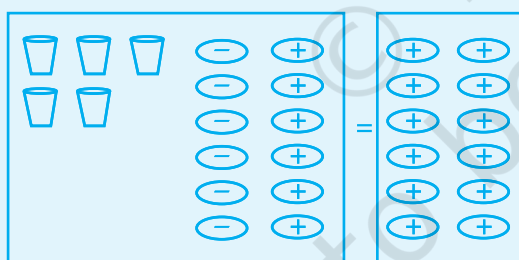
Modelling the Equation

Material Required : Glasses and containers, Equation mat

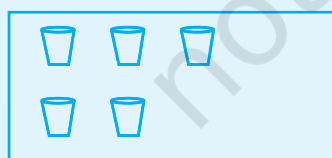
$$5x - 6 = 4$$



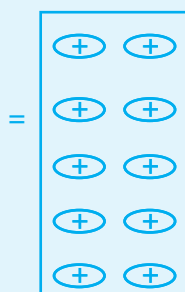
Symbols	
	x
	-1
	$+1$



$$5x - 6 + 6 = 4 + 6$$



$$5x = 10$$



Add 6 positive containers to each side of the equal and then remove the zero pairs.

Arrange the glasses and containers into five equal groups.

Each Cup is matched with 2 positive contents. So, $x = 2$.

Model the following equations:

(i) $3x - 3 = 12$

(ii) $12x + 4 = 24$

(iii) $7y + 14 = 7$

5. Cross word puzzle.

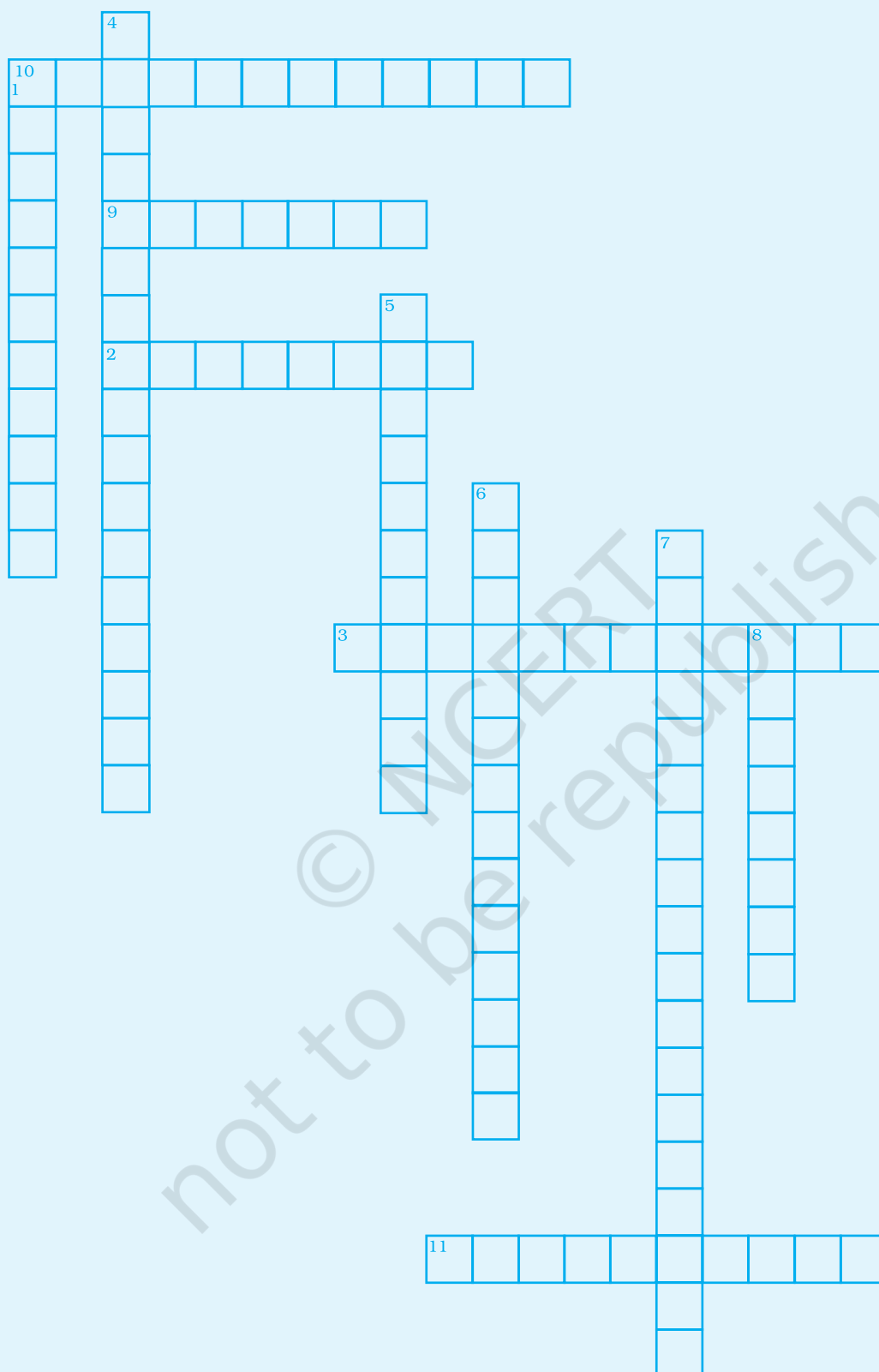
Solve the given crossword and then fill up the given boxes. Clues are given below for across as well as downward filling. Also, for across and down clues, clue number is written at the corner of the boxes. Answers of clues have to be filled up in their respective boxes.

Down

1. Inverse of addition.
4. A symbolic form made up of constants, variables and operation (other than algebraic expressions).
5. If a term of an expression consists of a number multiplied by one or more variables, this number is the _____ of the term.
6. Inverse of division.
7. Equations that have the same solution.
8. An _____ in an equality which is true for all values of the variable in the equality.

Across

2. A statement formed when an equal sign is placed between two expressions.
3. $2(x+5) = 2x+10$. This is an example of what property.
9. Branch of mathematics concerned with operation by symbolic numbers.
10. A linear equation of the form $Ax + By = C$ when A and B both are not zero is in the _____.
11. An expression is _____ if it has no grouping symbols and all the like terms have been combined.



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UNIT 5

UNDERSTANDING QUADRILATERALS AND PRACTICAL GEOMETRY

(A) Main Concepts and Results

- A simple closed curve made up of only line segments is called a **polygon**.
- A diagonal of a polygon is a line segment connecting two non-consecutive vertices.
- A convex polygon is a polygon in which no portion of its any diagonal is in its exterior.
- A **quadrilateral** is a polygon having only four sides.
- A **regular polygon** is a polygon whose all sides are equal and also all angles are equal.
- The sum of interior angles of a polygon of n sides is $(n-2)$ straight angles.
- The sum of interior angles of a quadrilateral is 360° .
- The sum of exterior angles, taken in an order, of a polygon is 360° .
- **Trapezium** is a quadrilateral in which a pair of opposite sides is parallel.
- **Kite** is a quadrilateral which has two pairs of equal consecutive sides.
- A **parallelogram** is a quadrilateral in which each pair of opposite sides is parallel.

- A **rhombus** is a parallelogram in which adjacent sides are equal.
- A **rectangle** is a parallelogram in which one angle is of 90° .
- A **square** is a parallelogram in which adjacent sides are equal and one angle is of 90° .
- In a parallelogram, opposite sides are equal, opposite angles are equal and diagonals bisect each other.
- In a rhombus diagonals intersect at right angles.
- In a rectangle diagonals are equal.
- Five measurements can determine a quadrilateral uniquely.
- A quadrilateral can be constructed uniquely if the lengths of its four sides and a diagonal are given.
- A quadrilateral can be constructed uniquely if the lengths of its three sides and two diagonals are given.
- A quadrilateral can be constructed uniquely if its two adjacent sides and three angles are given.
- A quadrilateral can be constructed uniquely if its three sides and two included angles are given.

(B) Solved Examples

In examples 1 to 8, there are four options out of which one is correct. Write the correct answer.

Example 1 : The number of diagonals in a polygon of n sides is

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n-2)}{2}$ (c) $\frac{n(n-3)}{2}$ (d) $n(n-3)$.

Solution : The correct answer is (c).

Example 2 : The angles of a quadrilateral ABCD taken in an order are in the ratio 3 : 7 : 6 : 4. Then ABCD is a

- (a) kite (b) parallelogram
(c) rhombus (d) trapezium

Solution : The correct answer is (d).

Example 3 : If the diagonals of a quadrilateral bisect each other at right angles, it will be a

- (a) rhombus (b) trapezium
(c) rectangle (d) kite

Solution : The correct answer is (a).

Example 4 : The sum of the angles of a quadrilateral is

- (a) 180° (b) 270° (c) 360° (d) 300°

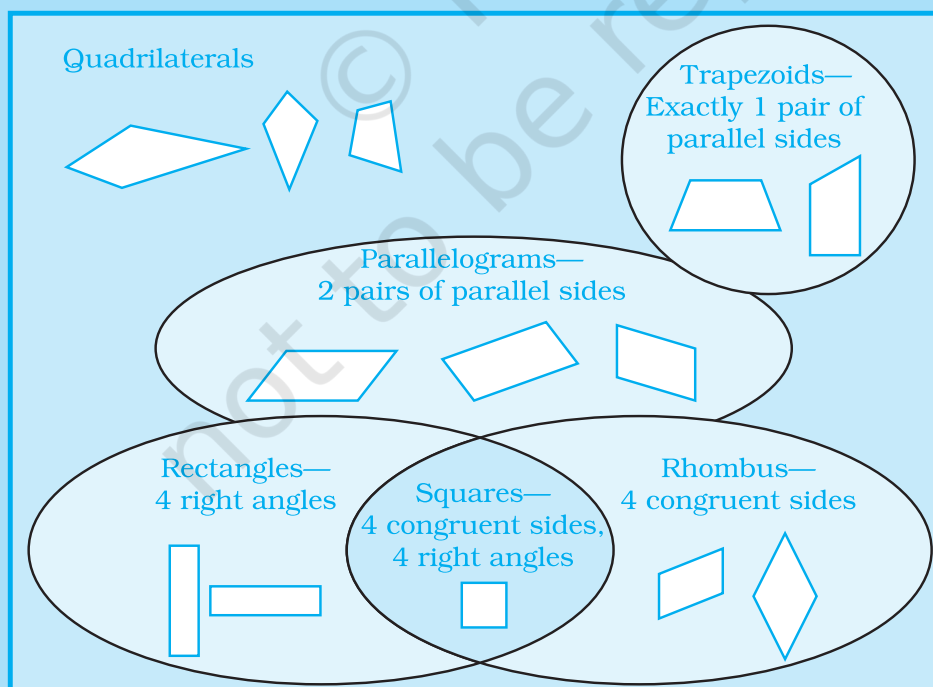
Solution : The correct answer is (c).

Example 5 : In a square ABCD, the diagonals meet at point O. The $\triangle AOB$ is

- (a) isosceles right triangle
(b) equilateral triangle
(c) isosceles triangle but not right triangle
(d) scalene right triangle.

Solution : The correct answer is (a).

Quadrilaterals with certain properties are given additional names. A trapezium has exactly 1 pair of parallel sides. A parallelogram has 2 pairs of parallel sides. A rectangle has 4 right angles. A rhombus has 4 congruent sides. A square has 4 congruent sides and 4 right angles.



Example 6 : ABCD is a quadrilateral in which $AB = 5$ cm, $CD = 8$ cm and the sum of angle A and angle D is 180° . What is the name of this quadrilateral?

- (a) Parallelogram (b) Trapezium
(c) Rhombus (d) Can not be determined

Solution : The correct answer is (b).

Example 7 : Rukmini has a farm land which is triangular in shape. What is the sum of all the exterior angles taken in an order of the farm land?

- (a) 90° (b) 180° (c) 360°
(d) Can not be determined.

Solution : The correct answer is (c).

Example 8 : How many sides does an octagon have?

- (A) 7 (b) 8 (c) 9 (d) 10

Solution : The correct answer is (b)

In examples 9 and 13, fill in the blanks to make the statements true.

Example 9 : The diagonals of a rhombus bisect each other at _____ angles.

Solution : Right.

Example 10 : For getting diagonals through vertex A of a pentagon ABCDE, A is joined to _____.

Solution : C and D.

Example 11 : For constructing a unique quadrilateral at least _____ measurements are required.

Solution : Five.

Example 12 : If diagonals of a quadrilateral bisect at right angles it is a _____.

Solution : Rhombus (or square).

Example 13 : The diagonals of a _____ intersect at right angles.

Solution : Kite.

In examples 14 to 23, state whether the statements are true (T) or false (F).

Example 14 : Every rectangle is a parallelogram.

Solution : True.

Example 15 : Every rhombus is a kite.

Solution : True.

Example 16 : Every parallelogram is a trapezium.

Solution : True.

Example 17 : Every kite is a trapezium.

Solution : False.

Example 18 : Every kite is a parallelogram.

Solution : False.

Example 19 : Diagonals of a rectangle are perpendicular to each other.

Solution : False.

Example 20 : For constructing a unique parallelogram lengths of only two sides should be given.

Solution : False.

Diagonals of a —		
Parallelogram	→	bisect each other
Rhombus	→	bisect each other are perpendicular to each other
Rectangle	→	are equal bisect each other
Square	→	bisect each other are perpendicular to each other are equal

Example 21 :  is a simple closed curve.

Solution : False.

Example 22 :  is a concave polygon.

Solution : True.

Example 23 : A triangle is not a polygon.

Solution : False.

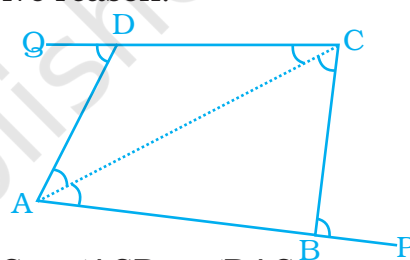
Example 24 : The sides AB and CD of a quadrilateral ABCD are extended to points P and Q respectively. Is $\angle ADQ + \angle CBP = \angle A + \angle C$? Give reason.

Solution : Join AC, then

$$\angle CBP = \angle BCA + \angle BAC \text{ and}$$

$$\angle ADQ = \angle ACD + \angle DAC$$

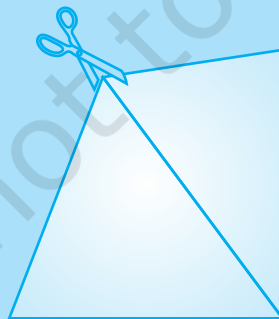
(Exterior angles of triangles)



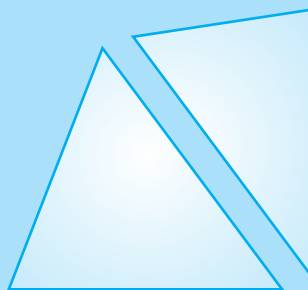
$$\begin{aligned} \text{Therefore, } \angle CBP + \angle ADQ &= \angle BCA + \angle BAC + \angle ACD + \angle DAC \\ &= (\angle BCA + \angle ACD) + (\angle BAC + \angle DAC) \\ &= \angle C + \angle A \end{aligned}$$

Angles in a Quadrilateral

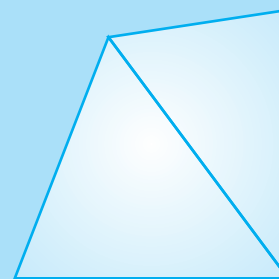
A diagonal of a quadrilateral is a segment that joins two vertices of the quadrilateral but is not a side. You can use a diagonal of a quadrilateral to show that the sum of the angle measures in a quadrilateral is 360° .



Cut a quadrilateral along a diagonal to form two triangles.



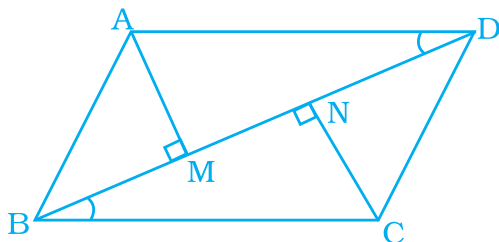
The sum of the angle measures in each triangle is 180° .



Quadrilateral with 2 pairs of parallel sides.

Example 25 : If AM and CN are perpendiculars on the diagonal BD of a parallelogram ABCD, Is $\triangle AMD \cong \triangle CNB$? Give reason.

Solution :



In triangles AMD and CNB,

$AD = BC$ (opposite sides of parallelogram)

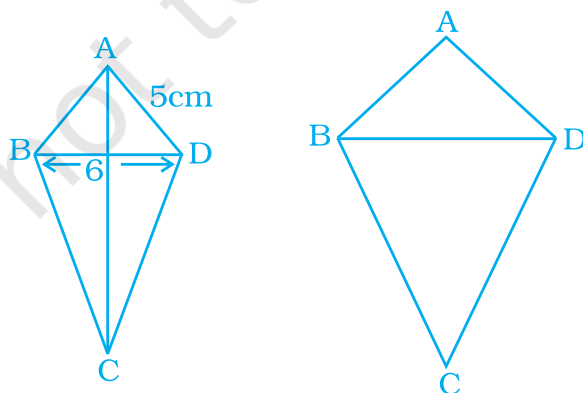
$\angle AMB = \angle CNB = 90^\circ$

$\angle ADM = \angle NBC$ ($AD \parallel BC$ and BD is transversal.)

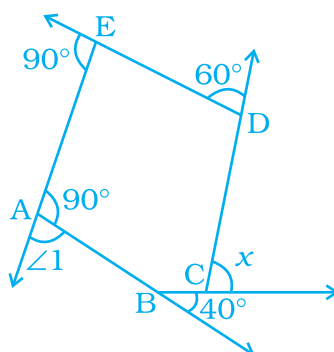
So, $\triangle AMD \cong \triangle CNB$ (AAS)

Example 26 : Construct a quadrilateral ABCD in which $AB = AD = 5\text{cm}$, $BC = CD = 7\text{cm}$ and $BD = 6\text{cm}$. What type of quadrilateral is this?

Solution : Looking at the rough figure, draw a line segment $BD = 6\text{cm}$. Taking B and D as centres and 5 cm radius, draw arcs to intersect at the point A, then taking B and D as centres and 7 cm radius, draw arcs in the opposite side of A to intersect at the point C. Join AB, AD and BC, DC. Then ABCD is the required quadrilateral. It is a kite.



Example 27 : Find x in the following figure.



Solution : In the given figure $\angle 1 + 90^\circ = 180^\circ$ (linear pair)

$$\angle 1 = 90^\circ$$

Now, sum of exterior angles of a polygon is 360° ,

therefore, $x + 60^\circ + 90^\circ + 90^\circ + 40^\circ = 360^\circ$

$$x + 280^\circ = 360^\circ$$

$$x = 80^\circ$$

Classifying Plane Figures

Triangle	Trapezoid	Parallelogram	
Closed figure with 3 straight sides that connect 3 points	Quadrilateral with 1 pair of parallel sides	Quadrilateral with 2 pairs of parallel sides	
Rhombus	Rectangle	Square	Circle
Parallelogram with 4 sides of equal length	Parallelogram with 4 right angles	Parallelogram with 4 sides of equal length and 4 right angles	Set of all points in a plane that are at the same distance from a fixed point

Example 28 : Two adjacent angles of a parallelogram are in the ratio 4:5. Find their measures.

Solution : Let the angles be $4x$ and $5x$.

$$\text{Then, } 4x + 5x = 180^\circ$$

$$9x = 180^\circ$$

$$x = 20^\circ$$

$$\text{So, angles are } 4 \times 20^\circ = 80^\circ \text{ and } 5 \times 20^\circ = 100^\circ.$$

Example 29 : The four angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. Find the angles.

Solution : Let angles be $3x$, $4x$, $5x$, $6x$.

Thus, $3x + 4x + 5x + 6x = 360^\circ$ since sum of the angles of a quadrilateral is 360° .

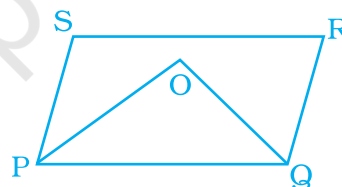
$$\text{So, } 18x = 360^\circ$$

$$\text{or, } x = 20^\circ$$

$$\text{Thus, angles are } 60^\circ, 80^\circ, 100^\circ, 120^\circ.$$

Example 30 : In a parallelogram PQRS, the bisectors of $\angle P$ and $\angle Q$ meet at O. Find $\angle POQ$.

Solution : Since OP and OQ are the bisectors of $\angle P$ and $\angle Q$ respectively (see figure on the right),



$$\text{so, } \angle OPQ = \frac{1}{2} \angle P \text{ and } \angle OQP = \frac{1}{2} \angle Q$$

In $\triangle POQ$,

$$\angle OPQ + \angle PQO + \angle POQ = 180^\circ \text{ (Angle sum property)}$$

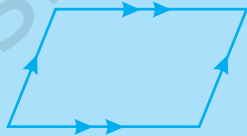
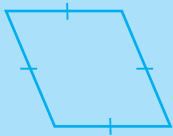
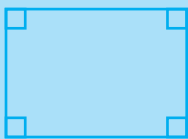
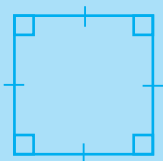
$$\text{i.e. } \frac{1}{2} \angle P + \angle POQ + \frac{1}{2} \angle Q = 180^\circ$$

$$\begin{aligned} \text{i.e. } \angle POQ &= 180^\circ - \frac{1}{2} (\angle P + \angle Q) \\ &= 180^\circ - \frac{1}{2} \times 180^\circ \\ &= 90^\circ \end{aligned}$$

Example 31 : Three angles of a quadrilateral are 50° , 40° and 123° .
Find its fourth angle.

Solution : Let fourth angle be x . Then $50^\circ + 40^\circ + 123^\circ + x = 360^\circ$.
or $x = 360^\circ - 50^\circ - 40^\circ - 123^\circ$
 $= 360^\circ - 213^\circ = 147^\circ$.

A quadrilateral is a closed plane figure with four sides that are line segments. The figures below are special types of quadrilaterals.

Special Quadrilaterals	Diagram
Trapezium A trapezium is a quadrilateral with exactly 1 pair of parallel sides.	
Parallelogram A Parallelogram is a quadrilateral with 2 pairs of parallel sides.	
Rhombus A rhombus is a parallelogram with 4 sides of equal length.	
Rectangle A rectangle is a parallelogram with 4 right angles.	
Square A square is a parallelogram with 4 sides of equal length and 4 right angles.	

Example 32 : The ratio of exterior angle to interior angle of a regular polygon is 1:4. Find the number of sides of the polygon.

Solution : Let the exterior angle of the polygon be x
Then, the interior angle of polygon $= 180^\circ - x$
According to question,

$$\frac{x}{180^\circ - x} = \frac{1}{4}$$

$$\text{or, } 4x = 180^\circ - x$$

$$\text{or, } 5x = 180^\circ$$

$$\text{or, } x = \frac{180^\circ}{5}$$

$$\text{So, } x = 36^\circ$$

$$\text{Number of sides of polygon} = \frac{360^\circ}{\text{exterior angle}}$$

$$= \frac{360^\circ}{36^\circ} = 10$$

Example 33 : Each interior angle of a polygon is 108° . Find the number of sides of the polygon.

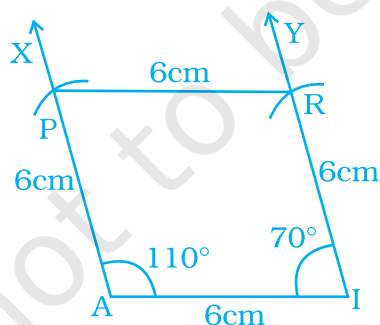
Solution : Since interior angle = 108°

$$\text{so, exterior angle} = 180^\circ - 108^\circ = 72^\circ$$

$$\text{Number of sides} = \frac{360^\circ}{\text{exterior angle}} = \frac{360^\circ}{72^\circ} = 5$$

Example 34 : Construct a rhombus PAIR, given that PA = 6 cm and angle $\angle A = 110^\circ$.

Solution :



Since in a rhombus, all sides are equal so, PA = AI = IR = RP = 6cm

Also, rhombus is a parallelogram

so, adjacent angle, $\angle I = 180^\circ - 110^\circ = 70^\circ$

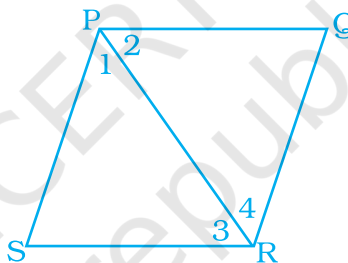
Steps of construction

1. Draw $AI = 6$ cm
2. Draw ray $\overrightarrow{AX}$ such that $\angle IAX = 110^\circ$ and draw $\overrightarrow{IY}$ such that $\angle AIY = 70^\circ$.
3. With A and I as centres and radius 6cm draw arcs intersecting AX and IY at P and R respectively.
4. Join PR.

Thus, PAIR is the required rhombus.

Example 35 : One of the diagonals of a rhombus and its sides are equal. Find the angles of the rhombus.

Solution : Let PQRS be a rhombus such that its diagonal PR is equal to its side, that is, $PQ = QR = RS = PS = PR$
So, $\triangle PRS$ and $\triangle PQR$ are equilateral.



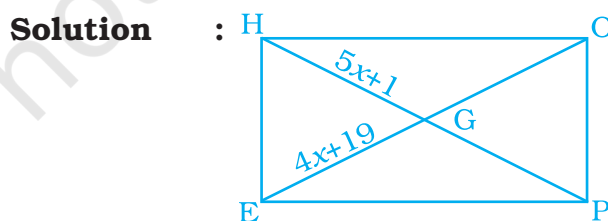
$\angle S = \angle Q = 60^\circ$ [Each angle of an equilateral triangle is 60° .]

and

$$\angle P = \angle 1 + \angle 2 = 60^\circ + 60^\circ = 120^\circ = \angle R$$

Hence $\angle S = \angle Q = 60^\circ$ and $\angle P = \angle R = 120^\circ$

Example 36 : In the figure, HOPE is a rectangle. Its diagonals meet at G. If $HG = 5x + 1$ and $EG = 4x + 19$, find x .



Since diagonals of a rectangle bisect each other,

$$HP = 2HG = 2(5x + 1) = 10x + 2$$

and

$$OE = 2EG = 2(4x + 19) = 8x + 38$$

Diagonals of a rectangle are equal. So $HP = OE$

$$\text{or } 10x + 2 = 8x + 38$$

$$\text{or } 2x = 36 \text{ or } x = 18$$

Example 37 : Application on the problem strategy

RICE is a rhombus. Find x , y , z . Justify your findings. Hence, find the perimeter of the rhombus.

Solution : Understand and explore the problem

We have to find the values of x , y , z .
i.e. OE , OY and side IR of the rhombus and perimeter of the rhombus.

What do we know?

RICE is a rhombus and

$$OC = 12, OE = 5, OI = x + 2, OR = x + y$$

Plan a strategy

- (1) We have to find the parts of the diagonal. Use diagonals of a rhombus bisect each other.
- (2) We have to find the side of the rhombus. We use diagonals intersect at right angles and apply pythagoras theorem.
- (3) Since all sides of a rhombus are equal, perimeter of the rhombus = $4 \times \text{side}$.

Solve

$$\text{Step 1. } OI = OE \Rightarrow x + 2 = 5 \text{ or } x = 5 - 2 = 3.$$

$$OC = OR \Rightarrow 12 = y + x \text{ or } y = 12 - x$$

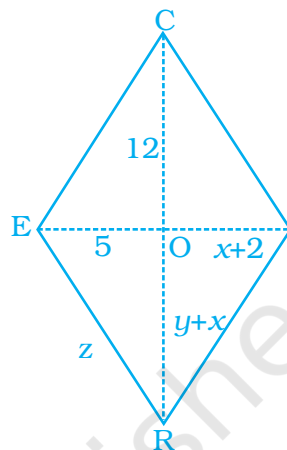
$$12 - 3 = 9$$

$$\text{Step 2. } EOR \text{ is a right triangle}$$

$$ER^2 = OE^2 + OR^2$$

$$= 5^2 + 12^2$$

$$= 25 + 144 = 169$$



$$ER = \sqrt{169} = 13\text{cm}$$

Step 3. Since all sides of a rhombus are equal.

$$\therefore RE = RI = IC = CE = 13 \text{ cm.}$$

$$\begin{aligned}\text{Perimeter of RICE} &= 4 \times RE = 4 \times 13 \text{ cm} \\ &= 52 \text{ cm}\end{aligned}$$

Revise

We have been asked to find x , y and z and we have found that.

Checking

$$x + 2 = 5 \text{ and } x = 3 \Rightarrow 3 + 2 = 5$$

Hence value of x is correct.

$$x + y = 12 \quad \therefore x = 3 \text{ and } y = 9$$

and $3 + 9 = 12 \Rightarrow$ value of y is correct.

$$\begin{aligned}\text{Perimeter of rhombus} &= 2\sqrt{d_1^2 + d_2^2} \text{ (where } d_1 \text{ and } d_2 \\ &\text{are diagonals)}\end{aligned}$$

$$= 2\sqrt{24^2 + 10^2}$$

$$= 2\sqrt{576 + 100}$$

$$= 2\sqrt{676} = 52 \text{ cm}$$

Think and Discuss

- (i) If RICE is a parallelogram, not a rhombus can you find x , y and z ?
- (ii) If RICE is a rhombus with $EC = 20 \text{ cm}$ and $OC = 12 \text{ cm}$, can you find x , y , z ?

Example 38 : Application on the problem solution strategy

Construct a rhombus with side 4.5cm and diagonal 6cm .

Solution : Understand and explore the problem

What do you know?

Here, side of rhombus = 4.5 cm .

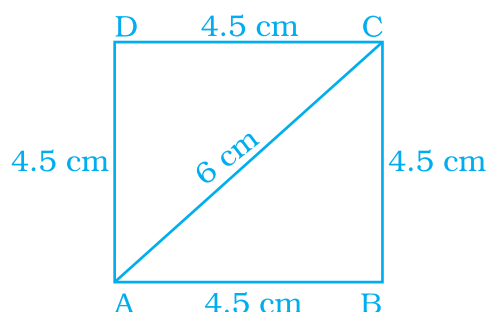
Diagonal of rhombus = 6 cm .

What do we need to make rhombus?

4 sides and its one diagonal

Plan a strategy

- (1) Use property of rhombus— all sides are equal.
- (2) Make a free hand rough sketch and name it ABCD.



Solve

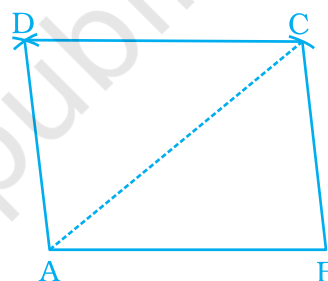
Step-1. Draw $AB = 4.5$ cm.



Step-2. With A as centre and radius 6 cm draw an arc above AB.



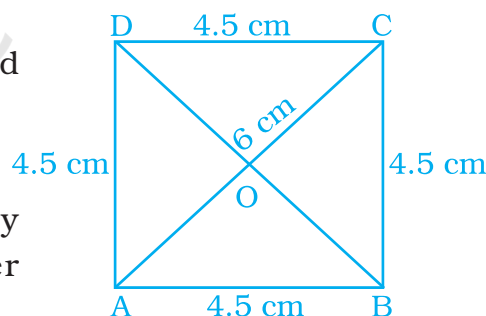
Step-3. With B as centre draw an arc to cut the arc drawn in step 2 at pt C.



Step-4. Join AC and BC.

Step-5. With A and C as centre and radius 4.5 cm draw arcs to intersect each other at D.

Step-6. ABCD is required rhombus.



Checking:

Verify your figure by adopting some other property of rhombus.

Step 1. Join BD to intersect AC as O.

Step 2. Measure $\angle AOB$. Is it 90° ?

Step 3. Measure OA and OC. Are they equal?

Step 4. Measure OB and OD. Are they equal?

If your answer to 2, 3, 4 is yes it means what you have constructed is a right angle.

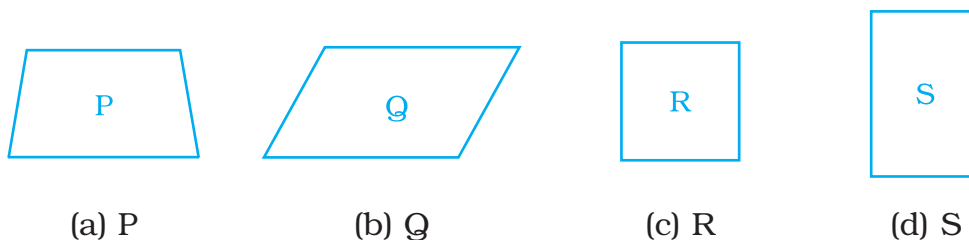
Think and Discuss

1. Can you draw this rhombus by using some other property?
2. Can you draw a parallelogram with given measurement?
3. How will you construct this rhombus if instead of side 4.5 cm diagonal 4.5 cm is given?

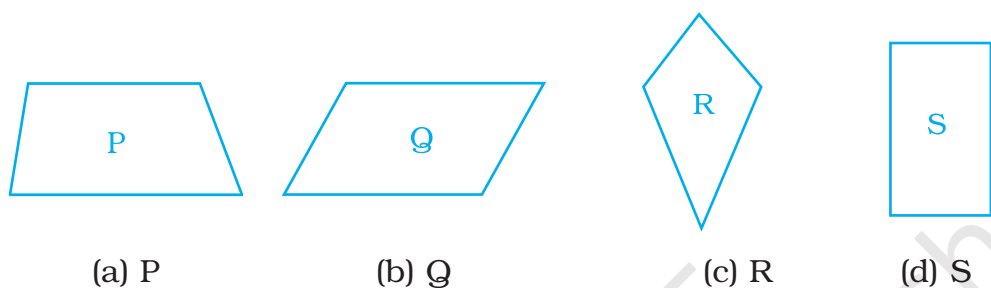
(C) Exercises

In questions 1 to 52, there are four options, out of which one is correct. Write the correct answer.

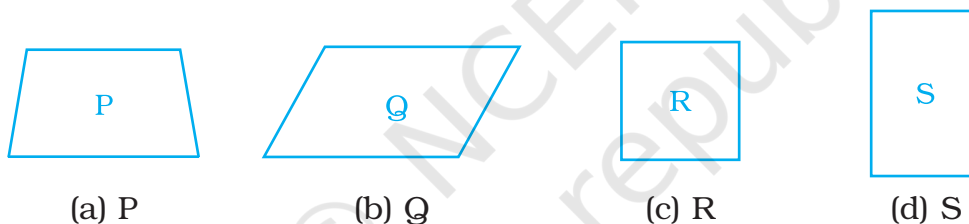
1. If three angles of a quadrilateral are each equal to 75° , the fourth angle is
(a) 150° (b) 135° (c) 45° (d) 75°
2. For which of the following, diagonals bisect each other?
(a) Square (b) Kite
(c) Trapezium (d) Quadrilateral
3. For which of the following figures, all angles are equal?
(a) Rectangle (b) Kite
(c) Trapezium (d) Rhombus
4. For which of the following figures, diagonals are perpendicular to each other?
(a) Parallelogram (b) Kite
(c) Trapezium (d) Rectangle
5. For which of the following figures, diagonals are equal?
(a) Trapezium (b) Rhombus
(c) Parallelogram (d) Rectangle
6. Which of the following figures satisfy the following properties?
- All sides are congruent.
- All angles are right angles.
- Opposite sides are parallel.



7. Which of the following figures satisfy the following property?
- Has two pairs of congruent adjacent sides.

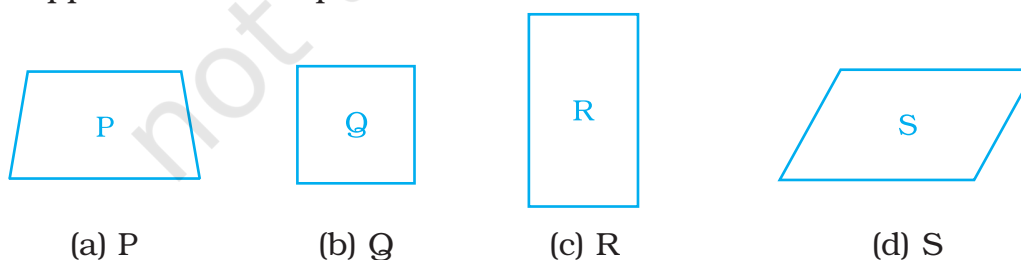


8. Which of the following figures satisfy the following property?
- Only one pair of sides are parallel.



9. Which of the following figures do not satisfy any of the following properties?

- All sides are equal.
- All angles are right angles.
- Opposite sides are parallel.

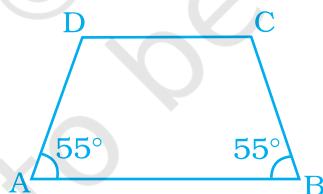


10. Which of the following properties describe a trapezium?

- (a) A pair of opposite sides is parallel.

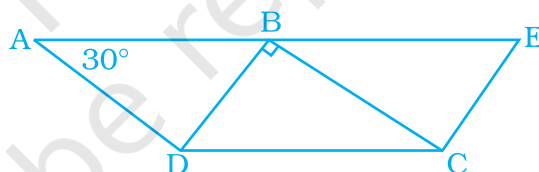
- (b) The diagonals bisect each other.
 (c) The diagonals are perpendicular to each other.
 (d) The diagonals are equal.
- 11.** Which of the following is a property of a parallelogram?
 (a) Opposite sides are parallel.
 (b) The diagonals bisect each other at right angles.
 (c) The diagonals are perpendicular to each other.
 (d) All angles are equal.
- 12.** What is the maximum number of obtuse angles that a quadrilateral can have ?
 (a) 1 (b) 2 (c) 3 (d) 4
- 13.** How many non-overlapping triangles can we make in a n -gon (polygon having n sides), by joining the vertices?
 (a) $n - 1$ (b) $n - 2$ (c) $n - 3$ (d) $n - 4$
- 14.** What is the sum of all the angles of a pentagon?
 (a) 180° (b) 360° (c) 540° (d) 720°
- 15.** What is the sum of all angles of a hexagon?
 (a) 180° (b) 360° (c) 540° (d) 720°
- 16.** If two adjacent angles of a parallelogram are $(5x - 5)^\circ$ and $(10x + 35)^\circ$, then the ratio of these angles is
 (a) 1 : 3 (b) 2 : 3 (c) 1 : 4 (d) 1 : 2
- 17.** A quadrilateral whose all sides are equal, opposite angles are equal and the diagonals bisect each other at right angles is a _____.
 (a) rhombus (b) parallelogram (c) square (d) rectangle
- 18.** A quadrilateral whose opposite sides and all the angles are equal is a
 (a) rectangle (b) parallelogram (c) square (d) rhombus
- 19.** A quadrilateral whose all sides, diagonals and angles are equal is a
 (a) square (b) trapezium (c) rectangle (d) rhombus

- 20.** How many diagonals does a hexagon have?
 (a) 9 (b) 8 (c) 2 (d) 6
- 21.** If the adjacent sides of a parallelogram are equal then parallelogram is a
 (a) rectangle (b) trapezium (c) rhombus (d) square
- 22.** If the diagonals of a quadrilateral are equal and bisect each other, then the quadrilateral is a
 (a) rhombus (b) rectangle (c) square (d) parallelogram
- 23.** The sum of all exterior angles of a triangle is
 (a) 180° (b) 360° (c) 540° (d) 720°
- 24.** Which of the following is an equiangular and equilateral polygon?
 (a) Square (b) Rectangle (c) Rhombus (d) Right triangle
- 25.** Which one has all the properties of a kite and a parallelogram?
 (a) Trapezium (b) Rhombus (c) Rectangle (d) Parallelogram
- 26.** The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4. The smallest angle is
 (a) 72° (b) 144° (c) 36° (d) 18°
- 27.** In the trapezium ABCD, the measure of $\angle D$ is
 (a) 55° (b) 115° (c) 135° (d) 125°



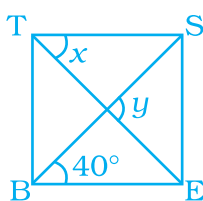
- 28.** A quadrilateral has three acute angles. If each measures 80° , then the measure of the fourth angle is
 (a) 150° (b) 120° (c) 105° (d) 140°
- 29.** The number of sides of a regular polygon where each exterior angle has a measure of 45° is
 (a) 8 (b) 10 (c) 4 (d) 6

- 30.** In a parallelogram PQRS, if $\angle P = 60^\circ$, then other three angles are
 (a) $45^\circ, 135^\circ, 120^\circ$ (b) $60^\circ, 120^\circ, 120^\circ$
 (c) $60^\circ, 135^\circ, 135^\circ$ (d) $45^\circ, 135^\circ, 135^\circ$
- 31.** If two adjacent angles of a parallelogram are in the ratio 2 : 3, then the measure of angles are
 (a) $72^\circ, 108^\circ$ (b) $36^\circ, 54^\circ$ (c) $80^\circ, 120^\circ$ (d) $96^\circ, 144^\circ$
- 32.** If PQRS is a parallelogram, then $\angle P - \angle R$ is equal to
 (a) 60° (b) 90° (c) 80° (d) 0°
- 33.** The sum of adjacent angles of a parallelogram is
 (a) 180° (b) 120° (c) 360° (d) 90°
- 34.** The angle between the two altitudes of a parallelogram through the same vertex of an obtuse angle of the parallelogram is 30° . The measure of the obtuse angle is
 (a) 100° (b) 150° (c) 105° (d) 120°
- 35.** In the given figure, ABCD and BDCE are parallelograms with common base DC. If $BC \perp BD$, then $\angle BEC =$
 (a) 60° (b) 30° (c) 150° (d) 120°

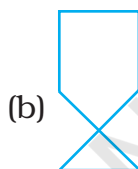


- 36.** Length of one of the diagonals of a rectangle whose sides are 10 cm and 24 cm is
 (a) 25 cm (b) 20 cm (c) 26 cm (d) 3.5 cm
- 37.** If the adjacent angles of a parallelogram are equal, then the parallelogram is a
 (a) rectangle (b) trapezium
 (c) rhombus (d) any of the three
- 38.** Which of the following can be four interior angles of a quadrilateral?
 (a) $140^\circ, 40^\circ, 20^\circ, 160^\circ$ (b) $270^\circ, 150^\circ, 30^\circ, 20^\circ$
 (c) $40^\circ, 70^\circ, 90^\circ, 60^\circ$ (d) $110^\circ, 40^\circ, 30^\circ, 180^\circ$

39. The sum of angles of a concave quadrilateral is
 (a) more than 360° (b) less than 360°
 (c) equal to 360° (d) twice of 360°
40. Which of the following can never be the measure of exterior angle of a regular polygon?
 (a) 22° (b) 36° (c) 45° (d) 30°
41. In the figure, BEST is a rhombus, Then the value of $y - x$ is
 (a) 40° (b) 50° (c) 20° (d) 10°



42. The closed curve which is also a polygon is



43. Which of the following is not true for an exterior angle of a regular polygon with n sides?

- (a) Each exterior angle = $\frac{360^\circ}{n}$
 (b) Exterior angle = $180^\circ - \text{interior angle}$
 (c) $n = \frac{360^\circ}{\text{exterior angle}}$
 (d) Each exterior angle = $\frac{(n-2) \times 180^\circ}{n}$

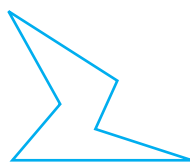
44. PQRS is a square. PR and SQ intersect at O. Then $\angle POQ$ is a
 (a) Right angle (b) Straight angle
 (c) Reflex angle (d) Complete angle

45. Two adjacent angles of a parallelogram are in the ratio 1:5. Then all the angles of the parallelogram are
 (a) 30° , 150° , 30° , 150° (b) 85° , 95° , 85° , 95°
 (c) 45° , 135° , 45° , 135° (d) 30° , 180° , 30° , 180°
46. A parallelogram PQRS is constructed with sides $QR = 6$ cm, $PQ = 4$ cm and $\angle PQR = 90^\circ$. Then PQRS is a
 (a) square (b) rectangle (c) rhombus (d) trapezium
47. The angles P, Q, R and S of a quadrilateral are in the ratio 1:3:7:9. Then PQRS is a
 (a) parallelogram (b) trapezium with $PQ \parallel RS$
 (c) trapezium with $QR \parallel PS$ (d) kite
48. PQRS is a trapezium in which $PQ \parallel SR$ and $\angle P = 130^\circ$, $\angle Q = 110^\circ$. Then $\angle R$ is equal to:
 (a) 70° (b) 50° (c) 65° (d) 55°
49. The number of sides of a regular polygon whose each interior angle is of 135° is
 (a) 6 (b) 7 (c) 8 (d) 9
50. If a diagonal of a quadrilateral bisects both the angles, then it is a
 (a) kite (b) parallelogram
 (c) rhombus (d) rectangle
51. To construct a unique parallelogram, the minimum number of measurements required is
 (a) 2 (b) 3 (c) 4 (d) 5
52. To construct a unique rectangle, the minimum number of measurements required is
 (a) 4 (b) 3 (c) 2 (d) 1

In questions 53 to 91, fill in the blanks to make the statements true.

53. In quadrilateral HOPE, the pairs of opposite sides are _____.
54. In quadrilateral ROPE, the pairs of adjacent angles are _____.
55. In quadrilateral WXYZ, the pairs of opposite angles are _____.

56. The diagonals of the quadrilateral DEFG are _____ and _____.
57. The sum of all _____ of a quadrilateral is 360° .
58. The measure of each exterior angle of a regular pentagon is _____.
59. Sum of the angles of a hexagon is _____.
60. The measure of each exterior angle of a regular polygon of 18 sides is _____.
61. The number of sides of a regular polygon, where each exterior angle has a measure of 36° , is _____.



62. _____ is a closed curve entirely made up of line segments. The another name for this shape is _____.
63. A quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure is _____.
64. The measure of each angle of a regular pentagon is _____.
65. The name of three-sided regular polygon is _____.
66. The number of diagonals in a hexagon is _____.
67. A polygon is a simple closed curve made up of only _____.
68. A regular polygon is a polygon whose all sides are equal and all _____ are equal.
69. The sum of interior angles of a polygon of n sides is _____ right angles.
70. The sum of all exterior angles of a polygon is _____.
71. _____ is a regular quadrilateral.
72. A quadrilateral in which a pair of opposite sides is parallel is _____.
73. If all sides of a quadrilateral are equal, it is a _____.
74. In a rhombus diagonals intersect at _____ angles.
75. _____ measurements can determine a quadrilateral uniquely.

76. A quadrilateral can be constructed uniquely if its three sides and _____ angles are given.
77. A rhombus is a parallelogram in which _____ sides are equal.
78. The measure of _____ angle of concave quadrilateral is more than 180° .
79. A diagonal of a quadrilateral is a line segment that joins two _____ vertices of the quadrilateral.
80. The number of sides in a regular polygon having measure of an exterior angle as 72° is _____.
81. If the diagonals of a quadrilateral bisect each other, it is a _____.
82. The adjacent sides of a parallelogram are 5 cm and 9 cm. Its perimeter is _____.
83. A nonagon has _____ sides.
84. Diagonals of a rectangle are _____.
85. A polygon having 10 sides is known as _____.
86. A rectangle whose adjacent sides are equal becomes a _____.
87. If one diagonal of a rectangle is 6 cm long, length of the other diagonal is _____.
88. Adjacent angles of a parallelogram are _____.
89. If only one diagonal of a quadrilateral bisects the other, then the quadrilateral is known as _____.
90. In trapezium ABCD with $AB \parallel CD$, if $\angle A = 100^\circ$, then $\angle D =$ _____.
91. The polygon in which sum of all exterior angles is equal to the sum of interior angles is called _____.

In questions 92 to 131 state whether the statements are true (T) or (F) false.

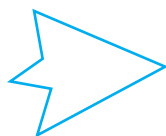
92. All angles of a trapezium are equal.
93. All squares are rectangles.
94. All kites are squares.
95. All rectangles are parallelograms.

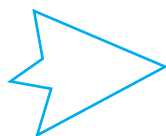
96. All rhombuses are squares.
97. Sum of all the angles of a quadrilateral is 180° .
98. A quadrilateral has two diagonals.
99. Triangle is a polygon whose sum of exterior angles is double the sum of interior angles.



100.  is a polygon.

101. A kite is not a convex quadrilateral.
102. The sum of interior angles and the sum of exterior angles taken in an order are equal in case of quadrilaterals only.
103. If the sum of interior angles is double the sum of exterior angles taken in an order of a polygon, then it is a hexagon.
104. A polygon is regular if all of its sides are equal.
105. Rectangle is a regular quadrilateral.
106. If diagonals of a quadrilateral are equal, it must be a rectangle.
107. If opposite angles of a quadrilateral are equal, it must be a parallelogram.
108. The interior angles of a triangle are in the ratio 1:2:3, then the ratio of its exterior angles is 3:2:1.



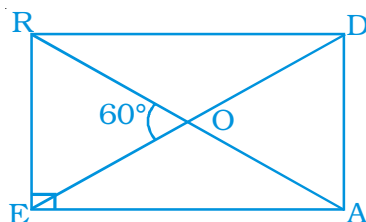
109.  is a concave pentagon.
110. Diagonals of a rhombus are equal and perpendicular to each other.
111. Diagonals of a rectangle are equal.
112. Diagonals of rectangle bisect each other at right angles.
113. Every kite is a parallelogram.

- 114.** Every trapezium is a parallelogram.
- 115.** Every parallelogram is a rectangle.
- 116.** Every trapezium is a rectangle.
- 117.** Every rectangle is a trapezium.
- 118.** Every square is a rhombus.
- 119.** Every square is a parallelogram.
- 120.** Every square is a trapezium.
- 121.** Every rhombus is a trapezium.
- 122.** A quadrilateral can be drawn if only measures of four sides are given.
- 123.** A quadrilateral can have all four angles as obtuse.
- 124.** A quadrilateral can be drawn if all four sides and one diagonal is known.
- 125.** A quadrilateral can be drawn when all the four angles and one side is given.
- 126.** A quadrilateral can be drawn if all four sides and one angle is known.
- 127.** A quadrilateral can be drawn if three sides and two diagonals are given.
- 128.** If diagonals of a quadrilateral bisect each other, it must be a parallelogram.
- 129.** A quadrilateral can be constructed uniquely if three angles and any two sides are given.
- 130.** A parallelogram can be constructed uniquely if both diagonals and the angle between them is given.
- 131.** A rhombus can be constructed uniquely if both diagonals are given.

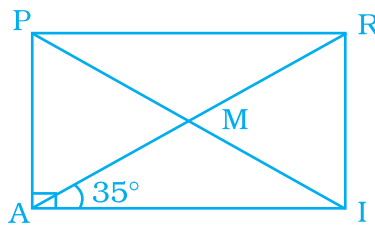
Solve the following :

- 132.** The diagonals of a rhombus are 8 cm and 15 cm. Find its side.
- 133.** Two adjacent angles of a parallelogram are in the ratio 1:3. Find its angles.
- 134.** Of the four quadrilaterals— square, rectangle, rhombus and trapezium— one is somewhat different from the others because of its design. Find it and give justification.

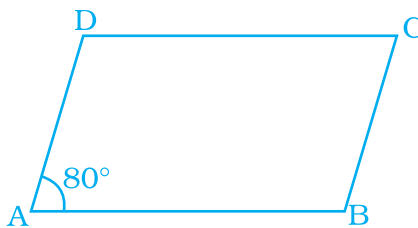
- 135.** In a rectangle ABCD, $AB = 25$ cm and $BC = 15$. In what ratio does the bisector of $\angle C$ divide AB?
- 136.** PQRS is a rectangle. The perpendicular ST from S on PR divides $\angle S$ in the ratio 2:3. Find $\angle TPQ$.
- 137.** A photo frame is in the shape of a quadrilateral. With one diagonal longer than the other. Is it a rectangle? Why or why not?
- 138.** The adjacent angles of a parallelogram are $(2x - 4)^\circ$ and $(3x - 1)^\circ$. Find the measures of all angles of the parallelogram.
- 139.** The point of intersection of diagonals of a quadrilateral divides one diagonal in the ratio 1:2. Can it be a parallelogram? Why or why not?
- 140.** The ratio between exterior angle and interior angle of a regular polygon is 1:5. Find the number of sides of the polygon.
- 141.** Two sticks each of length 5 cm are crossing each other such that they bisect each other. What shape is formed by joining their end points? Give reason.
- 142.** Two sticks each of length 7 cm are crossing each other such that they bisect each other at right angles. What shape is formed by joining their end points? Give reason.
- 143.** A playground in the town is in the form of a kite. The perimeter is 106 metres. If one of its sides is 23 metres, what are the lengths of other three sides?
- 144.** In rectangle READ, find $\angle EAR$, $\angle RAD$ and $\angle ROD$



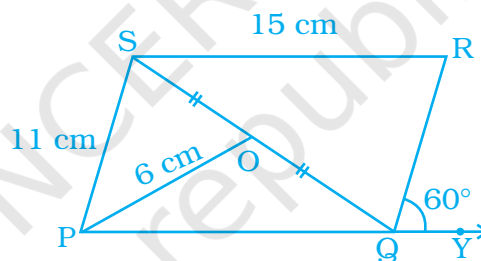
- 145.** In rectangle PAIR, find $\angle ARI$, $\angle RMI$ and $\angle PMA$.



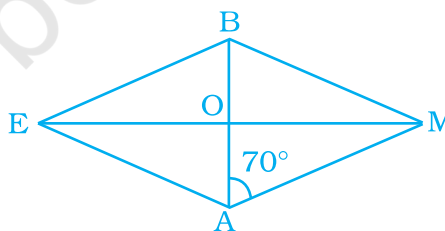
- 146.** In parallelogram ABCD, find $\angle B$, $\angle C$ and $\angle D$.



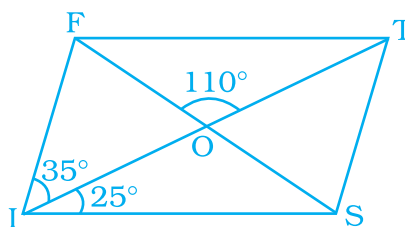
- 147.** In parallelogram PQRS, O is the mid point of SQ. Find $\angle S$, $\angle R$, PQ, QR and diagonal PR.



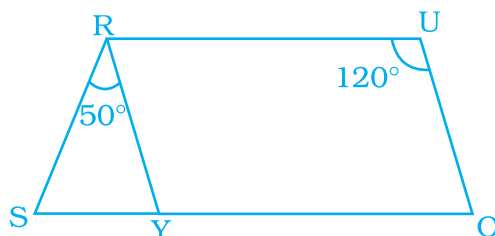
- 148.** In rhombus BEAM, find $\angle AME$ and $\angle AEM$.



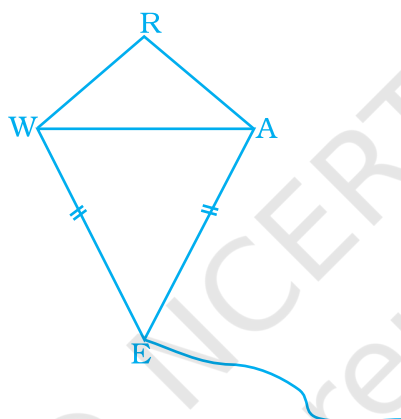
- 149.** In parallelogram FIST, find $\angle SFT$, $\angle OST$ and $\angle STO$.



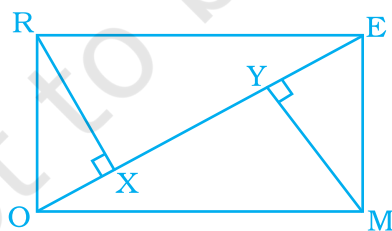
- 150.** In the given parallelogram YOUR, $\angle RUO = 120^\circ$ and OY is extended to point S such that $\angle SRY = 50^\circ$. Find $\angle YSR$.



- 151.** In kite WEAR, $\angle WEA = 70^\circ$ and $\angle ARW = 80^\circ$. Find the remaining two angles.



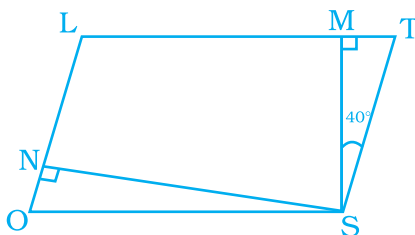
- 152.** A rectangular MORE is shown below:



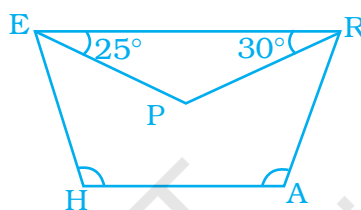
Answer the following questions by giving appropriate reason.

(i) Is $RE = OM$?	(ii) Is $\angle MYO = \angle RXE$?
(iii) Is $\angle MOY = \angle REX$?	(iv) Is $\triangle MYO \cong \triangle RXE$?
(v) Is $MY = RX$?	

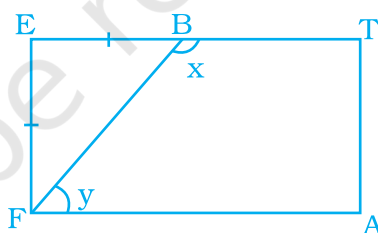
- 153.** In parallelogram $LOST$, $SN \perp OL$ and $SM \perp LT$. Find $\angle STM$, $\angle SON$ and $\angle NSM$.



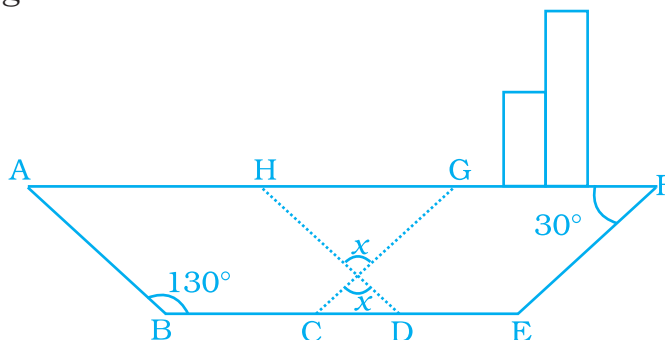
- 154.** In trapezium $HARE$, EP and RP are bisectors of $\angle E$ and $\angle R$ respectively. Find $\angle HAR$ and $\angle EHA$.



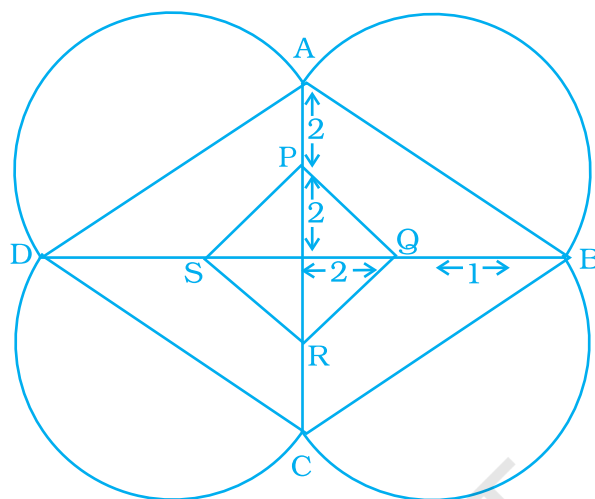
- 155.** In parallelogram $MODE$, the bisector of $\angle M$ and $\angle O$ meet at Q , find the measure of $\angle MQO$.
- 156.** A playground is in the form of a rectangle $ATEF$. Two players are standing at the points F and B where $EF = EB$. Find the values of x and y .



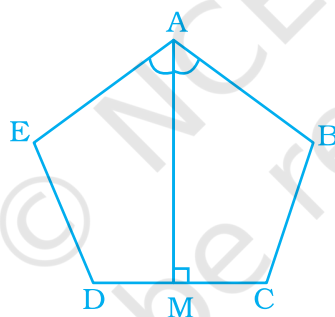
- 157.** In the following figure of a ship, $ABDH$ and $CEFG$ are two parallelograms. Find the value of x .



- 158.** A Rangoli has been drawn on a floor of a house. ABCD and PQRS both are in the shape of a rhombus. Find the radius of semicircle drawn on each side of rhombus ABCD.

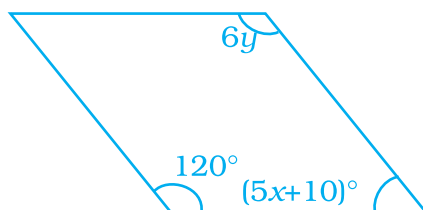


- 159.** ABCDE is a regular pentagon. The bisector of angle A meets the side CD at M. Find $\angle AMC$

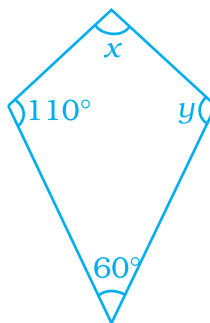


- 160.** Quadrilateral EFGH is a rectangle in which J is the point of intersection of the diagonals. Find the value of x if $JF = 8x + 4$ and $EG = 24x - 8$.

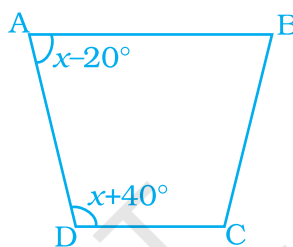
- 161.** Find the values of x and y in the following parallelogram.



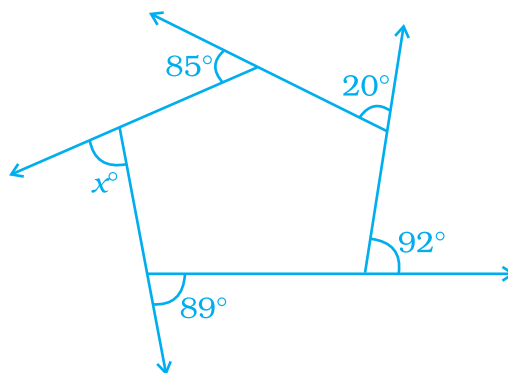
- 162.** Find the values of x and y in the following kite.



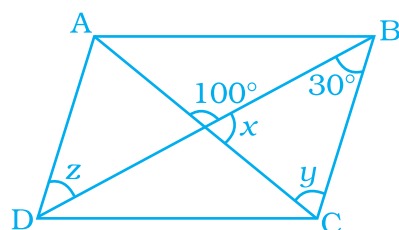
- 163.** Find the value of x in the trapezium ABCD given below.



- 164.** Two angles of a quadrilateral are each of measure 75° and the other two angles are equal. What is the measure of these two angles? Name the possible figures so formed.
- 165.** In a quadrilateral PQRS, $\angle P = 50^\circ$, $\angle Q = 50^\circ$, $\angle R = 60^\circ$. Find $\angle S$. Is this quadrilateral convex or concave?
- 166.** Both the pairs of opposite angles of a quadrilateral are equal and supplementary. Find the measure of each angle.
- 167.** Find the measure of each angle of a regular octagon.
- 168.** Find the measure of an exterior angle of a regular pentagon and an exterior angle of a regular decagon. What is the ratio between these two angles?
- 169.** In the figure, find the value of x .

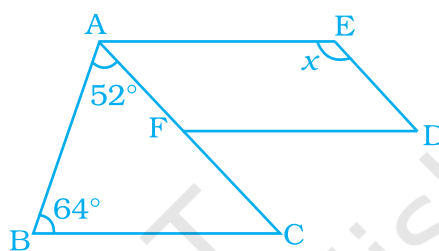


- 170.** Three angles of a quadrilateral are equal. Fourth angle is of measure 120° . What is the measure of equal angles?
- 171.** In a quadrilateral HOPE, PS and ES are bisectors of $\angle P$ and $\angle E$ respectively. Give reason.
- 172.** ABCD is a parallelogram. Find the value of x , y and z .

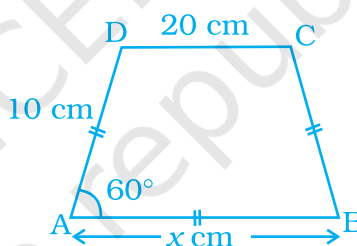


- 173.** Diagonals of a quadrilateral are perpendicular to each other. Is such a quadrilateral always a rhombus? Give a figure to justify your answer.
- 174.** ABCD is a trapezium such that $AB \parallel CD$, $\angle A : \angle D = 2 : 1$, $\angle B : \angle C = 7 : 5$. Find the angles of the trapezium.
- 175.** A line l is parallel to line m and a transversal p intersects them at X, Y respectively. Bisectors of interior angles at X and Y intersect at P and Q. Is PXQY a rectangle? Give reason.
- 176.** ABCD is a parallelogram. The bisector of angle A intersects CD at X and bisector of angle C intersects AB at Y. Is AXCY a parallelogram? Give reason.
- 177.** A diagonal of a parallelogram bisects an angle. Will it also bisect the other angle? Give reason.
- 178.** The angle between the two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 45° . Find the angles of the parallelogram.
- 179.** ABCD is a rhombus such that the perpendicular bisector of AB passes through D. Find the angles of the rhombus.
Hint: Join BD. Then $\triangle ABD$ is equilateral.
- 180.** ABCD is a parallelogram. Points P and Q are taken on the sides AB and AD respectively and the parallelogram PRQA is formed. If $\angle C = 45^\circ$, find $\angle R$.

- 181.** In parallelogram ABCD, the angle bisector of $\angle A$ bisects BC. Will angle bisector of B also bisect AD? Give reason.
- 182.** A regular pentagon ABCDE and a square ABFG are formed on opposite sides of AB. Find $\angle BCF$.
- 183.** Find maximum number of acute angles which a convex, a quadrilateral, a pentagon and a hexagon can have. Observe the pattern and generalise the result for any polygon.
- 184.** In the following figure, $FD \parallel BC \parallel AE$ and $AC \parallel ED$. Find the value of x .

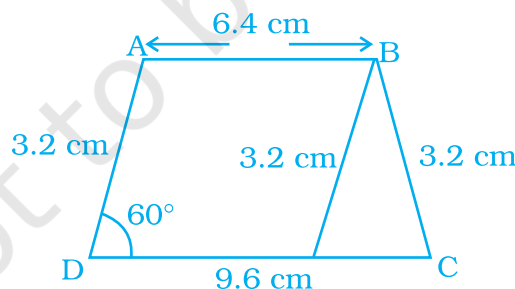


- 185.** In the following figure, $AB \parallel DC$ and $AD = BC$. Find the value of x .



- 186.** Construct a trapezium ABCD in which $AB \parallel DC$, $\angle A = 105^\circ$, $AD = 3$ cm, $AB = 4$ cm and $CD = 8$ cm.
- 187.** Construct a parallelogram ABCD in which $AB = 4$ cm, $BC = 5$ cm and $\angle B = 60^\circ$.
- 188.** Construct a rhombus whose side is 5 cm and one angle is of 60° .
- 189.** Construct a rectangle whose one side is 3 cm and a diagonal equal to 5 cm.
- 190.** Construct a square of side 4 cm.
- 191.** Construct a rhombus CLUE in which $CL = 7.5$ cm and $LE = 6$ cm.
- 192.** Construct a quadrilateral BEAR in which $BE = 6$ cm, $EA = 7$ cm, $RB = RE = 5$ cm and $BA = 9$ cm. Measure its fourth side.

- 193.** Construct a parallelogram POUR in which, $PO=5.5$ cm, $OU = 7.2$ cm and $\angle O = 70^\circ$.
- 194.** Draw a circle of radius 3 cm and draw its diameter and label it as AC. Construct its perpendicular bisector and let it intersect the circle at B and D. What type of quadrilateral is ABCD? Justify your answer.
- 195.** Construct a parallelogram HOME with $HO = 6$ cm, $HE = 4$ cm and $OE = 3$ cm.
- 196.** Is it possible to construct a quadrilateral ABCD in which $AB = 3$ cm, $BC = 4$ cm, $CD = 5.4$ cm, $DA = 5.9$ cm and diagonal $AC = 8$ cm? If not, why?
- 197.** Is it possible to construct a quadrilateral ROAM in which $RO=4$ cm, $OA = 5$ cm, $\angle O = 120^\circ$, $\angle R = 105^\circ$ and $\angle A = 135^\circ$? If not, why?
- 198.** Construct a square in which each diagonal is 5cm long.
- 199.** Construct a quadrilateral NEWS in which $NE = 7$ cm, $EW = 6$ cm, $\angle N = 60^\circ$, $\angle E = 110^\circ$ and $\angle S = 85^\circ$.
- 200.** Construct a parallelogram when one of its side is 4cm and its two diagonals are 5.6 cm and 7cm. Measure the other side.
- 201.** Find the measure of each angle of a regular polygon of 20 sides?
- 202.** Construct a trapezium RISK in which $RI \parallel KS$, $RI = 7$ cm, $IS = 5$ cm, $RK=6.5$ cm and $\angle I = 60^\circ$.
- 203.** Construct a trapezium ABCD where $AB \parallel CD$, $AD = BC = 3.2$ cm, $AB = 6.4$ cm and $CD = 9.6$ cm. Measure $\angle B$ and $\angle A$.



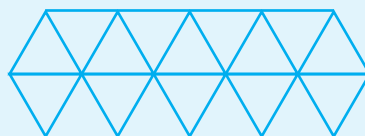
[Hint : Difference of two parallel sides gives an equilateral triangle.]

(D) Applications, Games and Puzzles

1 : Constructing a Tessellation

Tessellation: A tessellation is created when a shape is repeated over and over again covering a plane surface without any gaps or overlaps.

Regular Tessellations : It means a tessellation made up of congruent regular polygons. For example:



A tessellation of triangles

This arrangement can be extended to complete tiling of a floor (or tessellation).

Rules for Regular Tessellation:

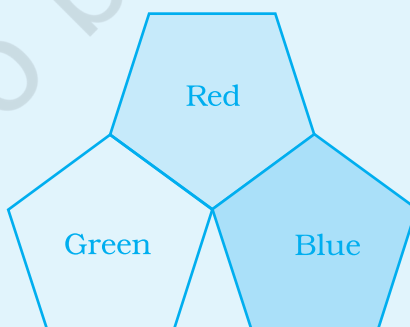
- (i) In tessellation there should be no overlappings/gaps between tiles.
- (ii) The tiles must be regular polygons.
- (iii) Design at each vertex must look the same.

Caution

Will pentagons work?

The interior angle of a pentagon is $108^\circ \dots$

$180^\circ + 108^\circ + 108^\circ = 324^\circ$ degrees $\dots$ No!

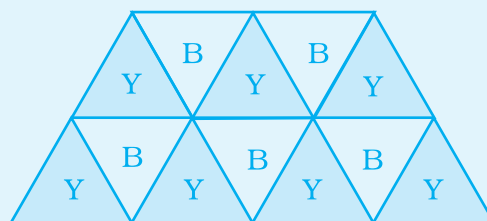


Thus, since the regular polygons must fill the plane at each vertex, the interior angle must be an exact divisor of 360° .

Now, find the regular polygon that can tessellate by trying a sample in table below.

Polygon**Tessellation**

1. Triangle



2. Square

3. Regular Pentagon

4. Regular Hexagon

5. Regular Heptagon

6. Regular Octagon

Conclusion

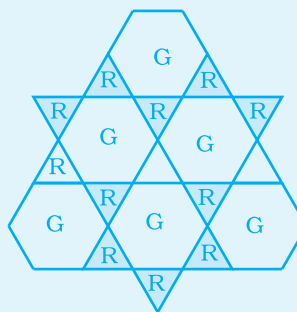
Thus, only regular polygons that can tessellate are

1. _____
2. _____
3. _____

Assignment

1. You can construct a tessellation on computer using following steps:
 - Hold down a basic images and copy it to paintbrush.
 - Keep on moving and pasting by positioning each to see a tessellation.

2. Semi Regular Tessellation : These are made by using two or more different regular polygons. Every vertex must have the same configuration, e.g.:



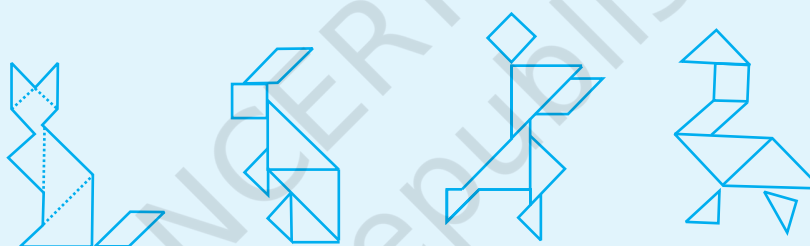
Y - yellow
B - Blue
G - Green
R - Red

Now discover some more tessellation of this type .

2 Constructing a TANGRAM

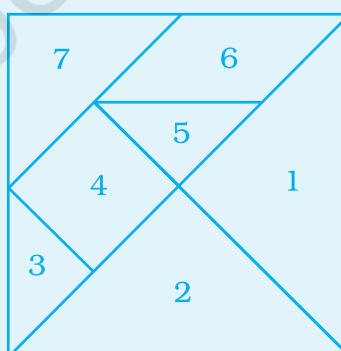
Cut the pieces of given square as shown on next page and make different shapes as shown below.

Different shapes can be made of Tangram Pieces



Try to form a story using different shapes of animals.

Required Square



3 Motivate the students to participate

Read the following description of a square before the students and let them draw what you have described.

Descriptions: My quadrilateral has opposite sides equal.

Let students compare their drawings with each other and with your square. Let students discuss what all their drawings have in common (they are all parallelograms) and what additional information is necessary to guarantee that they all would draw a square.

(e.g. All 4 sides equal and one right angle.)

- 4:** Place '✓' or '×' in the appropriate spaces according to the property of different quadrilaterals.

	Parallelogram	Rectangle	Rhombus	Square	Trapezium with non parallel sides equal	Trapezium	Kite
Opposite sides parallel	✓	✓	✓	✓	×	×	×
Opposite sides equal							
Opposite angles equal							
Diagonal forms congruent triangles							
Diagonals bisect each other							
Diagonals are perpendicular							
Diagonals are equal							
Diagonals bisect opposite angles							
All angles are right							
All sides are equal							

Use the quadrilateral chart at Page 167 to do the following activity and answer the following questions.

- (a) How can you use the properties shown in the quadrilateral chart to make a statement that you believe is true about all parallelograms?
 - (b) How can you use the properties shown in the quadrilateral chart to make a statement that you believe is true about all rhombuses?
 - (c) How can you use the properties shown in the quadrilateral chart to make a statement that you believe is true about all rhombuses, but not parallelograms?
 - (d) How can you use the properties shown in the quadrilateral chart to make a statement that you believe is true about only rhombuses?
 - (e) How are the properties of rhombuses like the properties of parallelograms in general?
 - (f) How are the properties of rhombuses different from the properties of parallelograms?
 - (g) Which quadrilaterals have exactly one line of symmetry? Exactly two? Exactly three? Exactly four?
 - (h) Make a 'Family Tree' to show the relationship among the quadrilaterals you have been investigating.
- 5:** Have students take each of the quadrilateral named below, join, in order, the mid points of the sides and describe the special kind of quadrilaterals they get each time:
- (a) Rhombus.
 - (b) Rectangle.
 - (c) Trapezium with non-parallel sides equal.
 - (d) Trapezium with non-parallel sides unequal.
 - (e) Kite.

6: Crossword Puzzle

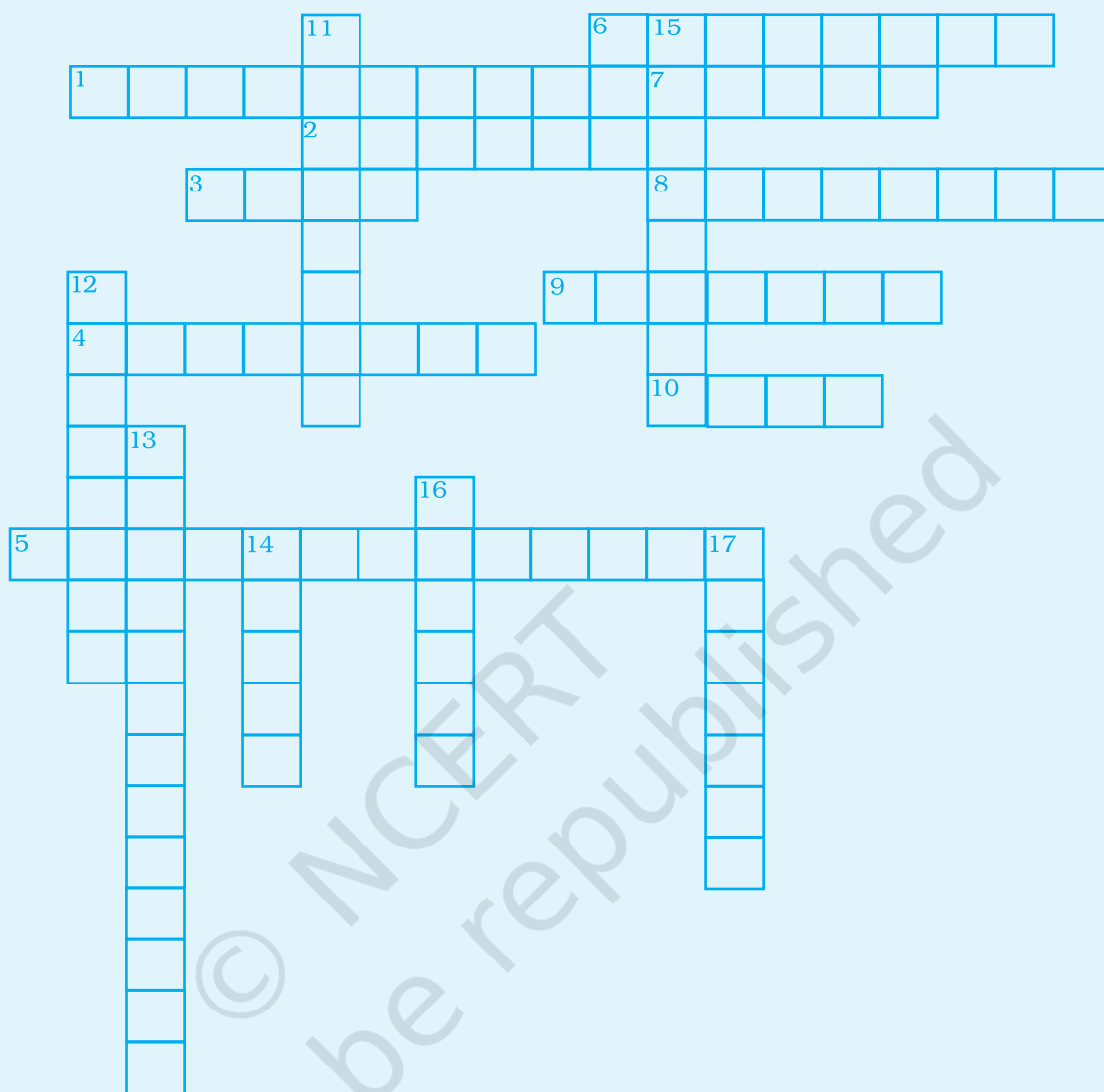
Solve the given crossword and then fill up the given boxes (on the next page). Clues are given below for across as well as downward filling. Also, for across and down clues, clue number is written at the corner of the boxes. Answers of clues have to be filled up in their respective boxes.

Clues**Across**

1. A quadrilateral with pair of parallel sides.
2. A simple closed curve made up of only line segments.
3. A quadrilateral which has exactly two distinct consecutive pairs of sides of equal length.
4. A line segment connecting two non-consecutive vertices of a polygon.
5. The diagonals of a rhombus are _____ bisectors of one another.
6. The _____ sides of a parallelogram are of equal length.
7. The number of sides of a regular polygon whose each exterior angle has a measure of 45° .
8. The sum of measure of the three angles of a _____ is 180° .
9. A polygon which is both equiangular and equilateral is called a _____ polygon.
10. Number of sides of a nonagon.

Down

11. Name of the figure
12. The _____ angles of a parallelogram are supplementary.
13. A _____ is a quadrilateral whose pair of opposite sides are parallel.
14. The diagonals of a rectangle are of _____ length.
15. A five sided polygon.
16. The diagonals of a parallelogram _____ each other.
17. A quadrilateral having all the properties of a parallelogram and also that of a kite.



UNIT 6

VISUALISING SOLID SHAPES

(A) Main Concepts and Results

- **3D shapes/objects** are those which do not lie completely in a plane.
- 3D objects have different views from different positions.
- A **solid** is a polyhedron if it is made up of only polygonal faces, the faces meet at edges which are line segments and the edges meet at a point called vertex.
- Euler's formula for any polyhedron is,

$$F + V - E = 2$$

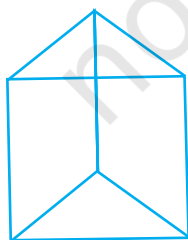
Where F stands for number of faces, V for number of vertices and E for number of edges.

- Types of polyhedrons:

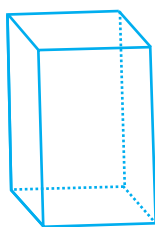
(a) Convex polyhedron

A convex polyhedron is one in which all faces make it convex.

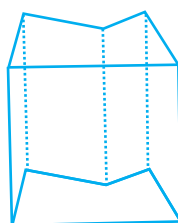
e.g.



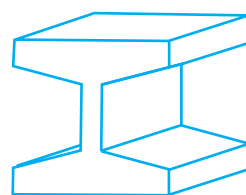
(1)



(2)



(3)



(4)

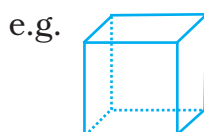
(1) and (2) are convex polyhedrons whereas

(3) and (4) are non convex polyhedron.

(b) **Regular polyhedra** or platonic solids:

A polyhedron is regular if its faces are congruent regular polygons and the same number of faces meet at each vertex.

For example, a cube is a platonic solid because all six of its faces are congruent squares. There are five such solids—tetrahedron, cube, octahedron, dodecahedron and icosahedron.



Cube

- A **prism** is a polyhedron whose bottom and top faces (known as bases) are congruent polygons and faces known as lateral faces are parallelograms (when the side faces are rectangles, the shape is known as right prism).
- A **pyramid** is a polyhedron whose base is a polygon and lateral faces are triangles.
- A **map** depicts the location of a particular object/place in relation to other objects/places.

Try This

The front, top and side of a figure are shown. Use centimetre cubes to build the figure. Then sketch the figure.

1.



Front

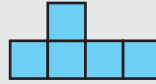


Top



Side

2.



Front



Top



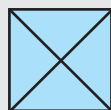
Side

3.

The views below represent a three-dimensional figure that cannot be built from cubes. Determine which three-dimensional figures match the views.



Front



Top



Side



A



B



C



D

- Scale is the relationship between the drawing's/model's dimensions to the actual object's dimensions.
- In a map, symbols are used to depict the different objects and places.
- Maps involve a scale which is fixed for a particular map.

Think and Discuss

1. **Explain** how you would find the surface area of an open-top box that is shaped like a rectangular prism.
2. **Describe** the shapes in a net used to cover a cylinder.

(B) Solved Examples

In examples 1 and 2, write the correct answer from the given four options.

Example 1 : A prism is a polyhedron whose lateral faces are
 (a) Circles (b) Triangles
 (c) Parallelograms (d) Rhombuses or Rhombi

Solution : Correct answer is (c).

Example 2 : A pyramid is a polyhedron whose lateral faces are
 (a) Rectangles (b) Triangles
 (c) Parallelograms (d) Rhombuses or Rhombi

Solution : Correct answer is (b).

In examples 3 and 4, fill in the blanks to make the statements true

Example 3 : In a regular polyhedron _____ number of faces meet at each vertex.

Solution : same.

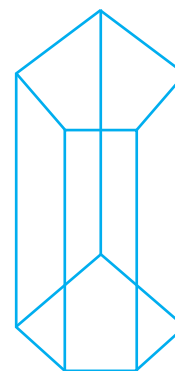
Example 4 : A pentagonal prism has _____ edges.

Solution : 15.

In examples 5 and 6, state whether the statements are true or false.

Example 5 : A sphere is a polyhedron.

Solution : False.



Pentagonal prism

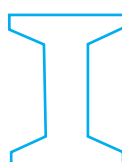
Example 6 : In a prism the lateral faces need not be congruent

Solution : True.

Example 7 : Draw the top, front and side views of the given solid.



Solution :



Front view



Top view



Side view

Activity

Use a compass and straight edge to create a larger version of each net on a cardboard. Fold each net into a polyhedron.

REGULAR POLYHEDRONS			
NAME	FACES	EXAMPLE	NET
Tetrahedron	4 triangles		
Octahedron	8 triangles		
Icosahedron	20 triangles		
Cube	6 squares		
Dodecahedron	12 pentagons		

Example 8 : Use isometric dot paper to sketch a rectangular prism with length 4 units, height 2 units and width 3 units.

Solution : Steps:

- (1) Draw a parallelogram with sides 4 units and 3 units.

This is top of the prism (Fig 1).

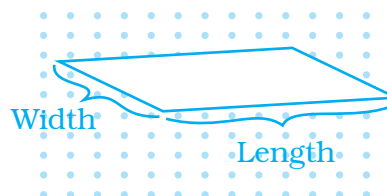


Fig. 1

- (2) Start at one vertex. Draw a line passing through two dots. Repeat for other three vertices. Draw the hidden edges as dashed line (Fig 2).

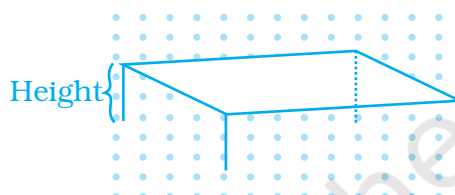


Fig. 2

- (3) Connect the ends of the lines to complete the prism (Fig 3).

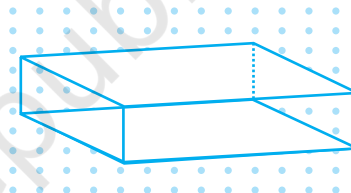


Fig. 3

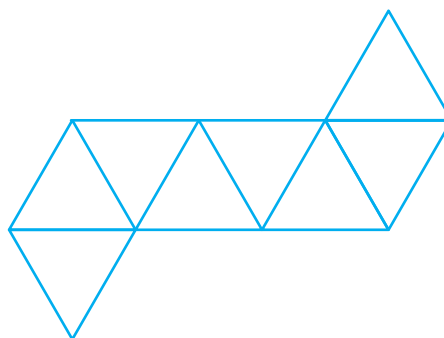
Try This

1. Complete the table for the number of vertices V, edges E and faces F for each of the polyhedrons you made.

2. **Make a Conjecture**
What do you think is true about the relationship between the number of vertices, edges and faces of a polyhedron?

POLYHEDRON	V	E	F	$V - E + F$
Tetrahedron				
Octahedron				
Icosahedron				
Cube				
Dodecahedron				

Example 9 : Identify the shape whose net is given below.

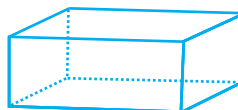


Solution : This shape is entirely made of equilateral triangles. When folded, it results in a regular octahedron. Note that since these are all equilateral and congruent faces, it is a **regular polyhedron**.

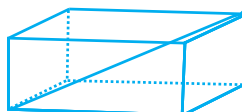
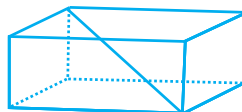
Use Nets to Create Polyhedrons

A polyhedron is formed by four or more polygons that intersect only at their edges. The faces of a regular polyhedron are all congruent regular polygons and the same number of faces intersect at each vertex. Regular polyhedrons are also called Platonic solid. There are exactly five regular polyhedrons.

Example 10 : The solid given below is a rectangular prism or cuboid. Make all the diagonals of this shape.

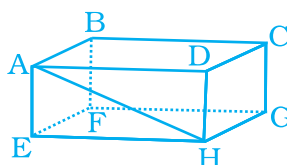


Solution : There are only four diagonals as shown below.

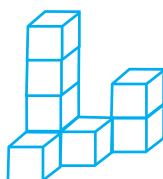


Note that in a 3D shape, diagonals connect two vertices that do not lie on the same face.

E.g. the line segment from A to H in figure below is not a diagonal for the solid. Diagonals must pass through the inside of the shape. However, AH is diagonal of face ADHE.



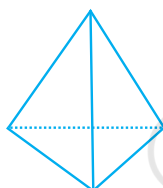
Example 11: Count the number of cubes in the given shapes.



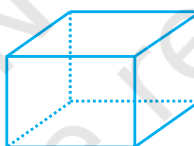
Solution : (i) 8 cubes

(ii) 6 cubes

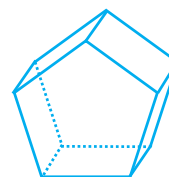
Example 12 : Name the following polyhedrons and verify the Euler's formula for each of them.



(a)



(b)



(c)

Solution :

S. No	Polyhedron	F	V	F + V	E	F + V - E
(a)	Tetrahedron	4	4	8	6	2
(b)	Cube	6	8	14	12	2
(c)	Pentagonal prism	7	10	17	15	2

Volume, the space inside a three-dimensional object, is measured in cubic unit. If the blocks you build are each 1 cubic unit, then the volume of a block structure is equal to the number of blocks in the structure. For example, a structure made from eight blocks has a volume of 8 cubic units. If the blocks have an edge length of 1 cm, the structure's volume is 8 cm³.

Example 13 : A polyhedron has 7 faces and 10 vertices. How many edges does the polyhedron have?

Solution : For any polyhedron,

$$F + V - E = 2$$

$$\text{Here, } F = 7, V = 10, E = ?$$

Using above formula,

$$\Rightarrow 7 + 10 - E = 2$$

$$\Rightarrow 17 - E = 2$$

$$\Rightarrow 17 - 2 = E$$

$$\Rightarrow E = 15$$

Example 14 : Find the number of vertices in a polyhedron which has 30 edges and 12 faces.

Solution : For any polyhedron,

$$F + V - E = 2$$

$$\text{Here, } F = 12, V = ?, E = 30$$

Using above formula,

$$12 + V - 30 = 2$$

$$V - 18 = 2$$

$$V = 2 + 18$$

$$V = 20$$

Example 15 : The distance between City A and City B on a map is given as 6 cm. If the scale represents 1 cm = 200 km, then find the actual distance between City A and City B.

Solution : Actual distance represented by 1 cm = 200 km

Think and Discuss



What is the surface area of a single block in square units?



If the edge lengths of a block are 2 cm, what is the block's surface area?

What is the volume of the structure at the right in cubic units?



What is the surface area of the structure above in square units?

(Remember: Count only the squares on the outside of the structure.)

Actual distance represented by 6 cm = 6×200 km
= 1200 km

So, actual distance between City A and City B is 1200 km.

Example 16 : Height of a building is 9 m and this building is represented by 9 cm on a map. What is the scale used for the map?

Solution : Scale of map = $\frac{\text{Size drawn}}{\text{Actual size}}$
= $\frac{9 \text{ cm}}{900 \text{ cm}}$ (because 9 m = 900 cm)
= $\frac{1}{100}$

Thus, scale is 1:100.

Example 17 : The scale on a map is 1 mm : 4 m. Find the distance on the map for an actual distance of 52 m.

Solution : Distance on map for an actual distance of 4 m = 1 mm
Distance on map for actual distance of 52 m = $\frac{1}{4} \times 52$
= 13 mm

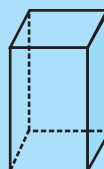
Thus, distance on map for actual distance of 52 m is 13 mm.

All **prisms** have two identical, parallel faces. These two faces are always polygons. A prism's other faces are always parallelograms.

A prism is sometimes referred to by the shape of the two identical faces on its ends. For example, a triangular prism has triangular faces on its ends, and a *rectangular prism* has rectangular faces on its ends.



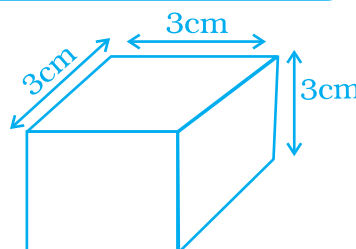
Triangular Prism



Rectangular Prism

Example 18 : Application of problem solving strategy

Determine the number of edges, vertices and in the following figure:



Solution : Understand and Explore the problem

- What information is given in the question?
A cube with one of its corner cut.
- What are you trying to find?
The no. of edges, vertices and faces.
- Is there any information that is not needed?
The measures of edges are not needed.

Plan a strategy

- Think of the definitions of an edge, vertex and faces and try to co-relate them to the figure given above.

Solve

- The polygonal regions are called faces, hence there are 7 faces.
- The line segment formed by the intersection of two faces is called edges, hence there are 15 edges.
- Edges meet at vertices which are points, hence there are 10 vertices.

Revise

- The above answer is verified using Euler's Formula.

$$F + V = E + 2$$

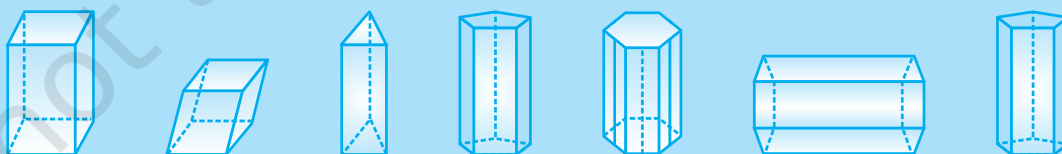
For the above problem, $F = 7$, $V = 10$, $E = 15$

$$F + V = 7 + 10 = 17$$

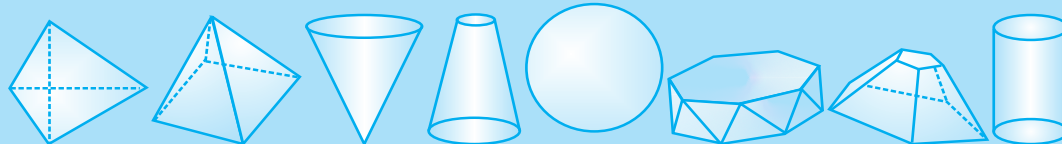
$$E + 2 = 15 + 2 = 17$$

Hence $F + V = E + 2$ is verified.

These figures are prisms.



These figures are not prisms.

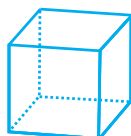


(C) Exercise

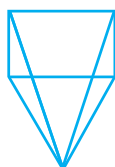
In each of the questions 1 to 21, out of four options only one is correct.

Write the correct answer.

1. Which amongst the following is not a polyhedron?



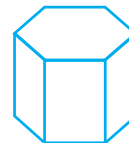
(a)



(b)



(c)



(d)

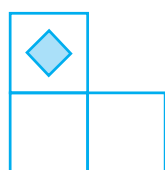
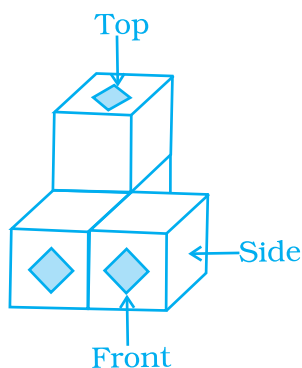
2. Which of the following will not form a polyhedron?
 (a) 3 triangles (b) 2 triangles and 3 parallelogram
 (c) 8 triangles (d) 1 pentagon and 5 triangles
3. Which of the following is a regular polyhedron?
 (a) Cuboid (b) Triangular prism
 (c) Cube (d) Square prism
4. Which of the following is a two Dimensional figure?
 (a) Rectangle (b) Rectangular Prism
 (c) Square Pyramid (d) Square Prism
5. Which of the following can be the base of a pyramid?
 (a) Line segment (b) Circle (c) Octagon (d) Oval
6. Which of the following 3D shapes does not have a vertex?
 (a) Pyramid (b) Prism (c) Cone (d) Sphere
7. Solid having only line segments as its edges is a
 (a) Polyhedron (b) Cone (c) Cylinder (d) Polygon

Think and Discuss

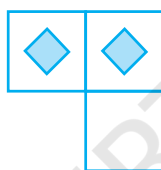
What do all the prisms have in common?

How are the non-prisms different from the prisms?

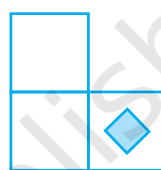
8. In a solid if $F = V = 5$, then the number of edges in this shape is
 (a) 6 (b) 4 (c) 8 (d) 2
9. Which of the following is the top view of the given shape?



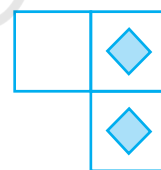
(a)



(b)

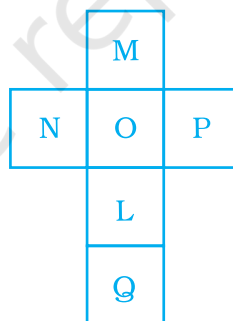


(c)



(d)

10. The net shown below can be folded into the shape of a cube. The face marked with the letter L is opposite to the face marked with which letter?



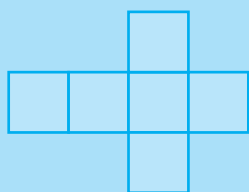
(a) M

(b) N

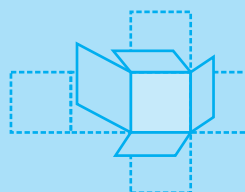
(c) Q

(d) O

A net is a flat figure that can be folded to form a closed, three-dimensional object. Such an object is called a solid.



Net

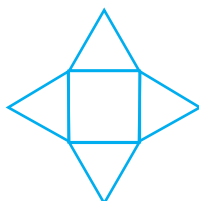


Solid Cube

11. Which of the nets given below will generate a cone?



(a)



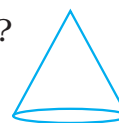
(b)



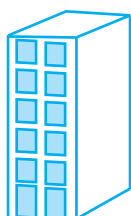
(c)



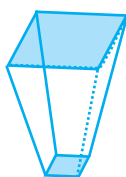
(d)



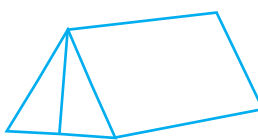
12. Which of the following is not a prism?



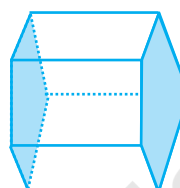
(a)



(b)



(c)



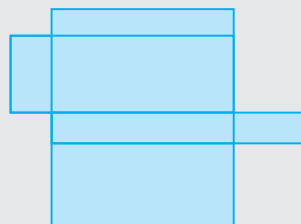
(d)

13. We have 4 congruent equilateral triangles. What do we need more to make a pyramid?

- (a) An equilateral triangle.
- (b) A square with same side length as of triangle.
- (c) 2 equilateral triangles with side length same as triangle.
- (d) 2 squares with side length same as triangle.

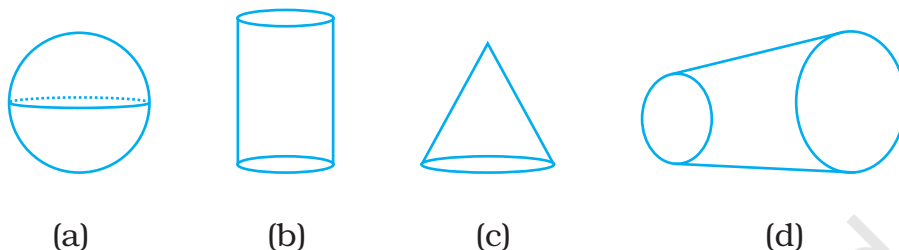
Share and Summarise

1. Explain how to find the surface area of a solid from the net for that solid.
2. Here is a net for a rectangular solid. Take whatever measurements you think are necessary to find the solid's volume and surface area. Explain what measurements you took and what you did with them.

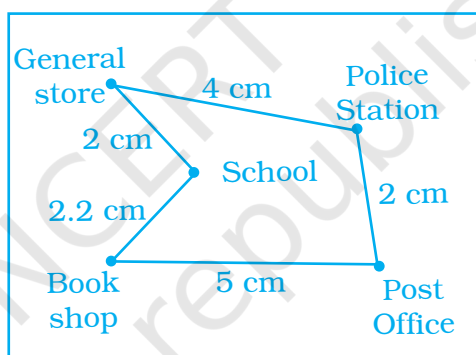


The **surface area** of a three-dimensional object is the space covering the object's surface. If you could pick up the object and flatten it so you could see all sides at once, the area of the flat figure would be the surface area. (Don't forget to count the bottom surface!) Surface area is also measured in square units.

- 14.** Side of a square garden is 30 m. If the scale used to draw its picture is 1 cm: 5 m, the perimeter of the square in the picture is
 (a) 20 cm (b) 24 cm (c) 28 cm (d) 30 cm
- 15.** Which of the following shapes has a vertex.



- 16.** In the given map, the distance between the places is shown using the scale 1 cm : 0.5 km. Then the actual distance (in km) between school and the book shop is



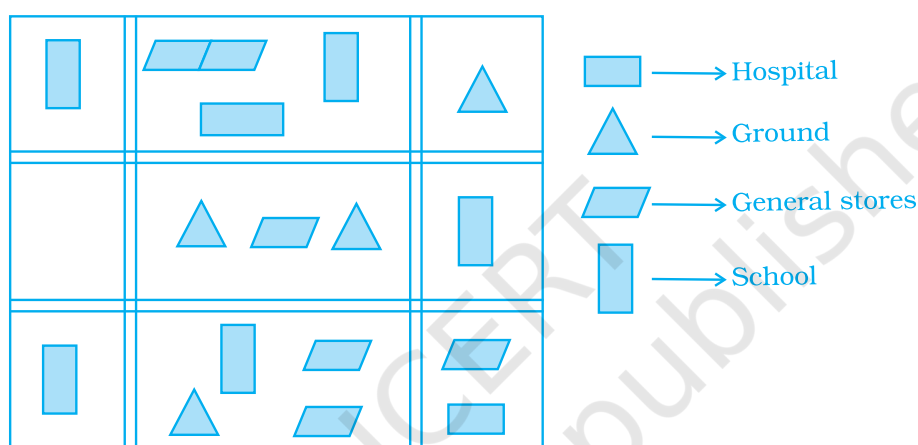
- (a) 1.25 (b) 2.5 (c) 2 (d) 1.1

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following.

1. What does the term area total tell you about the meaning of surface area?
2. The word edge comes from the Latin word *acer*, meaning *sharp*. How does the Latin root help you define an edge of a three-dimensional figure?
3. The word vertex can mean *peak* or *highest* point. What part of a cone or pyramid is the vertex?
4. The word prism comes from the Greek word *priein*, meaning *to saw*. How might you describe a prism in terms of something sawn or cut off?

17. Which of the following cannot be true for a polyhedron?
- (a) $V = 4, F = 4, E = 6$ (b) $V = 6, F = 8, E = 12$
(c) $V = 20, F = 12, E = 30$ (d) $V = 4, F = 6, E = 6$
18. In a blueprint of a room, an architect has shown the height of the room as 33 cm. If the actual height of the room is 330 cm, then the scale used by her is
- (a) 1:11 (b) 1:10 (c) 1:100 (d) 1:3
19. The following is the map of a town. Based on it answer question 19-21.



The number of hospitals in the town is

- (a) 1 (b) 2 (c) 3 (d) 4



There are two rectangular bases.
There are four rectangular faces.
The figure is a rectangular prism.



There is one rectangular base.
There are four triangular faces.
The figure is a rectangular pyramid.



There are two triangular bases.
There are three rectangular faces.
The figure is a triangular prism.



There is one hexagonal base.
There are six triangular faces.
The figure is a hexagonal pyramid.

20. The ratio of the number of general stores and that of the ground is

- (a) 1 : 2 (b) 2 : 1 (c) 2 : 3 (d) 3 : 2

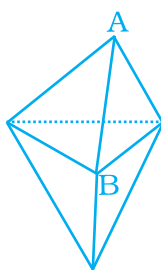
21. According to the map, the number of schools in the town is

- (a) 4 (b) 3 (c) 5 (d) 2

In questions 22 to 41, fill in the blanks to make the statements true.

22. Square prism is also called a _____.

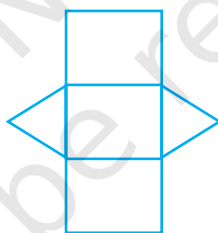
23. Rectangular prism is also called a _____.



24. In the figure, the number of faces meeting at B is _____.

25. A pyramid on an n sided polygon has _____ faces.

26. If a solid shape has 12 faces and 20 vertices, then the number of edges in this solid is _____.



27. The given net can be folded to make a _____.

28. A solid figure with only 1 vertex is a _____.

29. Total number of faces in a pyramid which has eight edges is _____.

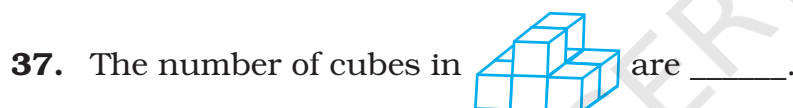
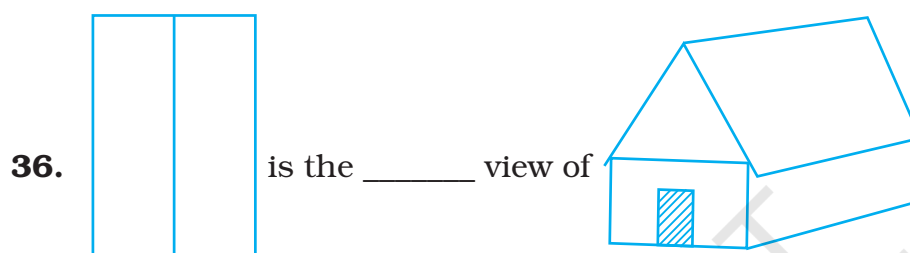
30. The net of a rectangular prism has _____ rectangles.

(Hint: Every square is a rectangle but every rectangle is not a square.)

Three-dimensional figures have three dimensions: length, width and height. A flat surface of a three-dimensional figure is a **face**. An **edge** is where two faces meet.

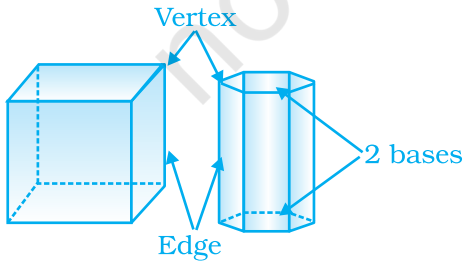
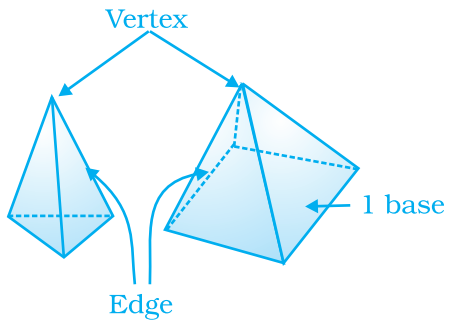
A **polyhedron** is a three-dimensional figure whose faces are all polygons. A **vertex** of a polyhedron is a point where three or more edges meet. The face that is used to name a polyhedron is called a **base**.

31. In a three-dimensional shape, diagonal is a line segment that joins two vertices that do not lie on the _____ face.
32. If 4 km on a map is represented by 1 cm, then 16 km is represented by _____ cm.
33. If actual distance between two places A and B is 110 km and it is represented on a map by 25 mm. Then the scale used is _____.
34. A pentagonal prism has _____ faces.
35. If a pyramid has a hexagonal base, then the number of vertices is _____.



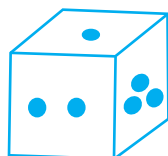
38. If the sum of number of vertices and faces in a polyhedron is 14, then the number of edges in that shape is _____.
39. Total number of regular polyhedra is _____.

A *prism* has two bases, and a *pyramid* has one base.

Prisms	Pyramids
A Prism is a polyhedron that has two parallel, congruent bases. The bases can be any polygon. The other faces are parallelograms. 	A pyramid is a polyhedron that has one base. The base can be any polygon. The other faces are triangles. 

40. A regular polyhedron is a solid made up of _____ faces.
41. For each of the following solids, identify the front, side and top views and write it in the space provided.

(a)



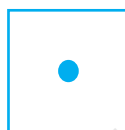
(i)



(ii)



(iii)



(b)



(i)



(ii)



(iii)





The faces are all polygons, so the figure is a polyhedron.

There is one triangular base.

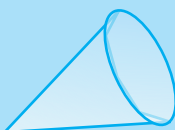
The figure is a triangular pyramid.



The faces are not all polygons, so the figure is not a polyhedron.

There are two circular bases.

The figure is a cylinder.

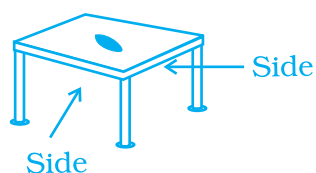


The faces are not all polygons, so the figure is not a polyhedron.

There is one circular base.

The figure is a cone.

(c)



(i)



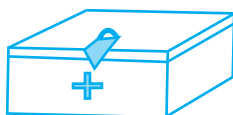
(ii)



(iii)



(d)



(i)



(ii)



(iii)



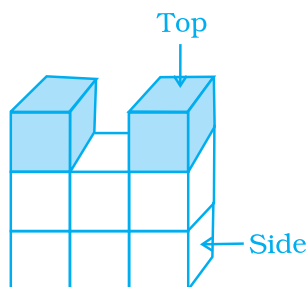
In each of the questions 42 to 61, state whether the following statements are true (T) or false (F).

42. The other name of cuboid is tetrahedron.
43. A polyhedron can have 3 faces.
44. A polyhedron with least number of faces is known as a triangular pyramid.
45. Regular octahedron has 8 congruent faces which are isosceles triangles.

Think and Discuss

1. **Explain** how to identify a prism or a pyramid.
2. **Compare and contrast** cylinders and prisms. How are they alike? How are they different?
3. **Compare and contrast** pyramids and cones. How are they alike? How are they different?

46. Pentagonal prism has 5 pentagons.
47. Every cylinder has 2 opposite faces as congruent circles, so it is also a prism.
48. Euler's formula is true for all three-dimensional shapes.
49. A polyhedron can have 10 faces, 20 edges and 15 vertices.
50. The top view of



is



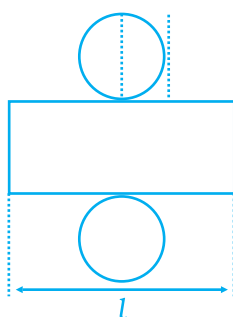
51. The number of edges in a parallelogram is 4.
52. Every solid shape has a unique net.
53. Pyramids do not have a diagonal.
54. The given shape is a cylinder.

55. A cuboid has atleast 4 diagonals.
56. All cubes are prisms.

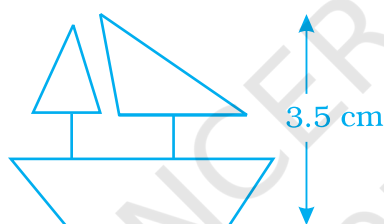
Cylinders and **cones** are not polyhedrons because they are not made of faces that are all polygons.

Cylinders	Cones
A cylinder has two parallel, congruent bases that are circles.	A cone has one base that is a circle and a surface that comes to a point called the vertex.

- 57.** A cylinder is a 3-D shape having two circular faces of different radii.
- 58.** On the basis of the given figure, the length of a rectangle in the net of a cylinder is same as circumference of circles in its net.



- 59.** If a length of 100 m is represented on a map by 1 cm, then the actual distance corresponding to 2 cm is 200 m.
- 60.** The model of a ship shown is of height 3.5 cm. The actual height of the ship is 210 cm if the scale chosen is 1: 60.

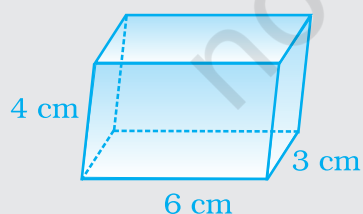


- 61.** The actual width of a store room is 280 cm. If the scale chosen to make its drawing is 1:7, then the width of the room in the drawing will be 40 cm.

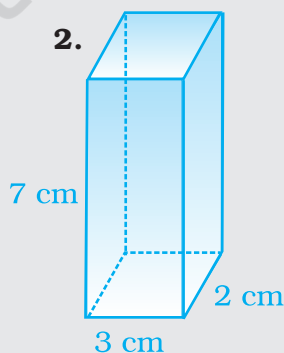
Try This

Find the volume of each prism.

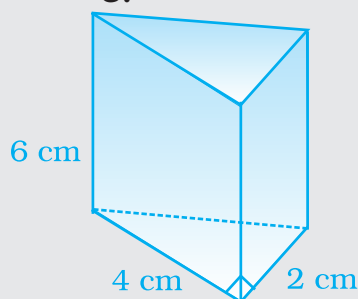
1.



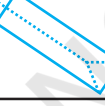
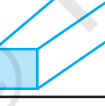
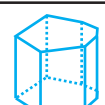
2.



3.



62. Complete the table given below:

S. No	Solid	Shape of Solid	Number of faces F	Number of Vertices V	Number of edges E	F + V	E + 2
a.	Cuboid						
b.	Triangular Pyramid						
c.	Square Pyramid						
d.	Rectangular Pyramid						
e.	Pentagonal Pyramid						
f.	Hexagonal Pyramid						
g.	Triangular Prism						
h.	Square Prism						
i.	Cube						
j.	Pentagonal Prism						
k.	Octagonal Prism						
l.	Heptagonal Prism						

63. How many faces does each of the following solids, have?

(a) Tetrahedron

(b) Hexahedron

(c) Octagonal Pyramid

(d) Octahedron

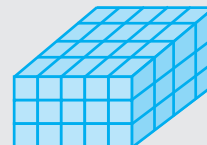
- 64.** Draw a prism with its base as regular hexagon with one of its face facing you. Now draw the top view, front view and side view of this solid.
- 65.** How many vertices does each of the following solids have?
- | | |
|-----------------|-----------------------|
| (a) Cone | (b) Cylinder |
| (c) Sphere | (d) Octagonal Pyramid |
| (e) Tetrahedron | (f) Hexagonal Prism |
- 66.** How many edges does each of following solids have?
- | | |
|---------------------|-----------------------|
| (a) Cone | (b) Cylinder |
| (c) Sphere | (d) Octagonal Pyramid |
| (e) Hexagonal Prism | (f) Kaleidoscope |

Explore the volume of Prisms and Cylinders

The volume of a three-dimensional figure is the number of cubes it can hold. One cube represents one cubic unit of volume.

Activity

- Use centimetre cubes to build the rectangular prism shown. What are the length, width and height of the prism? How many cubes does the prism hold?
- You can find out how many cubes the prism holds without counting every cube. First look at the prism from above. How can you find the number of cubes in the top layer without counting every cube?
- Now look at the prism from the side. How many layers does the prism have? How can you use this to find the total number of cubes in the prism?



Top

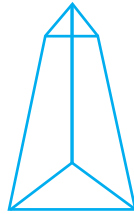


Side

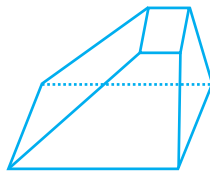
Think and Discuss

- Describe a shortcut for finding the number of cubes in a rectangular prism.
- Suppose you know the area of the base of a prism and the height of the prism. How can you find the prism's volume?
- Let the area of the base of a prism be B and the height of the prism be h . Write a formula for the prism's volume V .

67. Look at the shapes given below and state which of these are polyhedra using Euler's formula.



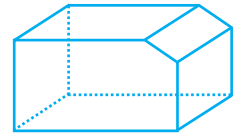
(a)



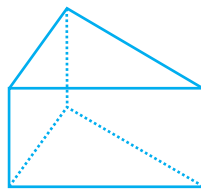
(b)



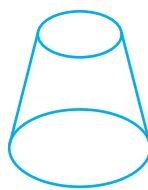
(c)



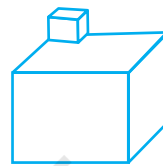
(d)



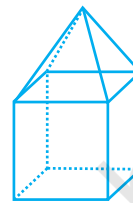
(e)



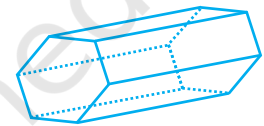
(f)



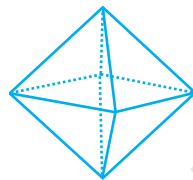
(g)



(h)



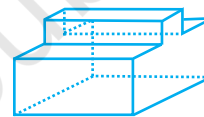
(i)



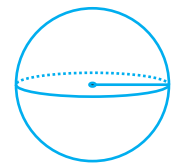
(j)



(k)



(l)

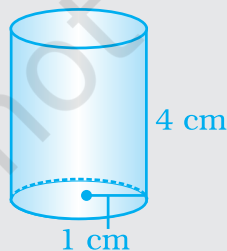


(m)

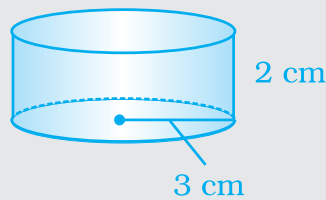
Try This

Find the volume of each cylinder.
Use 3.14 for π and round to the nearest tenth.

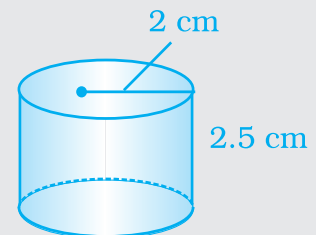
1.



2.

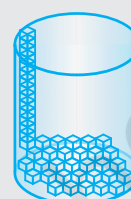
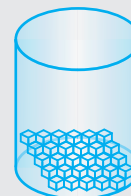
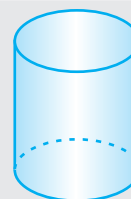


3.

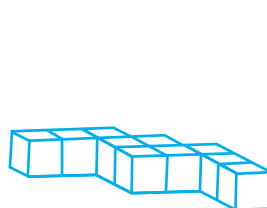


Activity

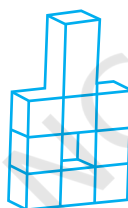
1. Use a similar process to that in earlier Activity to develop the formula for the volume of a cylinder. You will need an empty can or other cylindrical pot. Remove one of the bases.
2. Arrange centimetre cubes in a single layer at the bottom of the cylinder. Fit as many cubes into the layer as possible. How many cubes are in this layer?
3. To find how many layers of cubes would fit in the cylinder, make a stack of cubes along the inside of the cylinder. How many layers would fit in the cylinder?
4. How can you use what you know to find the approximate number of cubes that would fit in the cylinder?



68. Count the number of cubes in the given shapes.



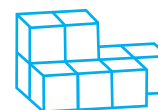
(a)



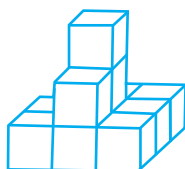
(b)



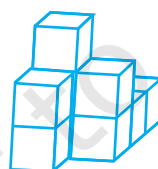
(c)



(d)



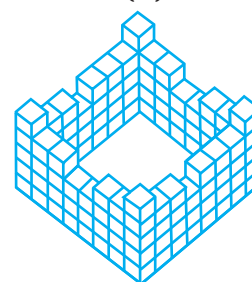
(e)



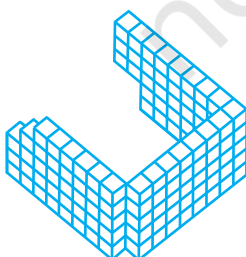
(f)



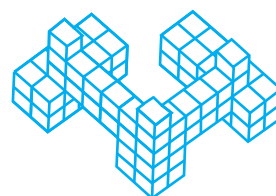
(g)



(h)



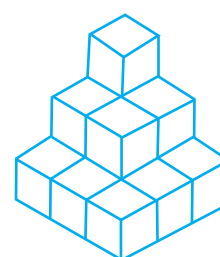
(i)



(j)



(k)

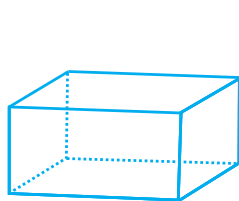


(l)

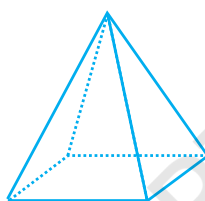
Think and Discuss

1. Suppose you know the area of the base of a cylinder and the height of the cylinder. How can you find the cylinder's volume?
2. Let the area of the base of a cylinder be B and the height of the cylinder be h . Write a formula for the cylinder's volume V .
3. The base of a cylinder is a circle with radius r . How can you find the area of the base? How can you use this in your formula for the volume of a cylinder?

69. Draw the front, side and top view of the given shapes.



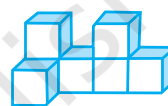
(a)



(b)



(c)



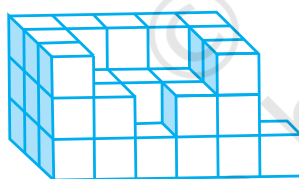
(d)



(e)



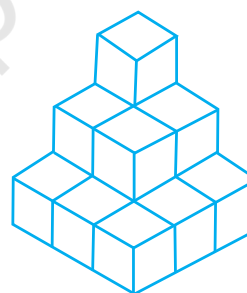
(f)



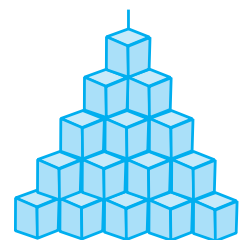
(g)



(h)



(i)



(j)

Try This

1. Use a net to construct a rectangular prism that is 3 cm by 6 cm by 9 cm.
2. Use a net to construct a cylinder with a height of 3 cm and a radius of 1.5 cm (Hint: The length of the rectangle in the net must match the circumference of the circles, so the length should be $2\pi r = 2\pi (1.5) \approx 9.42$ cm.)

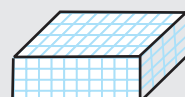
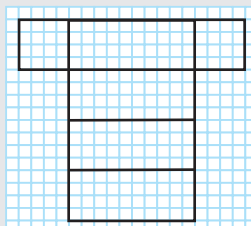
Use Nets to Build Prisms and Cylinders

A net is a pattern of two-dimensions that can be folded to make a three-dimensional figure. You can use 1 cm graph paper to help you make nets.

Activity

- Use a net to construct a rectangular prism.

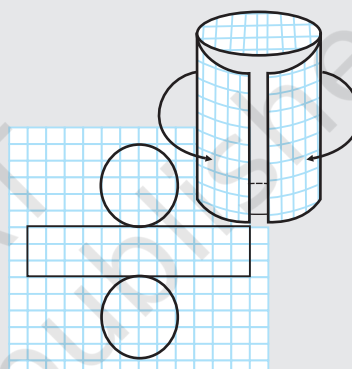
a. Draw the net at right on a piece of graph paper. Each rectangle is 10 squares by 4 squares. The two squares are 4 small squares on each side.



b. Cut out the net. Fold the net along the edges of each rectangle to make a rectangular prism. Tape the edges to hold them in place.

- Use a net to construct a cylinder.

a. Draw the net at right on a piece of graph paper. The rectangle is 25 squares by 8 squares. Use a compass to make the circles. Each circle has a radius of 4 squares.



b. Cut out the net. Fold the net as shown to make a cylinder. Tape the edges to hold them in place.

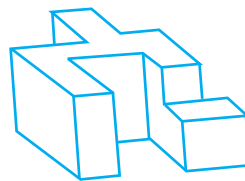
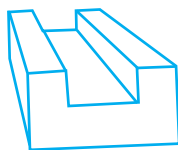
- Using Euler's formula, find the value of unknown x , y , z , p , q , r , in the following table.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Faces	7	y	9	p	6	8
Vertices	10	12	z	6	q	11
Edges	x	18	16	12	12	r

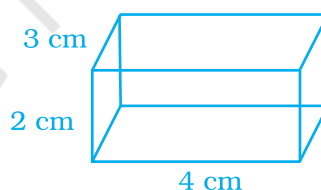
Think and Discuss

- What are the dimensions, in inches, of the rectangular prism that you built?
- What is the height, in cm, of the cylinder that you built? What is the cylinder's radius?

71. Can a polyhedron have $V = F = 9$ and $E = 16$? If yes, draw its figure.
72. Check whether a polyhedron can have $V = 12$, $E = 6$ and $F = 8$.
73. A polyhedron has 60 edges and 40 vertices. Find the number of its faces.
74. Find the number of faces in the given shapes:



75. A polyhedron has 20 faces and 12 vertices. Find the edges of the polyhedron.
76. A solid has forty faces and, sixty edges. Find the number of vertices of the solid.
77. Draw the net of a regular hexahedron with side 3 cm. (Hint: Regular hexahedron - cube)
78. Draw the net of a regular tetrahedron with side 6 cm.
79. Draw the net of the following cuboid:



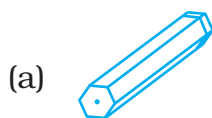
Think and Discuss

1. **Tell** whether a figure's surface area has increased or decreased if each dimension of the figure is changed by a factor of $\frac{1}{3}$.
2. **Explain** how the surface area of a figure is changed if each dimension is multiplied by a factor of 3.
3. **Explain** how the volume of a figure is changed if each dimension is multiplied by a factor of 2.

80. Match the following:

Figure

Name



(a) Hexahedron



(b) Hexagonal Prism



(c) Square Pyramid



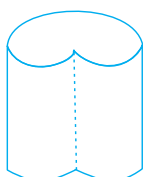
(d) Cone

81. Complete the table given below by putting tick mark across the respective property found in the solids mentioned.

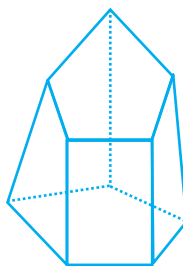
Solids

Properties	Cone	Cylinder	Prism	Pyramid
1. The figure is a Polyhedron.				
2. The figure has diagonals.				
3. The shape has curved edges				
4. The base of figure is a polygon.				
5. The bases are congruent				
6. The base of figure is a polygon and other faces meet at a single point.				
7. The base of figure is a curved edge and other faces meet at a single point.				

82. Draw the net of the following shape.

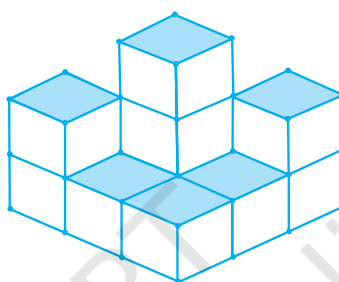


83. Draw the net of the following solid.

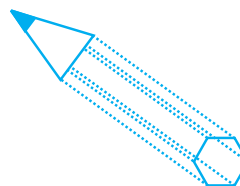


(Hint: Pentagons are not congruent.)

84. Find the number of cubes in the base layer of the following figure.

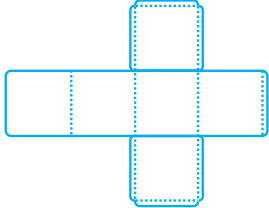
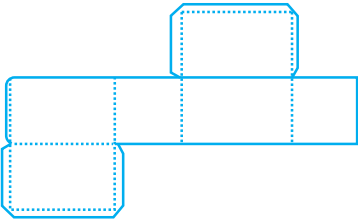
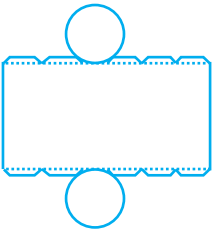
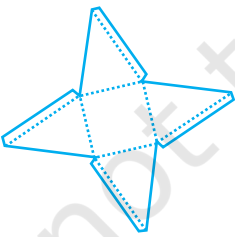
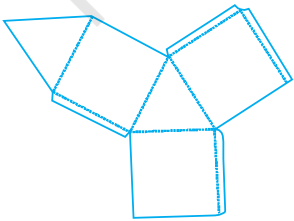


85. In the above figure, if only the shaded cubes are visible from the top, draw the base layer.
86. How many faces, edges and vertices does a pyramid have with n sided polygon as its base?
87. Draw a figure that represents your mathematics textbook. What is the name of this figure? Is it a prism?
88. In the given figures, identify the different shapes involved.

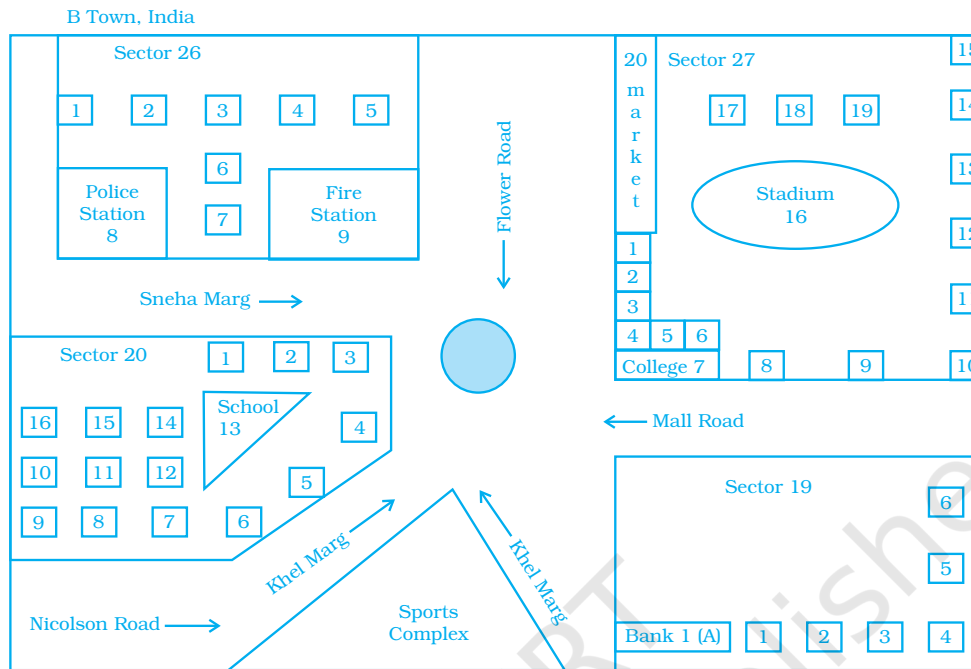


89. What figure is formed if only the height of a cube is increased or decreased?
90. Use isometric dot paper to draw each figure.
- A tetrahedron.
 - A rectangular prism with length 4 units, width 2 units and height 2 units.

91. Identify the nets given below and mention the name of the corresponding solid in the space provided.

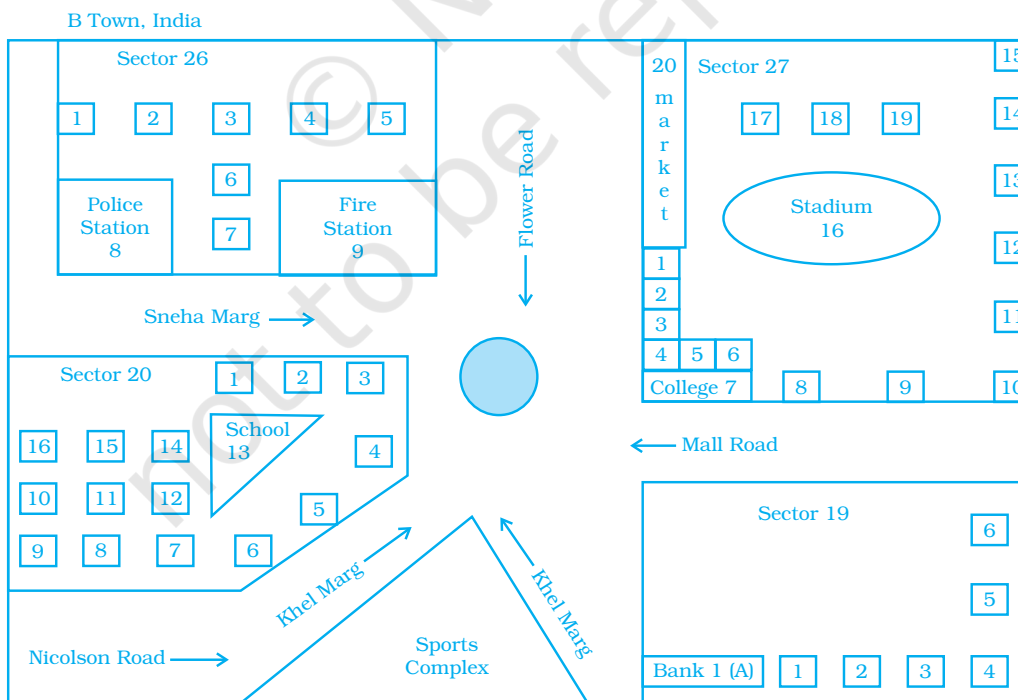
Nets	Name of Solid
(a) 	
(b) 	
(c) 	
(d) 	
(e) 	
(f) 	

(c) Name of which road is similar to the name of some month.



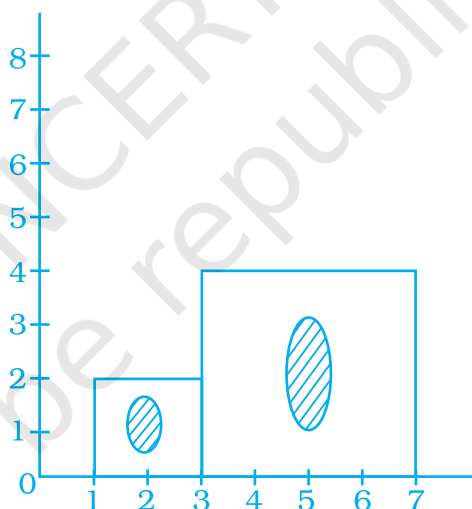
95. Look at the map given below.

☐ Houses



Now answer the following questions.

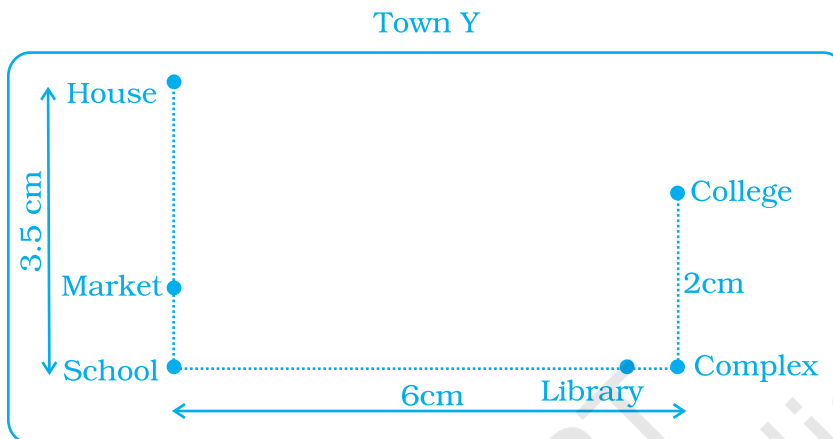
- (a) Name the roads that meet at round about.
 - (b) What is the address of the stadium?
 - (c) On which road is the Police Station situated?
 - (d) If Ritika stays adjacent to bank and you have to send her a card, what address will you write?
 - (e) Which sector has maximum number of houses?
 - (f) In which sector is Fire Station located?
 - (g) In the map, how many sectors have been shown?
- 96.** A photographer uses a computer program to enlarge a photograph. What is the scale according to which the width has enlarged?



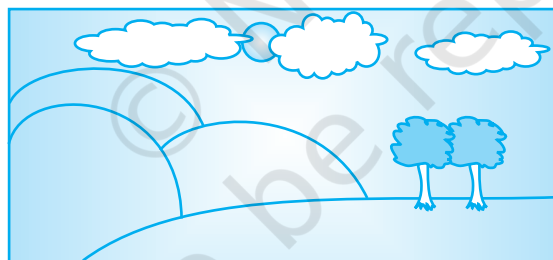
- 97.** The side of a square board is 50 cm. A student has to draw its image in her notebook. If the drawing of the square board in the notebook has perimeter of 40 cm, then by which scale the figure has been drawn?
- 98.** The distance between school and house of a girl is given by 5 cm in a picture, using the scale 1 cm : 5 km. Find the actual distance between the two places?

99. Use a ruler to measure the distance in cm between the places joined by dotted lines. If the map has been drawn using the scale 1 cm : 10 km, find the actual distances between

- (1) School and Library
- (2) College and Complex
- (3) House and School



100. The actual length of a painting was 2 m. What is its length in the photograph if the scale used is 1 mm : 20 cm.



101. Find the scale.

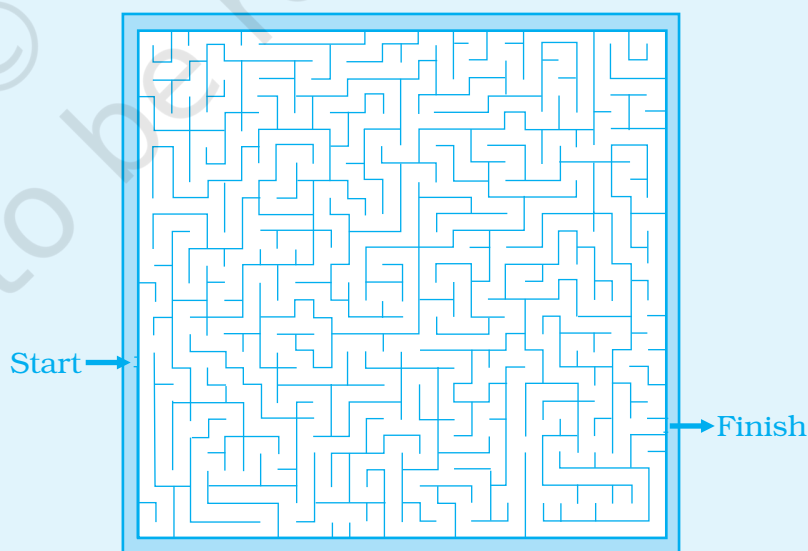
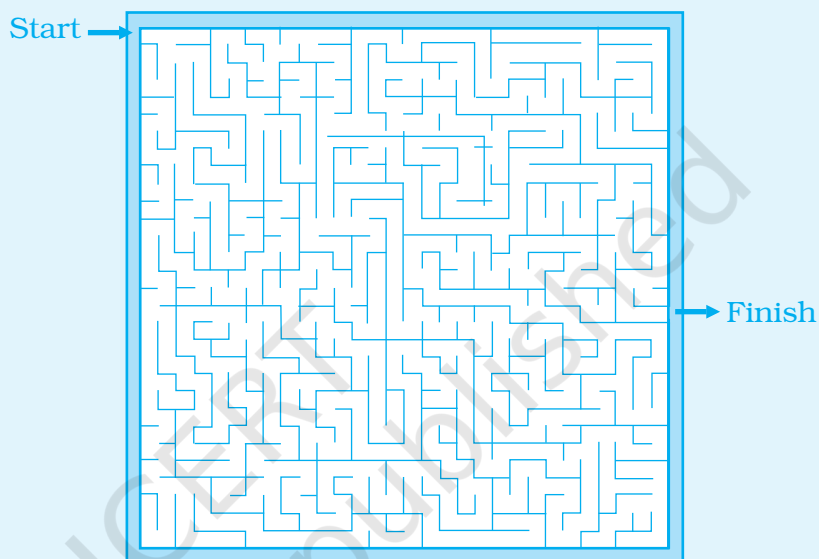
- (a) Actual size 12 m
Drawing size 3 cm
- (b) Actual size 45 feet
Drawing size 5 inches

102. In a town, an ice cream parlour has displayed an ice cream sculpture of height 360 cm. The parlour claims that these ice creams and the sculpture are in the scale 1:30. What is the height of the ice creams served?

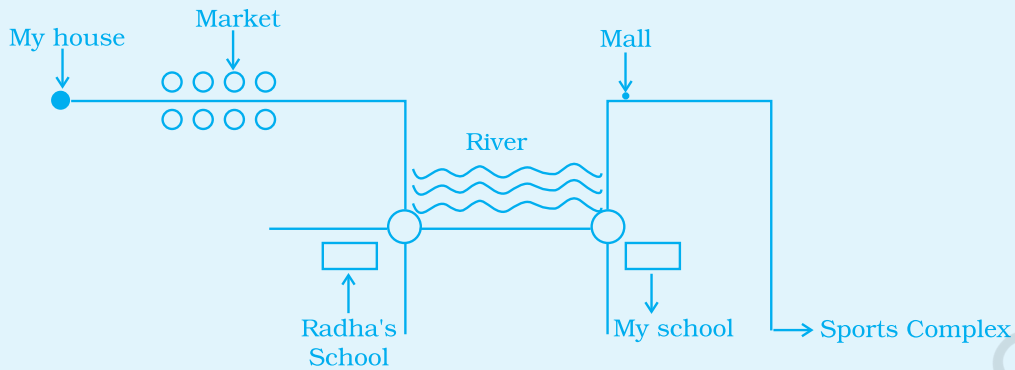
(D) Activities

Activity 1 : Find the most appropriate way to reach from start to finish.

(Use a coloured pen of your choice to show the path traced.)



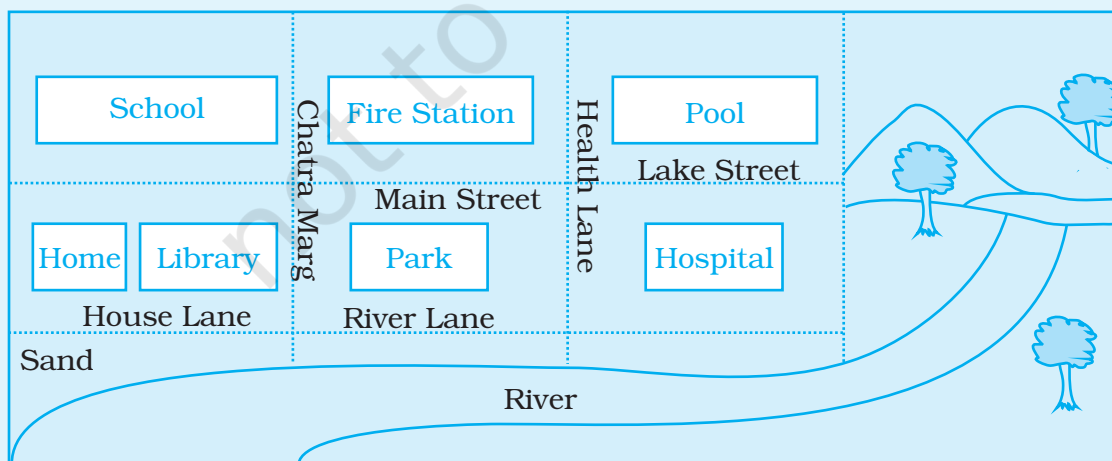
Activity 2 : Rohit is a 7 year old young boy. His uncle Raj asked him to draw a map to reach Sport's Complex from his (Rohit's) house. He drew the following map.



Can you help Uncle Raj by drawing a better map considering yourself as Rohit's Father/Mother.

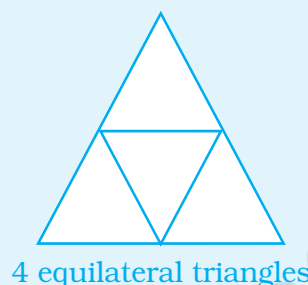
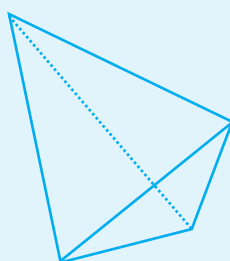
Activity 3 : Look at the map of city given on the next page:

- Colour the map using the given colour code as follows.
Blue-water, Red-Fire station, White-Hospital, Green-Parks, Cream-River Bed, Brown-Mountains.
- Mark the shortest route from House to Pool with the help of arrows.
- Put X at the intersection of Chatra Marg and House Lane.



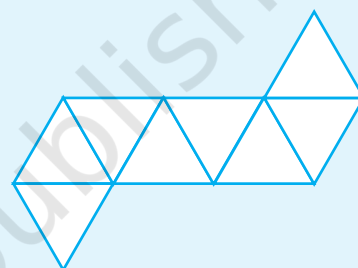
Activity 4 : Draw the nets given below on coloured sheets by taking each side of the triangle as 3cm, and try to form the shapes shown in front of it. Count the number of edges, faces and vertices in ends and verify the Euler's formula.

(a)



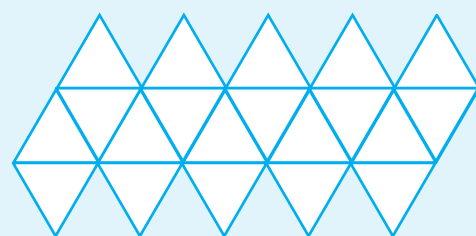
4 equilateral triangles

(b)



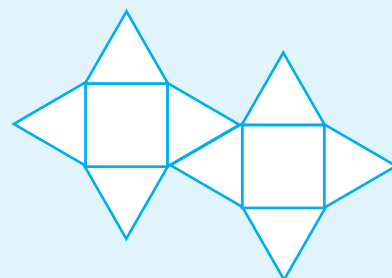
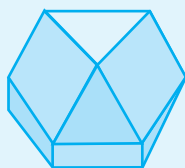
8 equilateral triangles

(c)



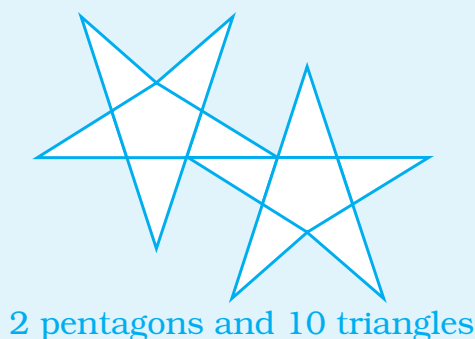
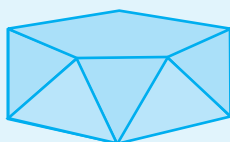
20 equilateral triangles

(d)



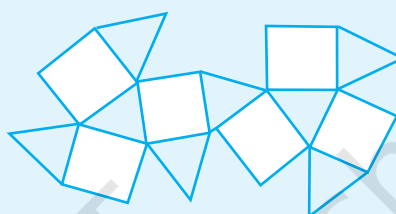
2 squares and 8 triangles

(e)



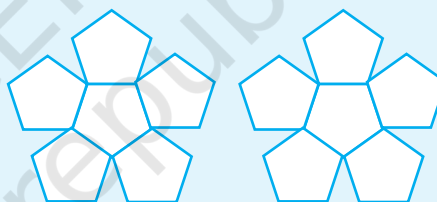
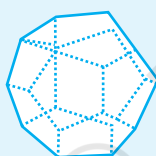
2 pentagons and 10 triangles

(f)



6 squares and 8 triangles

(g)

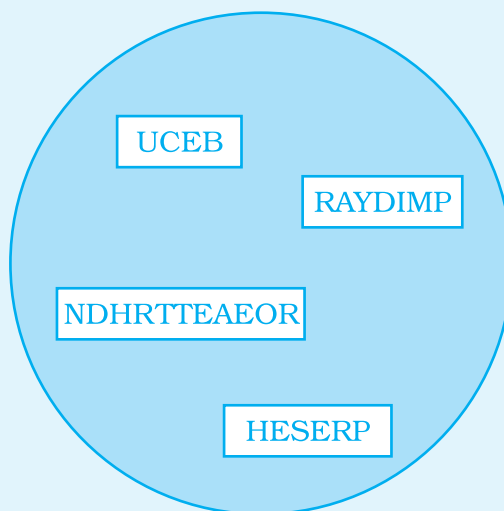


12 pentagons

Activity 5 : Guess who am I? My name is given in the box below.
Oops! Spelling of my name is jumbled up.

Try to identify it from the clues given below and write it in the blank space.

- (1) I am a polyhedron with least number of faces. _____
- (2) I am a prism whose all faces are square. _____
- (3) Looks like marbles but have no vertex. _____
- (4) I am a solid whose base is polygonal and other faces are triangles. _____



6. Draw the map of your locality, showing the details of your sector/ block. Also highlight the appropriate landmarks which will help your friend to locate your house in your sector/block.

Activity 7 : Crossword Puzzle

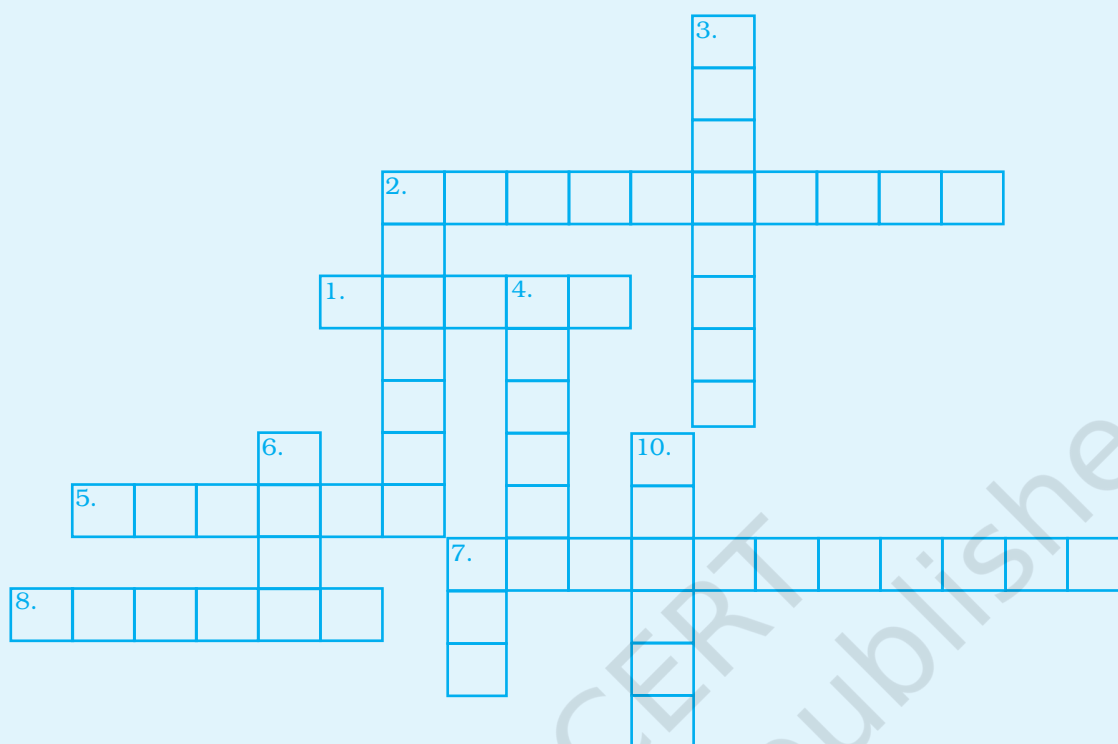
Answer the following based on the hints given below.

Across

1. Polyhedron whose lateral faces are parallelograms.
2. Prism having fifteen edges.
5. Another name for a square prism.
7. Polyhedron made up of four triangles.
9. Polyhedron made up of convex polygons.

Down

2. Polyhedron whose lateral faces are triangles.
3. In a solid shape, the line segment joining two vertices not lying on the same face.
4. A 3-D shape having no vertex.
6. A solid figure having only one vertex.
7. Number of pentagons in a pentagonal prism.
10. Point where edges of a solid shape meet.



Rough Work

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UNIT 7

ALGEBRAIC EXPRESSIONS, IDENTITIES AND FACTORISATION

(A) Main Concepts and Results

(i) Algebraic Expression

- Terms are formed by the product of variables and constants, e.g. $-3xy$, $2xyz$, $5x^2$, etc.
- Terms are added to form expressions, e.g. $-2xy + 5x^2$.
- Expressions that contain exactly one, two and three terms are called monomials, binomials and trinomials, respectively.
- In general, any expression containing one or more terms with non-zero coefficients (and with variables having non-negative exponents) is called a polynomial.
- Like terms are formed from the same variables and the powers of these variables are also the same. But coefficients of like terms need not be the same.
- There are number of situations like finding the area of rectangle, triangle, etc. in which we need to multiply algebraic expressions.
- Multiplication of two algebraic expressions is again an algebraic expression.
- A monomial multiplied by a monomial always gives a monomial.
- While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial using the distributive law $a(b + c) = ab + ac$.

- In the multiplication of a polynomial by a binomial (or trinomial), we multiply term by term, i.e. every term of the polynomial is multiplied by every term in the binomial (or trinomial) using the distributive property.
- An identity is an equality, which is true for all values of its variables in the equality, i.e. an identity is a universal truth.
- An equation is true only for certain values of its variables.
- Some standard identities:
 - (i) $(a + b)^2 = a^2 + 2ab + b^2$
 - (ii) $(a - b)^2 = a^2 - 2ab + b^2$
 - (iii) $(a + b)(a - b) = a^2 - b^2$
 - (iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(ii) Factorisation

- Representation of an algebraic expression as the product of two or more expressions is called factorisation. Each such expression is called a factor of the given algebraic expression.
- When we factorise an expression, we write it as a product of its factors. These factors may be numbers, algebraic (or literal) variables or algebraic expressions.

A **formula** is an equation stating a relationship between two or more variables. For example, the number of square units in the area (A) of a rectangle is equal to the number of units of length (l) multiplied by the number of units of width (w). Therefore, the formula for the area of a rectangle is $A = lw$.

Sometimes, you can evaluate a variable in a formula by using the given information.

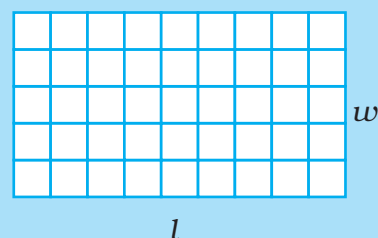
In the figure shown, the length is 9 units and the width is 5 units.

$$A = lw$$

$$A = 95$$

$$A = 45$$

The area is 45 square units or 45 units².



At other times, you must use your knowledge of equations to solve for a variable in a formula.

- An irreducible factor is a factor which cannot be expressed further as a product of factors. Such a factorisation is called an irreducible factorisation or complete factorisation.
- A factor which occurs in each term is called the common factor.
- The factorisation done by using the distributive law (property) is called the common factor method of factorisation.
- Sometimes, many of the expressions to be factorised are of the form or can be put in the form: $a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$, $a^2 - b^2$ or $x^2 + (a + b)x + ab$. These expressions can be easily factorised using identities:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

- In the division of a polynomial by a monomial, we carry out the division by dividing each term of the polynomial by the monomial.
- In the division of a polynomial by a polynomial, we factorise both the polynomials and cancel their common factors.

(B) Solved Examples

In examples 1 to 4, there are four options given out of which one is correct. Write the correct answer.

Example 1 : Which is the like term as $24a^2bc$?

- (a) $13 \times 8a \times 2b \times c \times a$ (b) $8 \times 3 \times a \times b \times c$
 (c) $3 \times 8 \times a \times b \times c \times c$ (d) $3 \times 8 \times a \times b \times b \times c$

Solution : The correct answer is (a).

Example 2 : Which of the following is an identity?

- (a) $(p + q)^2 = p^2 + q^2$ (b) $p^2 - q^2 = (p - q)^2$
 (c) $p^2 - q^2 = p^2 + 2pq - q^2$ (d) $(p + q)^2 = p^2 + 2pq + q^2$

Solution : The correct answer is (d).

Example 3 : The irreducible factorisation of $3a^3 + 6a$ is

- (a) $3a(a^2 + 2)$ (b) $3(a^3 + 2)$
 (c) $a(3a^2 + 6)$ (d) $3 \times a \times a \times a + 2 \times 3 \times a$

Solution : The correct answer is (a).

Example 4 : $a(b + c) = ab + ac$ is

- (a) commutative property (b) distributive property
 (c) associative property (d) closure property

Solution : The correct answer is (b).

In examples 5 and 6, fill in the blanks to make the statements true.

Example 5 : The representation of an expression as the product of its factors is called _____.

Solution : Factorisation.

Example 6 : $(x + a)(x + b) = x^2 + (a + b)x + \underline{\hspace{2cm}}$.

Solution : ab .

In examples 7 to 9, state whether the statements are true (T) or false (F).

Example 7 : An identity is true for all values of its variables.

Solution : True.

Example 8 : Common factor of x^2y and $-xy^2$ is xy .

Solution : True.

Example 9 : $(3x + 3x^2) \div 3x = 3x^2$

Solution : False.

Example 10 : Simplify (i) $-pqr(p^2 + q^2 + r^2)$

(ii) $(px + qy)(ax - by)$

Solution :

$$\begin{aligned} \text{(i)} \quad & -pqr(p^2 + q^2 + r^2) \\ &= -(pqr) \times p^2 - (pqr) \times q^2 - (pqr) \times r^2 \\ &= -p^3qr - pq^3r - pqr^3 \end{aligned}$$

(ii) $(px + qy)(ax - by)$

$$\begin{aligned} &= px(ax - by) + qy(ax - by) \\ &= apx^2 - pbxy + aqxy - qby^2 \end{aligned}$$

Real-Life Math

Algebra in the Strongest Places : You might think that algebra is a topic found only in textbooks, but you can find algebra all around you – in some of the strongest places.

Did you know there is a relationship between the speed at which ants crawl and the air temperature? If you were to find some ants outside and time them as they crawled, you could actually estimate the temperature. Here is the algebraic equation that describes this relationship.

Celsius temperature

$$t = 15s + 3$$

ant speed in centimetres per seconds

There are many ordinary and extraordinary places where you will encounter algebra.

Think About it : What do you think is the speed of a typical ant?

Example 11 : Find the expansion of the following using suitable identity.

$$(i) (3x + 7y)(3x - 7y) \quad (ii) \left(\frac{4x}{5} + \frac{y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$$

Solution : (i) $(3x + 7y)(3x - 7y)$

Since $(a + b)(a - b) = a^2 - b^2$, therefore

$$\begin{aligned} (3x + 7y)(3x - 7y) &= (3x)^2 - (7y)^2 \\ &= 9x^2 - 49y^2 \end{aligned}$$

$$(ii) \left(\frac{4x}{5} + \frac{y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$$

Since $(x + a)(x + b) = x^2 + (a + b)x + ab$, therefore

$$\begin{aligned} &\left(\frac{4x}{5} + \frac{y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right) \\ &= \left(\frac{4x}{5}\right)^2 + \left(\frac{y}{4} + \frac{3y}{4}\right) \times \frac{4x}{5} + \frac{y}{4} \times \frac{3y}{4} \\ &= \left[\text{Here, } x = \frac{4x}{5}, a = \frac{y}{4} \text{ and } b = \frac{3y}{4} \right] \\ &= \frac{16x^2}{25} + \frac{4y}{4} \times \frac{4x}{5} + \frac{3y^2}{16} \\ &= \frac{16x^2}{25} + \frac{4xy}{5} + \frac{3y^2}{16} \end{aligned}$$

Example 12 : Factorise the following.

(i) $21x^2y^3 + 27x^3y^2$

(ii) $a^3 - 4a^2 + 12 - 3a$

(iii) $4x^2 - 20x + 25$

(iv) $\frac{y^2}{9} - 9$

(v) $x^4 - 256$

Solution

: (i) $21x^2y^3 + 27x^3y^2$

$$= 3 \times 7 \times x \times x \times y \times y \times y + 3 \times 3 \times 3 \times x \times x \times x \times y \times y$$

$$= 3 \times x \times x \times y \times y (7y + 9x) \text{ (Using } ab + ac = a(b + c))$$

$$= 3x^2y^2 (7y + 9x)$$

(ii) $a^3 - 4a^2 + 12 - 3a$

$$= a^2 (a - 4) - 3a + 12$$

$$= a^2 (a - 4) - 3(a - 4)$$

$$= (a - 4)(a^2 - 3)$$

(iii) $4x^2 - 20x + 25$

$$= (2x)^2 - 2 \times 2x \times 5 + (5)^2$$

$$= (2x - 5)^2 \text{ (Since } a^2 - 2ab + b^2 = (a - b)^2 \text{)}$$

$$= (2x - 5)(2x - 5)$$

(iv) $\frac{y^2}{9} - 9$

$$= \left(\frac{y}{3}\right)^2 - (3)^2$$

If there are two numbers you don't know, that's not a problem. You can use two different variables, one for each unknown number.

In Words	Numbers
The sum of a and b	$a + b$
The product of v and w	$v \times w$, or vw
p is subtracted from $9q$	$9q - p$

You can use expressions with two q - p (or more) variables to represent situations with more than one unknown quantity.

An equation involving variables can be true for all values of the variable – for example, $y + y = 2y$ (this kind of equation is usually called an identity).

Or it can be true for only particular values of the variable – for example, $2y + 3 = 11$, which is true only if $y = 4$.

Finding the values that make an equation true is called solving the equation.

$$= \left(\frac{y}{3} + 3\right) \left(\frac{y}{3} - 3\right) \text{ (Since } a^2 - b^2 = (a + b)(a - b) \text{)}$$

$$\begin{aligned} \text{(v) } x^4 - 256 &= (x^2)^2 - (16)^2 \\ &= (x^2 + 16)(x^2 - 16) \text{ (using } a^2 - b^2 = (a + b)(a - b) \text{)} \\ &= (x^2 + 16)(x^2 - 4^2) \\ &= (x^2 + 16)(x + 4)(x - 4) \text{ (using } a^2 - b^2 = (a + b)(a - b) \text{)} \end{aligned}$$

Example 13 : Evaluate using suitable identities.

$$\text{(i) } (48)^2 \qquad \text{(ii) } 181^2 - 19^2$$

$$\text{(iii) } 497 \times 505 \qquad \text{(iv) } 2.07 \times 1.93$$

Solution : (i) $(48)^2$

$$= (50 - 2)^2$$

Since $(a - b)^2 = a^2 - 2ab + b^2$, therefore

$$\begin{aligned} (50 - 2)^2 &= (50)^2 - 2 \times 50 \times 2 + (2)^2 \\ &= 2500 - 200 + 4 \\ &= 2504 - 200 \\ &= 2304 \end{aligned}$$

$$\begin{aligned} \text{(ii) } 181^2 - 19^2 &= (181 - 19)(181 + 19) \\ &\text{[using } a^2 - b^2 = (a - b)(a + b)] \\ &= 162 \times 200 \\ &= 32400 \end{aligned}$$

$$\begin{aligned} \text{(iii) } 497 \times 505 &= (500 - 3)(500 + 5) \\ &= 500^2 + (-3 + 5) \times 500 + (-3)(5) \text{ [using} \\ &\quad (x + a)(x + b) = x^2 + (a + b)x + ab] \\ &= 250000 + 1000 - 15 \\ &= 250985 \end{aligned}$$

$$\begin{aligned} \text{(iv) } 2.07 \times 1.93 &= (2 + 0.07)(2 - 0.07) \\ &= 2^2 - (0.07)^2 \\ &= 3.9951 \end{aligned}$$

Example 14 : Verify that

$$(3x + 5y)^2 - 30xy = 9x^2 + 25y^2$$

Solution : L.H.S = $(3x + 5y)^2 - 30xy$

$$= (3x)^2 + 2 \times 3x \times 5y + (5y)^2 - 30xy$$

$$[\text{Since } (a + b)^2 = a^2 + 2ab + b^2]$$

$$= 9x^2 + 30xy + 25y^2 - 30xy$$

$$= 9x^2 + 25y^2$$

$$= \text{R.H.S}$$

Hence, verified.

Example 15 : Verify that $(11pq + 4q)^2 - (11pq - 4q)^2 = 176pq^2$

Solution : L.H.S. $(11pq + 4q)^2 - (11pq - 4q)^2$

$$= (11pq + 4q + 11pq - 4q) \times (11pq + 4q - 11pq + 4q)$$

$$[\text{using } a^2 - b^2 = (a - b)(a + b), \text{ here } a = 11pq + 4q \text{ and } b = 11pq - 4q]$$

$$= (22pq)(8q)$$

$$= 176pq^2$$

R.H.S. Hence Verified

To convert a Celsius temperature to a Fahrenheit temperature, find nine-fifths of the Celsius temperature and then add 32.

$$F = \frac{9}{5}C + 32$$

While the statement on the left may be easier to read and understand at first, the statement on the right has several advantages. It is shorter and easier to write, it shows clearly how the quantities – Celsius temperature and Fahrenheit temperature – are related, and it allows you to try different Celsius temperatures and compute their Fahrenheit equivalents.

Example 16 : The area of a rectangle is $x^2 + 12xy + 27y^2$ and its length is $(x + 9y)$. Find the breadth of the rectangle.

Solution : Breadth = $\frac{\text{Area}}{\text{Length}}$

$$= \frac{x^2 + 12xy + 27y^2}{(x + 9y)}$$

$$\begin{aligned}
 &= \frac{x^2 + 9xy + 3xy + 27y^2}{(x + 9y)} \\
 &= \frac{x(x + 9y) + 3y(x + 9y)}{x + 9y} \\
 &= \frac{(x + 9y)(x + 3y)}{(x + 9y)} \\
 &= (x + 3y)
 \end{aligned}$$

Example 17 : Divide $15(y + 3)(y^2 - 16)$ by $5(y^2 - y - 12)$.

Solution : Factorising $15(y + 3)(y^2 - 16)$,

we get $5 \times 3 \times (y + 3)(y - 4)(y + 4)$

On factorising $5(y^2 - y - 12)$, we get $5(y^2 - 4y + 3y - 12)$

$$= 5[y(y - 4) + 3(y - 4)]$$

$$= 5(y - 4)(y + 3)$$

Therefore, on dividing the first expression by the second

expression, we get $\frac{15(y + 3)(y^2 - 16)}{5(y^2 - y - 12)}$

$$= \frac{5 \times 3 \times (y + 3)(y - 4)(y + 4)}{5 \times (y - 4)(y + 3)}$$

$$= 3(y + 4)$$

Example 18 : By using suitable identity, evaluate $x^2 + \frac{1}{x^2}$, if $x + \frac{1}{x} = 5$.

Solution : Given that $x + \frac{1}{x} = 5$

$$\text{So, } \left(x + \frac{1}{x}\right)^2 = 25$$

$$\text{Now, } \left(x + \frac{1}{x}\right)^2 = x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 \quad [\text{Using identity}]$$

$$(a + b)^2 = a^2 + 2ab + b^2, \text{ with } a = x \text{ and } b = \frac{1}{x}$$

$$= x^2 + 2 + \left(\frac{1}{x^2}\right)$$

$$= x^2 + \left(\frac{1}{x^2}\right) + 2$$

$$\text{Since } \left(x + \frac{1}{x}\right)^2 = 25, \text{ therefore } x^2 + \frac{1}{x^2} + 2 = 25$$

$$\text{or } x^2 + \frac{1}{x^2} = 25 - 2 = 23$$

Example 19 : Find the value of $\frac{38^2 - 22^2}{16}$, using a suitable identity.

Solution : Since $a^2 - b^2 = (a + b)(a - b)$, therefore

$$\begin{aligned} 38^2 - 22^2 &= (38 - 22)(38 + 22) \\ &= 16 \times 60 \end{aligned}$$

$$\begin{aligned} \text{So, } \frac{38^2 - 22^2}{16} &= \frac{16 \times 60}{16} \\ &= 60 \end{aligned}$$

Example 20 : Find the value of x , if

$$10000x = (9982)^2 - (18)^2$$

$$\begin{aligned} \text{Solution : R.H.S.} &= (9982)^2 - (18)^2 \\ &= (9982 + 18)(9982 - 18) \text{ [Since } a^2 - b^2 = (a + b)(a - b)] \\ &= (10000) \times (9964) \\ \text{L.H.S.} &= (10000) \times x \end{aligned}$$

Comparing L.H.S. and R.H.S., we get

$$10000x = 10000 \times 9964$$

$$\text{or } x = \frac{10000 \times 9964}{10000} = 9964$$

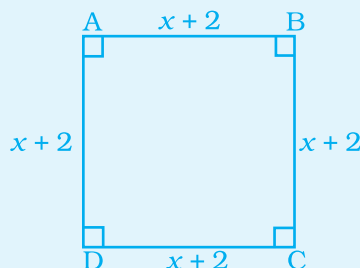
Think and Discuss

1. Can you find the reciprocal of $\frac{2}{11} \times \frac{5}{55}$?
2. Can you compare the ratio of this reciprocal with the earlier one?



Application on Problem Solving Strategy

Find each side of a figure given below, if its area is 64 cm^2 .



Understand and Explore the problem

- What information is given in the question?
 $AB = BC = DC = AD$, and $\angle A = \angle B = \angle C = \angle D = 90^\circ$
 Hence ABCD is a square.
- What are you trying to find?
 The value of one of the sides of the square ABCD.
- Is there any information that is not needed?
 No.



Make a Plan

- In a square all sides are equal, therefore, square of a side gives the area.



Solve

$$\begin{aligned}
 (\text{Side})^2 &= \text{Area} \\
 \Rightarrow (x + 2)^2 &= 64 \\
 \Rightarrow (x + 2)^2 &= 8^2 \\
 \Rightarrow x + 2 &= 8 \\
 \Rightarrow x &= 8 - 2 \\
 \therefore x &= 6 \\
 \therefore \text{Side} &= x + 2 = 6 + 2 = 8 \text{ cm}
 \end{aligned}$$



Revise

- The above answer is verified by squaring the side and comparing the result with the given area.
 $\therefore (\text{Side})^2 = 8^2 = 64 = \text{given area}.$

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following:

1. The word *equivalent* contains the same root as the word *equal*. What do you think equivalent expressions are?
2. The word *simplify* means *make less complicated*. What do you think it means to simplify an expression?
3. The adjective *like* means *alike*. What do you suppose like terms are?
4. A system is a group of related objects. What do you think a system of equations is?

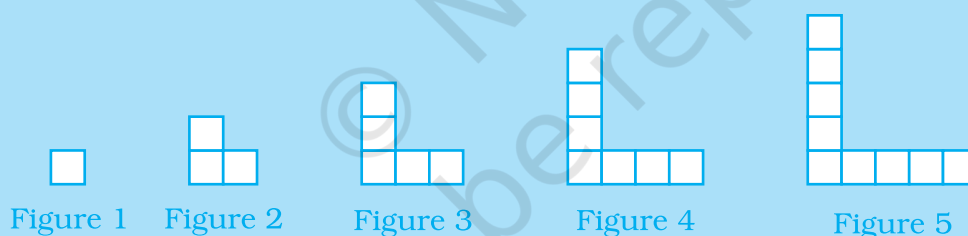
(C) Exercise

In questions 1 to 33, there are four options out of which one is correct. Write the correct answer.

1. The product of a monomial and a binomial is a
 - (a) monomial
 - (b) binomial
 - (c) trinomial
 - (d) none of these
2. In a polynomial, the exponents of the variables are always
 - (a) integers
 - (b) positive integers
 - (c) non-negative integers
 - (d) non-positive integers
3. Which of the following is correct?
 - (a) $(a - b)^2 = a^2 + 2ab - b^2$
 - (b) $(a - b)^2 = a^2 - 2ab + b^2$
 - (c) $(a - b)^2 = a^2 - b^2$
 - (d) $(a + b)^2 = a^2 + 2ab - b^2$
4. The sum of $-7pq$ and $2pq$ is
 - (a) $-9pq$
 - (b) $9pq$
 - (c) $5pq$
 - (d) $-5pq$
5. If we subtract $-3x^2y^2$ from x^2y^2 , then we get
 - (a) $-4x^2y^2$
 - (b) $-2x^2y^2$
 - (c) $2x^2y^2$
 - (d) $4x^2y^2$
6. Like term as $4m^3n^2$ is
 - (a) $4m^2n^2$
 - (b) $-6m^3n^2$
 - (c) $6pm^3n^2$
 - (d) $4m^3n$

7. Which of the following is a binomial?
- (a) $7 \times a + a$ (b) $6a^2 + 7b + 2c$
(c) $4a \times 3b \times 2c$ (d) $6(a^2 + b)$
8. Sum of $a - b + ab$, $b + c - bc$ and $c - a - ac$ is
- (a) $2c + ab - ac - bc$ (b) $2c - ab - ac - bc$
(c) $2c + ab + ac + bc$ (d) $2c - ab + ac + bc$
9. Product of the following monomials $4p$, $-7q^3$, $-7pq$ is
- (a) $196 p^2 q^4$ (b) $196 p q^4$ (c) $-196 p^2 q^4$ (d) $196 p^2 q^3$
10. Area of a rectangle with length $4ab$ and breadth $6b^2$ is
- (a) $24a^2 b^2$ (b) $24ab^3$ (c) $24ab^2$ (d) $24ab$
11. Volume of a rectangular box (cuboid) with length = $2ab$, breadth = $3ac$ and height = $2ac$ is
- (a) $12a^3 bc^2$ (b) $12a^3 bc$ (c) $12a^2 bc$
(d) $2ab + 3ac + 2ac$

The five figures form a pattern.



1. Copy and complete the table to find the perimeter of each figure. Each side of each individual square is 1 unit.

Figure	1	2	3	4	5
Perimeter					

2. Without drawing a picture, describe what the sixth figure will look like and predict its perimeter.
3. If you continue this pattern, what will be the perimeter of the 35th figure?
4. Explain how the perimeter of each figure is related to its figure number.
5. Using the variables n for the figure number and P for the perimeter, write an equation for the relationship in Question 4.

- 12.** Product of $6a^2 - 7b + 5ab$ and $2ab$ is
 (a) $12a^3b - 14ab^2 + 10ab$ (b) $12a^3b - 14ab^2 + 10a^2b^2$
 (c) $6a^2 - 7b + 7ab$ (d) $12a^2b - 7ab^2 + 10ab$
- 13.** Square of $3x - 4y$ is
 (a) $9x^2 - 16y^2$ (b) $6x^2 - 8y^2$
 (c) $9x^2 + 16y^2 + 24xy$ (d) $9x^2 + 16y^2 - 24xy$
- 14.** Which of the following are like terms?
 (a) $5xyz^2, -3xy^2z$ (b) $-5xyz^2, 7xyz^2$
 (c) $5xyz^2, 5x^2yz$ (d) $5xyz^2, x^2y^2z^2$
- 15.** Coefficient of y in the term $\frac{-y}{3}$ is
 (a) -1 (b) -3 (c) $\frac{-1}{3}$ (d) $\frac{1}{3}$
- 16.** $a^2 - b^2$ is equal to
 (a) $(a - b)^2$ (b) $(a - b)(a - b)$
 (c) $(a + b)(a - b)$ (d) $(a + b)(a + b)$
- 17.** Common factor of $17abc, 34ab^2, 51a^2b$ is
 (a) $17abc$ (b) $17ab$ (c) $17ac$ (d) $17a^2b^2c$
- 18.** Square of $9x - 7xy$ is
 (a) $81x^2 + 49x^2y^2$ (b) $81x^2 - 49x^2y^2$
 (c) $81x^2 + 49x^2y^2 - 126x^2y$ (d) $81x^2 + 49x^2y^2 - 63x^2y$
- 19.** Factorised form of $23xy - 46x + 54y - 108$ is
 (a) $(23x + 54)(y - 2)$ (b) $(23x + 54y)(y - 2)$
 (c) $(23xy + 54y)(-46x - 108)$ (d) $(23x + 54)(y + 2)$
- 20.** Factorised form of $r^2 - 10r + 21$ is
 (a) $(r - 1)(r - 4)$ (b) $(r - 7)(r - 3)$
 (c) $(r - 7)(r + 3)$ (d) $(r + 7)(r + 3)$
- 21.** Factorised form of $p^2 - 17p - 38$ is
 (a) $(p - 19)(p + 2)$ (b) $(p - 19)(p - 2)$
 (c) $(p + 19)(p + 2)$ (d) $(p + 19)(p - 2)$

- 22.** On dividing $57p^2qr$ by $114pq$, we get
 (a) $\frac{1}{4}pr$ (b) $\frac{3}{4}pr$ (c) $\frac{1}{2}pr$ (d) $2pr$
- 23.** On dividing $p(4p^2 - 16)$ by $4p(p - 2)$, we get
 (a) $2p + 4$ (b) $2p - 4$ (c) $p + 2$ (d) $p - 2$
- 24.** The common factor of $3ab$ and $2cd$ is
 (a) 1 (b) -1 (c) a (d) c
- 25.** An irreducible factor of $24x^2y^2$ is
 (a) x^2 (b) y^2 (c) x (d) $24x$
- 26.** Number of factors of $(a + b)^2$ is
 (a) 4 (b) 3 (c) 2 (d) 1
- 27.** The factorised form of $3x - 24$ is
 (a) $3x \times 24$ (b) $3(x - 8)$ (c) $24(x - 3)$ (d) $3(x - 12)$
- 28.** The factors of $x^2 - 4$ are
 (a) $(x - 2), (x - 2)$ (b) $(x + 2), (x - 2)$
 (c) $(x + 2), (x + 2)$ (d) $(x - 4), (x - 4)$
- 29.** The value of $(-27x^2y) \div (-9xy)$ is
 (a) $3xy$ (b) $-3xy$ (c) $-3x$ (d) $3x$
- 30.** The value of $(2x^2 + 4) \div 2$ is
 (a) $2x^2 + 2$ (b) $x^2 + 2$ (c) $x^2 + 4$ (d) $2x^2 + 4$
- 31.** The value of $(3x^3 + 9x^2 + 27x) \div 3x$ is
 (a) $x^2 + 9 + 27x$ (b) $3x^3 + 3x^2 + 27x$
 (c) $3x^3 + 9x^2 + 9$ (d) $x^2 + 3x + 9$
- 32.** The value of $(a + b)^2 + (a - b)^2$ is
 (a) $2a + 2b$ (b) $2a - 2b$ (c) $2a^2 + 2b^2$ (d) $2a^2 - 2b^2$
- 33.** The value of $(a + b)^2 - (a - b)^2$ is
 (a) $4ab$ (b) $-4ab$ (c) $2a^2 + 2b^2$ (d) $2a^2 - 2b^2$

In questions 34 to 58, fill in the blanks to make the statements true:

- 34.** The product of two terms with like signs is a _____ term.
- 35.** The product of two terms with unlike signs is a _____ term.

36. $a(b + c) = ax \text{ \_\_\_\_\_\_ } \times ax \text{ \_\_\_\_\_\_ }.$
37. $(a - b) \text{ \_\_\_\_\_\_ } = a^2 - 2ab + b^2$
38. $a^2 - b^2 = (a + b) \text{ \_\_\_\_\_\_ }.$
39. $(a - b)^2 + \text{ \_\_\_\_\_\_ } = a^2 - b^2$
40. $(a + b)^2 - 2ab = \text{ \_\_\_\_\_\_ } + \text{ \_\_\_\_\_\_ }$
41. $(x + a)(x + b) = x^2 + (a + b)x + \text{ \_\_\_\_\_\_ }.$
42. The product of two polynomials is a $\text{ \_\_\_\_\_\_ }.$
43. Common factor of $ax^2 + bx$ is $\text{ \_\_\_\_\_\_ }.$
44. Factorised form of $18mn + 10mnp$ is $\text{ \_\_\_\_\_\_ }.$
45. Factorised form of $4y^2 - 12y + 9$ is $\text{ \_\_\_\_\_\_ }.$
46. $38x^3y^2z \div 19xy^2$ is equal to $\text{ \_\_\_\_\_\_ }.$
47. Volume of a rectangular box with length $2x$, breadth $3y$ and height $4z$ is $\text{ \_\_\_\_\_\_ }.$
48. $67^2 - 37^2 = (67 - 37) \times \text{ \_\_\_\_\_\_ } = \text{ \_\_\_\_\_\_ }.$
49. $103^2 - 102^2 = \text{ \_\_\_\_\_\_ } \times (103 - 102) = \text{ \_\_\_\_\_\_ }.$
50. Area of a rectangular plot with sides $4x^2$ and $3y^2$ is $\text{ \_\_\_\_\_\_ }.$
51. Volume of a rectangular box with $l = b = h = 2x$ is $\text{ \_\_\_\_\_\_ }.$
52. The coefficient in $-37abc$ is $\text{ \_\_\_\_\_\_ }.$
53. Number of terms in the expression $a^2 + bc \times d$ is $\text{ \_\_\_\_\_\_ }.$
54. The sum of areas of two squares with sides $4a$ and $4b$ is $\text{ \_\_\_\_\_\_ }.$
55. The common factor method of factorisation for a polynomial is based on ______ property.
56. The side of the square of area $9y^2$ is $\text{ \_\_\_\_\_\_ }.$
57. On simplification $\frac{3x+3}{3} = \text{ \_\_\_\_\_\_ }$
58. The factorisation of $2x + 4y$ is $\text{ \_\_\_\_\_\_ }.$

In questions 59 to 80, state whether the statements are True (T) or False (F):

59. $(a + b)^2 = a^2 + b^2$
60. $(a - b)^2 = a^2 - b^2$
61. $(a + b)(a - b) = a^2 - b^2$
62. The product of two negative terms is a negative term.
63. The product of one negative and one positive term is a negative term.
64. The coefficient of the term $-6x^2y^2$ is -6 .
65. $p^2q + q^2r + r^2q$ is a binomial.
66. The factors of $a^2 - 2ab + b^2$ are $(a + b)$ and $(a + b)$.
67. h is a factor of $2\pi(h + r)$.
68. Some of the factors of $\frac{n^2}{2} + \frac{n}{2}$ are $\frac{1}{2}$, n and $(n+1)$.
69. An equation is true for all values of its variables.
70. $x^2 + (a + b)x + ab = (a + b)(x + ab)$
71. Common factor of $11pq^2$, $121p^2q^3$, $1331p^2q$ is $11p^2q^2$.
72. Common factor of $12a^2b^2 + 4ab^2 - 32$ is 4 .
73. Factorisation of $-3a^2 + 3ab + 3ac$ is $3a(-a - b - c)$.
74. Factorised form of $p^2 + 30p + 216$ is $(p + 18)(p - 12)$.
75. The difference of the squares of two consecutive numbers is their sum.
76. $abc + bca + cab$ is a monomial.
77. On dividing $\frac{p}{3}$ by $\frac{3}{p}$, the quotient is 9 .
78. The value of p for $51^2 - 49^2 = 100p$ is 2 .
79. $(9x - 51) \div 9$ is $x - 51$.
80. The value of $(a + 1)(a - 1)(a^2 + 1)$ is $a^4 - 1$.

81. Add:

- (i) $7a^2bc, -3abc^2, 3a^2bc, 2abc^2$
- (ii) $9ax, +3by - cz, -5by + ax + 3cz$
- (iii) $xy^2z^2 + 3x^2y^2z - 4x^2yz^2, -9x^2y^2z + 3xy^2z^2 + x^2yz^2$
- (iv) $5x^2 - 3xy + 4y^2 - 9, 7y^2 + 5xy - 2x^2 + 13$
- (v) $2p^4 - 3p^3 + p^2 - 5p + 7, -3p^4 - 7p^3 - 3p^2 - p - 12$
- (vi) $3a(a - b + c), 2b(a - b + c)$
- (vii) $3a(2b + 5c), 3c(2a + 2b)$

82. Subtract :

- (i) $5a^2b^2c^2$ from $-7a^2b^2c^2$
- (ii) $6x^2 - 4xy + 5y^2$ from $8y^2 + 6xy - 3x^2$
- (iii) $2ab^2c^2 + 4a^2b^2c - 5a^2bc^2$ from $-10a^2b^2c + 4ab^2c^2 + 2a^2bc^2$
- (iv) $3t^4 - 4t^3 + 2t^2 - 6t + 6$ from $-4t^4 + 8t^3 - 4t^2 - 2t + 11$
- (v) $2ab + 5bc - 7ac$ from $5ab - 2bc - 2ac + 10abc$
- (vi) $7p(3q + 7p)$ from $8p(2p - 7q)$
- (vii) $-3p^2 + 3pq + 3px$ from $3p(-p - a - r)$

83. Multiply the following:

- (i) $-7pq^2r^3, -13p^3q^2r$
- (ii) $3x^2y^2z^2, 17xyz$
- (iii) $15xy^2, 17yz^2$
- (iv) $-5a^2bc, 11ab, 13abc^2$
- (v) $-3x^2y, (5y - xy)$
- (vi) $abc, (bc + ca)$
- (vii) $7pqr, (p - q + r)$
- (viii) $x^2y^2z^2, (xy - yz + zx)$
- (ix) $(p + 6), (q - 7)$

- (x) $6mn, 0mn$
- (xi) a, a^5, a^6
- (xii) $-7st, -1, -13st^2$
- (xiii) $b^3, 3b^2, 7ab^5$
- (xiv) $-\frac{100}{9}rs; \frac{3}{4}r^3s^2$
- (xv) $(a^2 - b^2), (a^2 + b^2)$
- (xvi) $(ab + c), (ab + c)$
- (xvii) $(pq - 2r), (pq - 2r)$
- (xviii) $\left(\frac{3}{4}x - \frac{4}{3}y\right), \left(\frac{2}{3}x + \frac{3}{2}y\right)$
- (xix) $\frac{3}{2}p^2 + \frac{2}{3}q^2, (2p^2 - 3q^2)$
- (xx) $(x^2 - 5x + 6), (2x + 7)$
- (xxi) $(3x^2 + 4x - 8), (2x^2 - 4x + 3)$
- (xxii) $(2x - 2y - 3), (x + y + 5)$

84. Simplify

- (i) $(3x + 2y)^2 + (3x - 2y)^2$
- (ii) $(3x + 2y)^2 - (3x - 2y)^2$
- (iii) $\left(\frac{7}{9}a + \frac{9}{7}b\right)^2 - ab$
- (iv) $\left(\frac{3}{4}x - \frac{4}{3}y\right)^2 + 2xy$
- (v) $(1.5p + 1.2q)^2 - (1.5p - 1.2q)^2$
- (vi) $(2.5m + 1.5q)^2 + (2.5m - 1.5q)^2$
- (vii) $(x^2 - 4) + (x^2 + 4) + 16$
- (viii) $(ab - c)^2 + 2abc$
- (ix) $(a - b)(a^2 + b^2 + ab) - (a + b)(a^2 + b^2 - ab)$

$$(x) \quad (b^2 - 49)(b + 7) + 343$$

$$(xi) \quad (4.5a + 1.5b)^2 + (4.5b + 1.5a)^2$$

$$(xii) \quad (pq - qr)^2 + 4pq^2r$$

$$(xiii) \quad (s^2t + tq^2)^2 - (2stq)^2$$

85. Expand the following, using suitable identities.

$$(i) \quad (xy + yz)^2$$

$$(ii) \quad (x^2y - xy^2)^2$$

$$(iii) \quad \left(\frac{4}{5}a + \frac{5}{4}b\right)^2$$

$$(iv) \quad \left(\frac{2}{3}x - \frac{3}{2}y\right)^2$$

$$(v) \quad \left(\frac{4}{5}p + \frac{5}{3}q\right)^2$$

$$(vi) \quad (x + 3)(x + 7)$$

$$(vii) \quad (2x + 9)(2x - 7)$$

$$(viii) \quad \left(\frac{4x}{5} + \frac{y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$$

$$(ix) \quad \left(\frac{2x}{3} - \frac{2}{3}\right)\left(\frac{2x}{3} + \frac{2a}{3}\right)$$

$$(x) \quad (2x - 5y)(2x - 5y)$$

$$(xi) \quad \left(\frac{2a}{3} + \frac{b}{3}\right)\left(\frac{2a}{3} - \frac{b}{3}\right)$$

$$(xii) \quad (x^2 + y^2)(x^2 - y^2)$$

$$(xiii) \quad (a^2 + b^2)^2$$

$$(xiv) \quad (7x + 5)^2$$

$$(xv) \quad (0.9p - 0.5q)^2$$

$$(xvi) \quad x^2y^2 = (xy)^2$$

86. Using suitable identities, evaluate the following.

(i) $(52)^2$	(ii) $(49)^2$
(iii) $(103)^2$	(iv) $(98)^2$
(v) $(1005)^2$	(vi) $(995)^2$
(vii) 47×53	(viii) 52×53
(ix) 105×95	(x) 104×97
(xi) 101×103	(xii) 98×103
(xiii) $(9.9)^2$	(xiv) 9.8×10.2
(xv) 10.1×10.2	(xvi) $(35.4)^2 - (14.6)^2$
(xvii) $(69.3)^2 - (30.7)^2$	(xviii) $(9.7)^2 - (0.3)^2$
(xix) $(132)^2 - (68)^2$	(xx) $(339)^2 - (161)^2$
(xxi) $(729)^2 - (271)^2$	

87. Write the greatest common factor in each of the following terms.

(i) $-18a^2, 108a$	(ii) $3x^2y, 18xy^2, -6xy$
(iii) $2xy, -y^2, 2x^2y$	(iv) $l^2m^2n, lm^2n^2, l^2mn^2$
(v) $21pqr, -7p^2q^2r^2, 49p^2qr$	(vi) $qrx, y, pryz, rxyz$
(vii) $3x^3y^2z, -6xy^3z^2, 12x^2yz^3$	
(viii) $63p^2a^2r^2s, -9pq^2r^2s^2, 15p^2qr^2s^2, -60p^2a^2rs^2$	
(ix) $13x^2y, 169xy$	
(x) $11x^2, 12y^2$	

88. Factorise the following expressions.

(i) $6ab + 12bc$	(ii) $-xy - ay$
(iii) $ax^3 - bx^2 + cx$	(iv) $l^2m^2n - lm^2n^2 - l^2mn^2$
(v) $3pqr - 6p^2q^2r^2 - 15r^2$	(vi) $x^3y^2 + x^2y^3 - xy^4 + xy$
(vii) $4xy^2 - 10x^2y + 16x^2y^2 + 2xy$	
(viii) $2a^3 - 3a^2b + 5ab^2 - ab$	
(ix) $63p^2q^2r^2s - 9pq^2r^2s^2 + 15p^2qr^2s^2 - 60p^2q^2rs^2$	
(x) $24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz$	

- (xi) $a^3 + a^2 + a + 1$
 (xii) $lx + my + mx + ly$
 (xiii) $a^3x - x^4 + a^2x^2 - ax^3$
 (xiv) $2x^2 - 2y + 4xy - x$
 (xv) $y^2 + 8zx - 2xy - 4yz$
 (xvi) $ax^2y - bxyz - ax^2z + bxy^2$
 (xvii) $a^2b + a^2c + ab + ac + b^2c + c^2b$
 (xviii) $2ax^2 + 4axy + 3bx^2 + 2ay^2 + 6bxy + 3by^2$

89. Factorise the following, using the identity $a^2 + 2ab + b^2 = (a + b)^2$

- (i) $x^2 + 6x + 9$ (ii) $x^2 + 12x + 36$
 (iii) $x^2 + 14x + 49$ (iv) $x^2 + 2x + 1$
 (v) $4x^2 + 4x + 1$ (vi) $a^2x^2 + 2ax + 1$
 (vii) $a^2x^2 + 2abx + b^2$ (viii) $a^2x^2 + 2abxy + b^2y^2$
 (ix) $4x^2 + 12x + 9$ (x) $16x^2 + 40x + 25$
 (xi) $9x^2 + 24x + 16$ (xii) $9x^2 + 30x + 25$
 (xiii) $2x^3 + 24x^2 + 72x$ (xiv) $a^2x^3 + 2abx^2 + b^2x$
 (xv) $4x^4 + 12x^3 + 9x^2$ (xvi) $\frac{x^2}{4} + 2x + 4$
 (xvii) $9x^2 + 2xy + \frac{y^2}{9}$

90. Factorise the following, using the identity $a^2 - 2ab + b^2 = (a - b)^2$.

- (i) $x^2 - 8x + 16$ (ii) $x^2 - 10x + 25$
 (iii) $y^2 - 14y + 49$ (iv) $p^2 - 2p + 1$
 (v) $4a^2 - 4ab + b^2$ (vi) $p^2y^2 - 2py + 1$
 (vii) $a^2y^2 - 2aby + b^2$ (viii) $9x^2 - 12x + 4$
 (ix) $4y^2 - 12y + 9$ (x) $\frac{x^2}{4} - 2x + 4$

(xi) $a^2y^3 - 2aby^2 + b^2y$

(xii) $9y^2 - 4xy + \frac{4x^2}{9}$

91. Factorise the following.

(i) $x^2 + 15x + 26$

(ii) $x^2 + 9x + 20$

(iii) $y^2 + 18x + 65$

(iv) $p^2 + 14p + 13$

(v) $y^2 + 4y - 21$

(vi) $y^2 - 2y - 15$

(vii) $18 + 11x + x^2$

(viii) $x^2 - 10x + 21$

(ix) $x^2 = 17x + 60$

(x) $x^2 + 4x - 77$

(xi) $y^2 + 7y + 12$

(xii) $p^2 - 13p - 30$

(xiii) $a^2 - 16p - 80$

92. Factorise the following using the identity $a^2 - b^2 = (a + b)(a - b)$.

(i) $x^2 - 9$

(ii) $4x^2 - 25y^2$

(iii) $4x^2 - 49y^2$

(iv) $3a^2b^3 - 27a^4b$

(v) $28ay^2 - 175ax^2$

(vi) $9x^2 - 1$

(vii) $25ax^2 - 25a$

(viii) $\frac{x^2}{9} - \frac{y^2}{25}$

(ix) $\frac{2p^2}{25} - 32q^2$

(x) $49x^2 - 36y^2$

(xi) $y^3 - \frac{y}{9}$

(xii) $\frac{x^2}{25} - 625$

(xiii) $\frac{x^2}{8} - \frac{y^2}{18}$

(xiv) $\frac{4x^2}{9} - \frac{9y^2}{16}$

(xv) $\frac{x^3y}{9} - \frac{xy^3}{16}$

(xvi) $1331x^3y - 11y^3x$

(xvii) $\frac{1}{36}a^2b^2 - \frac{16}{49}b^2c^2$

(xviii) $a^4 - (a - b)^4$

(xix) $x^4 - 1$

(xx) $y^4 - 625$

- (xxi) $p^5 - 16p$ (xxii) $16x^4 - 81$
 (xxiii) $x^4 - y^4$ (xxiv) $y^4 - 81$
 (xxv) $16x^4 - 625y^4$ (xxvi) $(a - b)^2 - (b - c)^2$
 (xxvii) $(x + y)^4 - (x - y)^4$ (xxviii) $x^4 - y^4 + x^2 - y^2$
 (xxix) $8a^3 - 2a$ (xxx) $x^2 - \frac{y^2}{100}$
 (xxxi) $9x^2 - (3y + z)^2$

93. The following expressions are the areas of rectangles. Find the possible lengths and breadths of these rectangles.

- (i) $x^2 - 6x + 8$ (ii) $x^2 - 3x + 2$
 (iii) $x^2 - 7x + 10$ (iv) $x^2 + 19x - 20$
 (v) $x^2 + 9x + 20$

94. Carry out the following divisions:

- (i) $51x^3y^2z \div 17xyz$ (ii) $76x^3yz^3 \div 19x^2y^2$
 (iii) $17ab^2c^3 \div (-abc^2)$ (iv) $-121p^3q^3r^3 \div (-11xy^2z^3)$

95. Perform the following divisions:

- (i) $(3pqr - 6p^2q^2r^2) \div 3pq$ (ii) $(ax^3 - bx^2 + cx) \div (-dx)$
 (iii) $(x^3y^3 + x^2y^3 - xy^4 + xy) \div xy$ (iv) $(-qrx y + pryz - rxyz) \div (-xyz)$

96. Factorise the expressions and divide them as directed:

- (i) $(x^2 - 22x + 117) \div (x - 13)$ (ii) $(x^3 + x^2 - 132x) \div x(x - 11)$
 (iii) $(2x^3 - 12x^2 + 16x) \div (x - 2)(x - 4)$
 (iv) $(9x^2 - 4) \div (3x + 2)$
 (v) $(3x^2 - 48) \div (x - 4)$ (vi) $(x^4 - 16) \div x^3 + 2x^2 + 4x + 8$
 (vii) $(3x^4 - 1875) \div (3x^2 - 75)$

97. The area of a square is given by $4x^2 + 12xy + 9y^2$. Find the side of the square.

98. The area of a square is $9x^2 + 24xy + 16y^2$. Find the side of the square.

- 99.** The area of a rectangle is $x^2 + 7x + 12$. If its breadth is $(x + 3)$, then find its length.
- 100.** The curved surface area of a cylinder is $2\pi (y^2 - 7y + 12)$ and its radius is $(y - 3)$. Find the height of the cylinder (C.S.A. of cylinder = $2\pi rh$).
- 101.** The area of a circle is given by the expression $\pi x^2 + 6\pi x + 9\pi$. Find the radius of the circle.
- 102.** The sum of first n natural numbers is given by the expression $\frac{n^2}{2} + \frac{n}{2}$. Factorise this expression.
- 103.** The sum of $(x + 5)$ observations is $x^4 - 625$. Find the mean of the observations.
- 104.** The height of a triangle is $x^4 + y^4$ and its base is $14xy$. Find the area of the triangle.
- 105.** The cost of a chocolate is Rs $(x + y)$ and Rohit bought $(x + y)$ chocolates. Find the total amount paid by him in terms of x . If $x = 10$, find the amount paid by him.
- 106.** The base of a parallelogram is $(2x + 3)$ units and the corresponding height is $(2x - 3)$ units. Find the area of the parallelogram in terms of x . What will be the area of parallelogram of $x = 30$ units?
- 107.** The radius of a circle is $7ab - 7bc - 14ac$. Find the circumference of the circle. $\left(\pi = \frac{22}{7}\right)$
- 108.** If $p + q = 12$ and $pq = 22$, then find $p^2 + q^2$.
- 109.** If $a + b = 25$ and $a^2 + b^2 = 225$, then find ab .
- 110.** If $x - y = 13$ and $xy = 28$, then find $x^2 + y^2$.
- 111.** If $m - n = 16$ and $m^2 + n^2 = 400$, then find mn .
- 112.** If $a^2 + b^2 = 74$ and $ab = 35$, then find $a + b$.

113. Verify the following:

- (i) $(ab + bc)(ab - bc) + (bc + ca)(bc - ca) + (ca + ab)(ca - ab) = 0$
- (ii) $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$
- (iii) $(p - q)(p^2 + pq + q^2) = p^3 - q^3$
- (iv) $(m + n)(m^2 - mn + n^2) = m^3 + n^3$
- (v) $(a + b)(a + b)(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$
- (vi) $(a - b)(a - b)(a - b) = a^3 - 3a^2b + 3ab^2 - b^3$
- (vii) $(a^2 - b^2)(a^2 + b^2) + (b^2 - c^2)(b^2 + c^2) + (c^2 - a^2)(c^2 + a^2) = 0$
- (viii) $(5x + 8)^2 - 160x = (5x - 8)^2$
- (ix) $(7p - 13q)^2 + 364pq = (7p + 13q)^2$
- (x) $\left(\frac{3p}{7} + \frac{7}{6p}\right)^2 - \left(\frac{3}{7}p + \frac{7}{6p}\right)^2 = 2$

114. Find the value of a , if

- (i) $8a = 35^2 - 27^2$
- (ii) $9a = 76^2 - 67^2$
- (iii) $pqa = (3p + q)^2 - (3p - q)^2$
- (iv) $pq^2a = (4pq + 3q)^2 - (4pq - 3q)^2$

115. What should be added to $4c(-a + b + c)$ to obtain $3a(a + b + c) - 2b(a - b + c)$?

116. Subtract $b(b^2 + b - 7) + 5$ from $3b^2 - 8$ and find the value of expression obtained for $b = -3$.

117. If $x - \frac{1}{x} = 7$ then find the value of $x^2 + \frac{1}{x^2}$.

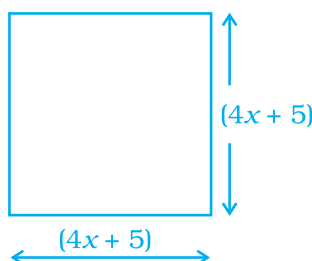
118. Factorise $x^2 + \frac{1}{x^2} + 2 - 3x - \frac{3}{x}$.

119. Factorise $p^4 + q^4 + p^2q^2$.

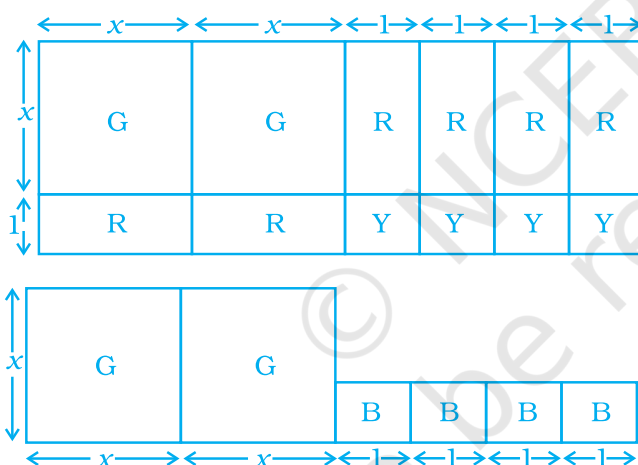
120. Find the value of

- (i) $\frac{6.25 \times 6.25 - 1.75 \times 1.75}{4.5}$
- (ii) $\frac{198 \times 198 - 102 \times 102}{96}$

- 121.** The product of two expressions is $x^5 + x^3 + x$. If one of them is $x^2 + x + 1$, find the other.
- 122.** Find the length of the side of the given square if area of the square is 625 square units and then find the value of x .

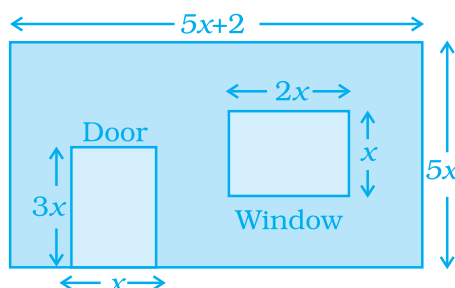


- 123.** Take suitable number of cards given in the adjoining diagram [$G(x \times x)$ representing x^2 , $R(x \times 1)$ representing x and $Y(1 \times 1)$ representing 1] to factorise the following expressions, by arranging the cards in the form of rectangles: (i) $2x^2 + 6x + 4$ (ii) $x^2 + 4x + 4$. Factorise $2x^2 + 6x + 4$ by using the figure.



Calculate the area of figure.

- 124.** The figure shows the dimensions of a wall having a window and a door of a room. Write an algebraic expression for the area of the wall to be painted.



125. Match the expressions of column I with that of column II:

Column I

- (1) $(21x + 13y)^2$
 (2) $(21x - 13y)^2$
 (3) $(21x - 13y)(21x + 13y)$

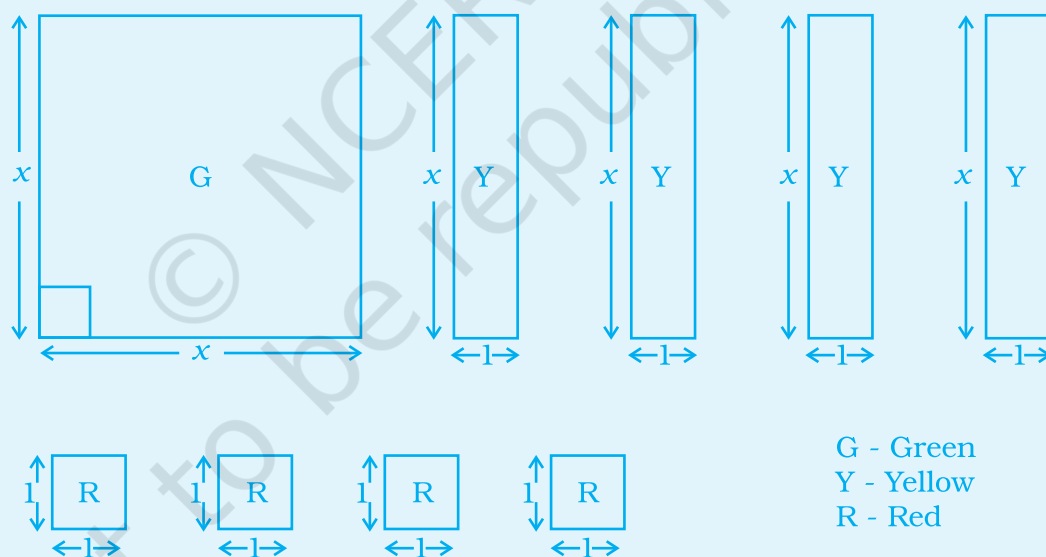
Column II

- (a) $441x^2 - 169y^2$
 (b) $441x^2 + 169y^2 + 546xy$
 (c) $441x^2 + 169y^2 - 546xy$
 (d) $441x^2 - 169y^2 + 546xy$

(D) ACTIVITIES

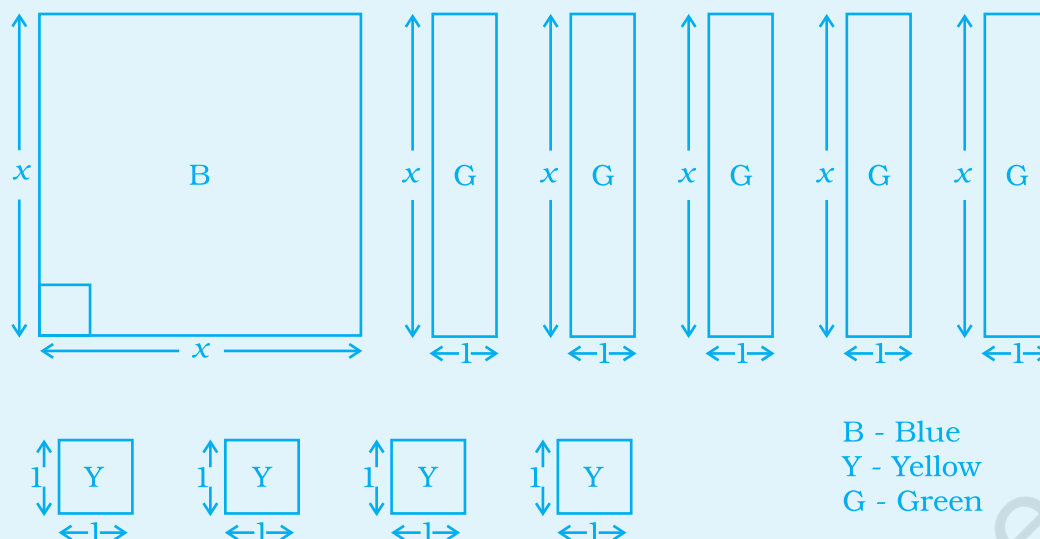
1. Algebraic Tiles

- (i) Cut the following tiles from a graph sheet. Now, colour the tiles as per the colour code. Arrange these algebraic tiles to form a square.



Find the length of the side of the square so formed. Also find the area of the square. Using the above result factorise $x^2 + 4x + 4$.

(ii)



2. Find the length of the side of the rectangle so formed. Also find the area of the rectangle. Using the above result factorise $x^2 + 5x + 4$.

Now choose and cut more algebraic tiles from the graph sheet. Create your own colour code and colour the tiles. Arrange them to form square/rectangle. Find the area of the figure so formed using it to factorise

- $x^2 + 4x + 3$
 - $x^2 + 9x + 18$
3. Build a square garden. Divide the square garden into four rectangular flower beds in such a way that each flower bed is as long as one side of the square. The perimeter of each flower bed is 40 m.
- Draw a diagram to represent the above information.
 - Mention the expression for perimeter of the entire garden.

Crossword Number Puzzle

Solve the given crossword and then fill up the given boxes. Clues are given below for across as well as downward filling. Also for across and down clues clue number is written at the corner of boxes. Answers of clues have to be filled in their respective boxes.

Down

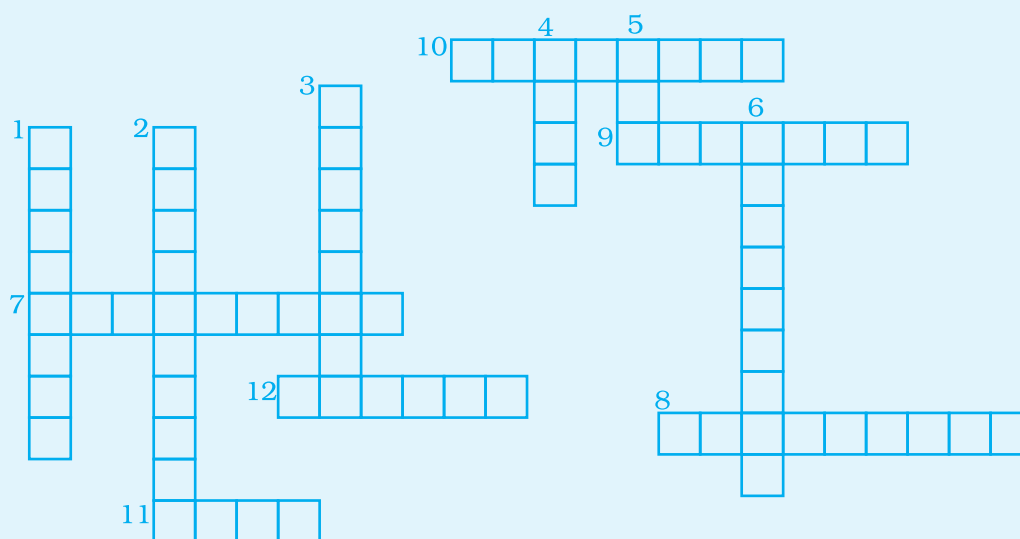
1. A polynomial with two terms.
2. An expression containing one or more terms with non-zero coefficient (with variables having non-negative exponents).
3. To find the value of a mathematical expression.
4. A _____ is formed by the product of variables and constants.
5. The abbreviation of the greatest no. (or expression) that is a factor of two or more numbers.
6. A polynomial with three terms.

Across

7. A polynomial with only one term.
8. An expression of the second degree.
9. Terms can be written as product of its _____.
10. The numbers $-3, -2, -1, 0, 1, 2, 3$ are known as _____.
11. _____ terms are formed from the same variables and the powers of these variables are the same term.
12. The highest power of a polynomial is called the _____ of the polynomial.

Solution

1. Binomial
2. Polynomial
3. Evaluate
4. Term
5. GCF
6. Trinomial
7. Monomial



8. Quadratic
9. Factors
10. Integers
11. Like
12. Degree

Rough Work

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UNIT 8

EXPONENTS AND POWERS

(A) Main Concepts and Results

- Exponential notation is a powerful way to express repeated multiplication of the same number. Specifically, powers of 10 express very large and very small numbers in a manner which is convenient to read, write and compare.
- For any non-zero integer a , $a^m = \frac{1}{a^{-m}}$
- Laws of exponents are
 - (a) $a^m \times a^n = a^{m+n}$
 - (b) $a^m \div a^n = a^{m-n}$
 - (c) $(a^m)^n = a^{mn}$
 - (d) $a^m \times b^m = (ab)^m$
 - (e) $a^0 = 1$, where $a \neq 0$
 - (f) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
- Numbers can be expressed in expanded form by using exponents.
- Very large and very small numbers can be expressed in standard form.
- Standard form is also called **scientific notation form**.

(B) Solved Examples

In example 1 and 2, there are four options given out of which one is correct. Write the correct answer.

Example 1 : Multiplicative inverse of 2^7 is

- (a) 2^{-7} (b) 7^2 (c) -2^7 (d) -2^7

Solution : The Correct answer is (a).

Example 2 : The human body has about 100 billion cells. This number can be written in exponential form as

- (a) 10^{-11} (b) 10^{11} (c) 10^9 (d) 10^{-9}

Solution : The correct answer is (b).

In examples 3 to 5, fill in the blanks to make the statements true.

Example 3 : $(-4)^4 \times \left(\frac{5}{4}\right)^4 = \underline{\hspace{2cm}}$

Solution : 5^4

Example 4 : $(2^{-3})^2 \times (3^{-2})^3 = \underline{\hspace{2cm}}$

Solution : 6^{-6}

Example 5 : The distance between earth and sun is 150 million kilometres which can be written in exponential form as $\underline{\hspace{2cm}}$.

Solution : 1.5×10^8 km

In examples 6 and 7, state whether the statements are true (T) or false (F):

Example 6 : Very small numbers can be expressed in standard form using positive exponents.

Solution : False.

Example 7 : $(-10) \times (-10) \times (-10) \times (-10) = 10^{-4}$

Solution : False.

Example 8 : Simplify $\frac{(-2)^3 \times (-2)^7}{3 \times 4^6}$

Solution : $\frac{(-2)^3 \times (-2)^7}{3 \times 4^6} = \frac{(-2)^{3+7}}{3 \times (2^2)^6} \{a^m \times a^n = a^{m+n}\}$

$$= \frac{(-2)^{10}}{3 \times 2^{12}} \{(a^m)^n = a^{m \times n}\}$$

$$= \frac{(-2)^{10}}{3 \times 2^{12}} = \frac{2^{10-12}}{3} \{a^m \div a^n = a^{m-n}, (-2)^{10} = 2^{10}\}$$

$$= \frac{2^{-2}}{3} = \frac{1}{3 \times 2^2} = \frac{1}{12}$$

Example 9 : Find x so that $(-5)^{x+1} \times (-5)^5 = (-5)^7$

Solution : $(-5)^{x+1} \times (-5)^5 = (-5)^7$

$$(-5)^{x+1+5} = (-5)^7 \quad \{a^m \times a^n = a^{m+n}\}$$

$$(-5)^{x+6} = (-5)^7$$

On both sides, powers have the same base, so their exponents must be equal.

Therefore, $x + 6 = 7$

$$x = 7 - 6 = 1$$

$$x = 1$$



Application on Problem Solving Strategy

Example 10 :

Find x so that $(-5)^{x+1} \times (-5)^5 = (-5)^7$



Understand and Explore the Problem

- What are you trying to find?

The value of x which satisfies the given equation.



Plan a Strategy

- You know the laws of exponents. Applying the laws of exponent in the given equation to find the value of x .



Solve

- Given $(-5)^{x+1} \times (-5)^5 = (-5)^7$

Using the Law of exponents, $a^m \times a^n = a^{m+n}$, we get

$$(-5)^{x+1+5} = (-5)^7$$

$$(-5)^{x+6} = (-5)^7$$

On both the sides, power has the same base, so their exponents must be equal,

$$\text{Therefore, } x + 6 = 7$$

$$x = 7 - 6$$

$$\text{So, } x = 1$$

Hence, the value of x is 1.



Revise

- Substitute the value of x in the equation and check if it satisfies the equation.

$$\begin{aligned} \text{LHS} &= (-5)^{x+1} \times (-5)^5 \\ &= (-5)^{1+1} \times (-5)^5 \\ &= (-5)^2 \times (-5)^5 \\ &= (-5)^{2+5} \\ &= (-5)^7 = \text{RHS} \end{aligned}$$

$x = 1$ satisfies the equation. Hence our answer is correct.

Think and Discuss



(a) Try to find the value of x in the question by changing -5 to 2 . What difference do you find in the value of x ? What do you infer from your answer?

(b) See if you can find the value of x if the equation is changed to

(i) $(-5)^{x+1} \times (5)^5 = (5)^7$

(ii) $(-5)^{2x} \times (5)^5 = (5)^7$

Getting Rid of Negative Exponents in Fractions

You might have to deal with fractions that have negative exponents in the numerator and denominator, like $\frac{2^{-4}}{3^{-7}}$. It's useful to be able to change them into fractions with only positive exponents because it's a simpler form. A number with negative exponent in the numerator is equivalent to the same number with positive exponent in the denominator $\Rightarrow 2^{-4} = \frac{2^{-4}}{1} = \frac{1}{2^4}$. A number with a negative exponent in the denominator is equivalent to the same number with positive exponent in the numerator $\Rightarrow \frac{1}{3^{-7}} = \frac{3^7}{1} = 3^7$.

So, 2^{-4} gets moved from the numerator to the denominator, where it is written as 2^4 . $\Rightarrow \frac{2^{-4}}{3^{-7}} = \frac{3^7}{2^4}$ $\leftarrow 3^{-7}$ moved from the denominator and becomes 3^7 in the numerator.

(C) Exercise

In questions 1 to 33, out of the four options, only one is correct. Write the correct answer.

- In 2^n , n is known as
 (a) Base (b) Constant (c) x (d) Variable
- For a fixed base, if the exponent decreases by 1, the number becomes
 (a) One-tenth of the previous number.
 (b) Ten times of the previous number.
 (c) Hundredth of the previous number.
 (d) Hundred times of the previous number.
- 3^{-2} can be written as
 (a) 3^2 (b) $\frac{1}{3^2}$ (c) $\frac{1}{3^{-2}}$ (d) $-\frac{2}{3}$
- The value of $\frac{1}{4^{-2}}$ is
 (a) 16 (b) 8 (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

5. The value of $3^5 \div 3^{-6}$ is
 (a) 3^5 (b) 3^{-6} (c) 3^{11} (d) 3^{-11}
6. The value of $\left(\frac{2}{5}\right)^{-2}$ is
 (a) $\frac{4}{5}$ (b) $\frac{4}{25}$ (c) $\frac{25}{4}$ (d) $\frac{5}{2}$
7. The reciprocal of $\left(\frac{2}{5}\right)^{-1}$ is
 (a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $-\frac{5}{2}$ (d) $-\frac{2}{5}$
8. The multiplicative inverse of 10^{-100} is
 (a) 10 (b) 100 (c) 10^{100} (d) 10^{-100}
9. The value of $(-2)^{2 \times 3 - 1}$ is
 (a) 32 (b) 64 (c) -32 (d) -64
10. The value of $\left(-\frac{2}{3}\right)^4$ is equal to
 (a) $\frac{16}{81}$ (b) $\frac{81}{16}$ (c) $-\frac{16}{81}$ (d) $\frac{81}{-16}$

The table shows several powers of 10 in various forms. Note the following:

- Powers of 10 with positive integer exponents involve repeated multiplication by 10.
- Power of 10 with negative integer exponents involve repeated multiplication by $\frac{1}{10}$ (the multiplicative inverse of 10), or repeated division by 10.
- The power of 10 with an exponent of 0 equals 1.

Power	Repeated multiplication	Standard form
10^3	$1 \times 10 \times 10 \times 10$	1000
10^2	$1 \times 10 \times 10$	100
10^1	1×10	10
10^0	1	1
10^{-1}	$1 \times \frac{1}{10}$	0.1
10^{-2}	$1 \times \frac{1}{10} \times \frac{1}{10}$	0.01
10^{-3}	$1 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$	0.001

11. The multiplicative inverse of $\left(-\frac{5}{9}\right)^{-99}$ is
 (a) $\left(-\frac{5}{9}\right)^{99}$ (b) $\left(\frac{5}{9}\right)^{99}$ (c) $\left(\frac{9}{-5}\right)^{99}$ (d) $\left(\frac{9}{5}\right)^{99}$
12. If x be any non-zero integer and m, n be negative integers, then $x^m \times x^n$ is equal to
 (a) x^m (b) x^{m+n} (c) x^n (d) x^{m-n}
13. If y be any non-zero integer, then y^0 is equal to
 (a) 1 (b) 0 (c) -1 (d) Not defined
14. If x be any non-zero integer, then x^{-1} is equal to
 (a) x (b) $\frac{1}{x}$ (c) $-x$ (d) $\frac{-1}{x}$
15. If x be any integer different from zero and m be any positive integer, then x^{-m} is equal to
 (a) x^m (b) $-x^m$ (c) $\frac{1}{x^m}$ (d) $\frac{-1}{x^m}$
16. If x be any integer different from zero and m, n be any integers, then $(x^m)^n$ is equal to
 (a) x^{m+n} (b) x^{mn} (c) $x^{\frac{m}{n}}$ (d) x^{m-n}
17. Which of the following is equal to $\left(-\frac{3}{4}\right)^{-3}$?
 (a) $\left(\frac{3}{4}\right)^{-3}$ (b) $-\left(\frac{3}{4}\right)^{-3}$ (c) $\left(\frac{4}{3}\right)^3$ (d) $\left(-\frac{4}{3}\right)^3$
18. $\left(-\frac{5}{7}\right)^{-5}$ is equal to
 (a) $\left(\frac{5}{7}\right)^{-5}$ (b) $\left(\frac{5}{7}\right)^5$ (c) $\left(\frac{7}{5}\right)^5$ (d) $-\frac{7}{5}^5$

19. $\left(\frac{-7}{5}\right)^{-1}$ is equal to
 (a) $\frac{5}{7}$ (b) $-\frac{5}{7}$ (c) $\frac{7}{5}$ (d) $\frac{-7}{5}$
20. $(-9)^3 \div (-9)^8$ is equal to
 (a) $(9)^5$ (b) $(9)^{-5}$ (c) $(-9)^5$ (d) $(-9)^{-5}$
21. For a non-zero integer x , $x^7 \div x^{12}$ is equal to
 (a) x^5 (b) x^{19} (c) x^{-5} (d) x^{-19}
22. For a non-zero integer x , $(x^4)^{-3}$ is equal to
 (a) x^{12} (b) x^{-12} (c) x^{64} (d) x^{-64}
23. The value of $(7^{-1} - 8^{-1})^{-1} - (3^{-1} - 4^{-1})^{-1}$ is
 (a) 44 (b) 56 (c) 68 (d) 12
24. The standard form for 0.000064 is
 (a) 64×10^4 (b) 64×10^{-4} (c) 6.4×10^5 (d) 6.4×10^{-5}
25. The standard form for 234000000 is
 (a) 2.34×10^8 (b) 0.234×10^9 (c) 2.34×10^{-8} (d) 0.234×10^{-9}
26. The usual form for 2.03×10^{-5}
 (a) 0.203 (b) 0.00203 (c) 203000 (d) 0.0000203

Explore

Use a pattern to raise 10 to a zero or negative power

Step 1: Copy the table and complete the next two rows by evaluating 10^2 and 10^1 .

Step 2: Look at the rows you have completed. How does the standard form changes each time the exponent decreases by 1?

Step 3: Use the pattern you identified in Step 2 to complete the remaining rows in the table.

Power	Standard
10^3	1000
10^2	?
10^1	?
10^0	?
10^{-1}	?
10^{-2}	?
10^{-3}	?

27. $\left(\frac{1}{10}\right)^0$ is equal to
 (a) 0 (b) $\frac{1}{10}$ (c) 1 (d) 10
28. $\left(\frac{3}{4}\right)^5 \div \left(\frac{5}{3}\right)^5$ is equal to
 (a) $\left(\frac{3}{4} \div \frac{5}{3}\right)^5$ (b) $\left(\frac{3}{4} \div \frac{5}{3}\right)^1$ (c) $\left(\frac{3}{4} \div \frac{5}{3}\right)^0$ (d) $\left(\frac{3}{4} \div \frac{5}{3}\right)^{10}$
29. For any two non-zero rational numbers x and y , $x^4 \div y^4$ is equal to
 (a) $(x \div y)^0$ (b) $(x \div y)^1$ (c) $(x \div y)^4$ (d) $(x \div y)^8$
30. For a non-zero rational number p , $p^{13} \div p^8$ is equal to
 (a) p^5 (b) p^{21} (c) p^{-5} (d) p^{-19}
31. For a non-zero rational number z , $(z^{-2})^3$ is equal to
 (a) z^6 (b) z^{-6} (c) z^1 (d) z^4
32. Cube of $-\frac{1}{2}$ is
 (a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $-\frac{1}{8}$ (d) $-\frac{1}{16}$
33. Which of the following is not the reciprocal of $\left(\frac{2}{3}\right)^4$?
 (a) $\left(\frac{3}{2}\right)^4$ (b) $\left(\frac{3}{2}\right)^{-4}$ (c) $\frac{2}{3}^{-4}$ (d) $\frac{3^4}{2^4}$

In questions 34 to 65, fill in the blanks to make the statements true.

34. The multiplicative inverse of 10^{10} is _____.

35. $\alpha^3 \times \alpha^{10} =$ _____.

Draw Conclusions

Use your observations to complete this exercise.

Write the power of 10 in standard form.

1. 10^4

2. 10^9

3. 10^{-4}

4. 10^{-6}

36. $5^0 =$ _____.
37. $5^5 \times 5^{-5} =$ _____.
38. The value of $\left(\frac{1}{2^3}\right)^2$ is equal to _____.
39. The expression for 8^{-2} as a power with the base 2 is _____.
40. Very small numbers can be expressed in standard form by using _____ exponents.
41. Very large numbers can be expressed in standard form by using _____ exponents.
42. By multiplying $(10)^5$ by $(10)^{-10}$ we get _____.
43. $\left[\left(\frac{2}{13}\right)^{-6} \div \left(\frac{2}{13}\right)^3\right]^3 \times \left(\frac{2}{13}\right)^{-9} =$ _____
44. Find the value $[4^{-1} + 3^{-1} + 6^{-2}]^{-1}$.
45. $[2^{-1} + 3^{-1} + 4^{-1}]^0 =$ _____
46. The standard form of $\left(\frac{1}{100000000}\right)$ is _____.
47. The standard form of 12340000 is _____.
48. The usual form of 3.41×10^6 is _____.
49. The usual form of 2.39461×10^6 is _____.
50. If $36 = 6 \times 6 = 6^2$, then $\frac{1}{36}$ expressed as a power with the base 6 is _____.

Key Concept

Scientific Notation

A number is written in scientific notation if it has the form $c \times 10^n$ where $c > 1$, $c < 10$, and n is an integer.

Standard form	Product form	Scientific notation
325,000	$3.25 \times 100,000$	3.25×10^5
0.0005	5×0.0001	5×10^{-4}

51. By multiplying $\left(\frac{5}{3}\right)^4$ by _____ we get 5^4 .
52. $3^5 \div 3^{-6}$ can be simplified as _____.
53. The value of 3×10^{-7} is equal to _____.
54. To add the numbers given in standard form, we first convert them into numbers with _____ exponents.
55. The standard form for 32,50,00,00,000 is _____.
56. The standard form for 0.000000008 is _____.
57. The usual form for 2.3×10^{-10} is _____.
58. On dividing 8^5 by _____ we get 8.
59. On multiplying _____ by 2^{-5} we get 2^5 .
60. The value of $[3^{-1} \times 4^{-1}]^2$ is _____.
61. The value of $[2^{-1} \times 3^{-1}]^{-1}$ is _____.
62. By solving $(6^0 - 7^0) \times (6^0 + 7^0)$ we get _____.
63. The expression for 3^5 with a negative exponent is _____.
64. The value for $(-7)^6 \div 7^6$ is _____.
65. The value of $[1^{-2} + 2^{-2} + 3^{-2}] \times 6^2$ is _____.

In questions 66 to 90, state whether the given statements are true (T) or false (F).

66. The multiplicative inverse of $(-4)^{-2}$ is $(4)^{-2}$.
67. The multiplicative inverse of $\left(\frac{3}{2}\right)^2$ is not equal to $\left(\frac{2}{3}\right)^{-2}$.
68. $10^{-2} = \frac{1}{100}$
69. $24.58 = 2 \times 10 + 4 \times 1 + 5 \times 10 + 8 \times 100$
70. $329.25 = 3 \times 10^2 + 2 \times 10^1 + 9 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$
71. $(-5)^{-2} \times (-5)^{-3} = (-5)^{-6}$
72. $(-4)^{-4} \times (4)^{-1} = (4)^5$

$$73. \left(\frac{2}{3}\right)^{-2} \times \left(\frac{2}{3}\right)^{-5} = \left(\frac{2}{3}\right)^{10}$$

$$74. 5^0 = 5$$

$$75. (-2)^0 = 2$$

$$76. \left(-\frac{8}{2}\right)^0 = 0$$

$$77. (-6)^0 = -1$$

$$78. (-7)^{-4} \times (-7)^2 = (-7)^{-2}$$

$$79. \text{The value of } \frac{1}{4^{-2}} \text{ is equal to } 16.$$

$$80. \text{The expression for } 4^{-3} \text{ as a power with the base } 2 \text{ is } 2^6.$$

$$81. a^p \times b^q = (ab)^{pq}$$

$$82. \frac{x^m}{y^m} = \left(\frac{y}{x}\right)^{-m}$$

$$83. a^m = \frac{1}{a^{-m}}$$

$$84. \text{The exponential form for } (-2)^4 \times \left(\frac{5}{2}\right)^4 \text{ is } 5^4.$$

$$85. \text{The standard form for } 0.000037 \text{ is } 3.7 \times 10^{-5}.$$

Key Concept

Definition of Zero and Negative Exponents

Let a be a non-zero number, and let n be an integer.

Words

a to the zero power is 1.

a^n is the reciprocal of a^n .

a^n is the reciprocal of a^n .

Algebra

$$a^0 = 1$$

$$a^n = \frac{1}{a^n}$$

$$a^n = \frac{1}{a^{-n}}$$

Example

$$5^0 = 1$$

$$2^{-3} = \frac{1}{2^3}$$

$$2 = \frac{1}{2^{-1}}$$

86. The standard form for 203000 is 2.03×10^5
87. The usual form for 2×10^{-2} is not equal to 0.02.
88. The value of 5^{-2} is equal to 25.
89. Large numbers can be expressed in the standard form by using positive exponents.
90. $a^m \times b^m = (ab)^m$

91. Solve the following:

(i) 100^{-10} (ii) $2^{-2} \times 2^{-3}$ (iii) $\frac{1}{2}^{-2} \div \frac{1}{2}^{-3}$

92. Express $3^{-5} \times 3^{-4}$ as a power of 3 with positive exponent.

93. Express 16^{-2} as a power with the base 2.

94. Express $\frac{27}{64}$ and $\frac{-27}{64}$ as powers of a rational number.

95. Express $\frac{16}{81}$ and $\frac{-16}{81}$ as powers of a rational number.

96. Express as a power of a rational number with negative exponent.

(a) $\left(\left(\frac{-3}{2}\right)^{-2}\right)^{-3}$ (b) $(2^5 \div 2^8) \times 2^{-7}$

97. Find the product of the cube of (-2) and the square of $(+4)$.

98. Simplify:

(i) $\left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2}$

(ii) $\left(\left(\frac{-2}{3}\right)^{-2}\right)^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

(iii) $\frac{49 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}} (z \neq 0)$

(iv) $(2^5 \div 2^8) \times 2^{-7}$

99. Find the value of x so that

$$(i) \left(\frac{5}{3}\right)^{-2} \times \left(\frac{5}{3}\right)^{-14} = \left(\frac{5}{3}\right)^{8x}$$

$$(ii) (-2)^3 \times (-2)^{-6} = (-2)^{2x-1}$$

$$(iii) (2^{-1} + 4^{-1} + 6^{-1} + 8^{-1})^x = 1$$

100. Divide 293 by 10,00,000 and express the result in standard form.

101. Find the value of x^{-3} if $x = (100)^{1-4} \div (100)^0$.

102. By what number should we multiply $(-29)^0$ so that the product becomes $(+29)^0$.

103. By what number should $(-15)^{-1}$ be divided so that quotient may be equal to $(-15)^{-1}$?



Plan Strategy :

- Identify too much/too little information.

When you read a problem, you must decide if the problem has too much or too little information. If the problem has too much information, you must decide what information to use to solve the problem. If the problem has too little information, then you should determine what additional information you need to solve the problem.

- Read the problems below and decide if there is too much or too little information in each problem. If there is too much information, tell what information you would use to solve the problem. If there is too little information, tell what additional information you would need to solve the problem.
- On Monday, 20 students took an examination. There were 10 students who scored above 85 and 10 students who scored below 85. What was the average score?
- Aayesha is practising for a marathon. She ran for 50 minutes on Monday, 70 minutes on Wednesday, and 45 minutes on Friday. On Tuesday and Thursday, she lifted weights at the gym for 45 minutes each day. She swam for 45 minutes over the weekend. What was the average amount of time per day Aayesha spent running last week?

- 104.** Find the multiplicative inverse of $(-7)^{-2} \div (90)^{-1}$.
- 105.** If $5^{3x-1} \div 25 = 125$, find the value of x .
- 106.** Write 39,00,00,000 in the standard form.
- 107.** Write 0.000005678 in the standard form.
- 108.** Express the product of 3.2×10^6 and 4.1×10^{-1} in the standard form.
- 109.** Express $\frac{1.5 \times 10^6}{2.5 \times 10^{-4}}$ in the standard form.
- 110.** Some migratory birds travel as much as 15,000 km to escape the extreme climatic conditions at home. Write the distance in metres using scientific notation.
- 111.** Pluto is 59,1,30,00, 000 m from the sun. Express this in the standard form.
- 112.** Special balances can weigh something as 0.00000001 gram. Express this number in the standard form.
- 113.** A sugar factory has annual sales of 3 billion 720 million kilograms of sugar. Express this number in the standard form.
- 114.** The number of red blood cells per cubic millimetre of blood is approximately 5.5 million. If the average body contains 5 litres of blood, what is the total number of red cells in the body? Write the standard form. (1 litre = 1,00,000 mm³)
- 115.** Express each of the following in standard form:

(a) The mass of a proton in gram is

$$\frac{1673}{100000000000000000000000000000000}$$

- (b) A Helium atom has a diameter of 0.000000022 cm.
- (c) Mass of a molecule of hydrogen gas is about 0.0000000000000000000334 tons.
- (d) Human body has 1 trillion of cells which vary in shapes and sizes.

- (e) Express 56 km in m.
- (f) Express 5 tons in g.
- (g) Express 2 years in seconds.
- (h) Express 5 hectares in cm^2 (1 hectare = 10000 m^2)

116. Find x so that $\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2x-1}$

117. By what number should $\left(\frac{-3}{2}\right)^{-3}$ be divided so that the quotient may be $\left(\frac{4}{27}\right)^{-2}$?

In questions 118 and 119, find the value of n .

118. $\frac{6^n}{6^{-2}} = 6^3$

119. $\frac{2^n \times 2^6}{2^{-3}} = 2^{18}$

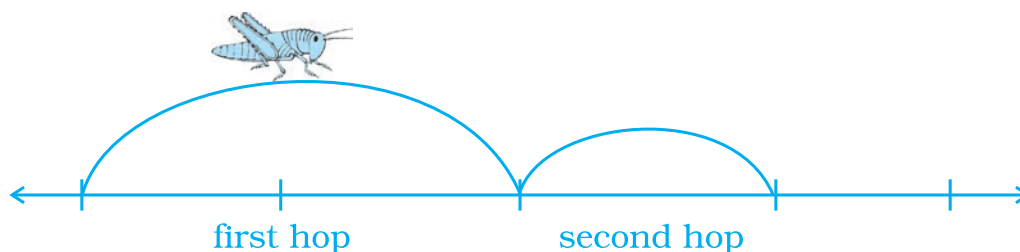
120. $\frac{125 \times x^{-3}}{5^{-3} \times 25 \times x^{-6}}$

121. $\frac{16 \times 10^2 \times 64}{2^4 \times 4^2}$

122. If $\frac{5^m \times 5^3 \times 5^{-2}}{5^{-5}} = 5^{12}$, find m .

- 123.** A new born bear weighs 4 kg. How many kilograms might a five year old bear weigh if its weight increases by the power of 2 in 5 years?
- 124.** The cells of a bacteria double in every 30 minutes. A scientist begins with a single cell. How many cells will be there after
 - (a) 12 hours
 - (b) 24 hours
- 125.** Planet A is at a distance of 9.35×10^6 km from Earth and planet B is 6.27×10^7 km from Earth. Which planet is nearer to Earth?
- 126.** The cells of a bacteria double itself every hour. How many cells will there be after 8 hours, if initially we start with 1 cell. Express the answer in powers.

- 127.** An insect is on the 0 point of a number line, hopping towards 1. She covers half the distance from her current location to 1 with each hop. So, she will be at $\frac{1}{2}$ after one hop, $\frac{3}{4}$ after two hops, and so on.



- (a) Make a table showing the insect's location for the first 10 hops.
 (b) Where will the insect be after n hops?
 (c) Will the insect ever get to 1? Explain.
- 128.** Predicting the ones digit, copy and complete this table and answer the questions that follow.

Powers Table										
x	1^x	2^x	3^x	4^x	5^x	6^x	7^x	8^x	9^x	10^x
1	1	2								
2	1	4								
3	1	8								
4	1	16								
5	1	32								
6	1	64								
7	1	128								
8	1	256								
Ones Digits of the Powers	1	2,4,8,6								

- (a) Describe patterns you see in the ones digits of the powers.
 (b) Predict the ones digit in the following:
 1. 4^{12} 2. 9^{20} 3. 3^{17} 4. 5^{100} 5. 10^{500}
 (c) Predict the ones digit in the following:
 1. 31^{10} 2. 12^{10} 3. 17^{21} 4. 29^{10}

- 129. Astronomy** The table shows the mass of the planets, the sun and the moon in our solar system.

Celestial Body	Mass (kg)	Mass (kg) Standard Notation
Sun	1,990,000,000,000,000,000,000,000,000	1.99×10^{30}
Mercury	330,000,000,000,000,000,000,000	
Venus	4,870,000,000,000,000,000,000,000	
Earth	5,970,000,000,000,000,000,000,000	
Mars	642,000,000,000,000,000,000,000,000	
Jupiter	1,900,000,000,000,000,000,000,000,000	
Saturn	568,000,000,000,000,000,000,000,000	
Uranus	86,800,000,000,000,000,000,000,000	
Neptune	102,000,000,000,000,000,000,000,000	
Pluto	12,700,000,000,000,000,000,000,000	
Moon	73,500,000,000,000,000,000,000,000	

- Write the mass of each planet and the Moon in scientific notation.
- Order the planets and the moon by mass, from least to greatest.
- Which planet has about the same mass as earth?

- 130. Investigating Solar System** The table shows the average distance from each planet in our solar system to the sun.

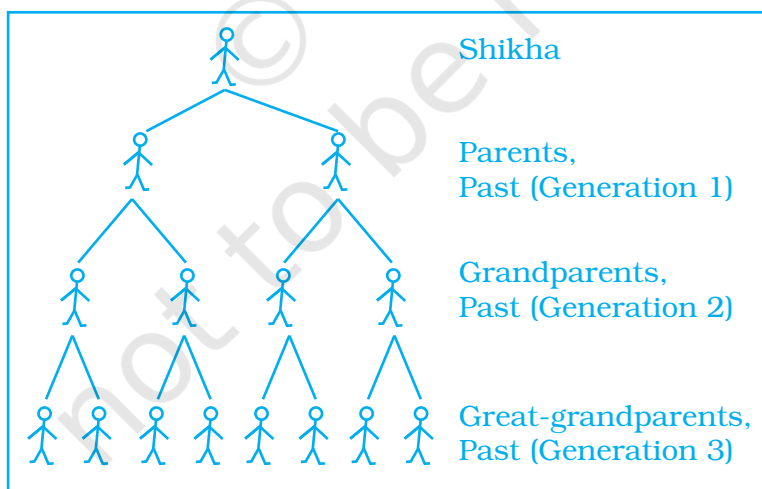
Planet	Distance from Sun (km)	Distance from Sun (km) Standard Notation
Earth	149,600,000	1.496×10^8
Jupiter	778,300,000	
Mars	227,900,000	
Mercury	57,900,000	
Neptune	4,497,000,000	
Pluto	5,900,000,000	
Saturn	1,427,000,000	
Uranus	2,870,000,000	
Venus	108,200,000	

- Complete the table by expressing the distance from each planet to the Sun in scientific notation.
 - Order the planets from closest to the sun to farthest from the sun.
- 131.** This table shows the mass of one atom for five chemical elements. Use it to answer the question given.

Element	Mass of atom (kg)
Titanium	7.95×10^{-26}
Lead	3.44×10^{-25}

Silver	1.79×10^{-25}
Lithium	1.15×10^{-26}
Hydrogen	1.674×10^{-27}

- (a) Which is the heaviest element?
 - (b) Which element is lighter, Silver or Titanium?
 - (c) List all five elements in order from lightest to heaviest.
- 132.** The planet Uranus is approximately 2,896,819,200,000 metres away from the Sun. What is this distance in standard form?
- 133.** An inch is approximately equal to 0.02543 metres. Write this distance in standard form.
- 134.** The volume of the Earth is approximately 7.67×10^{-7} times the volume of the Sun. Express this figure in usual form.
- 135.** An electron's mass is approximately $9.1093826 \times 10^{-31}$ kilograms. What is this mass in grams?
- 136.** At the end of the 20th century, the world population was approximately 6.1×10^9 people. Express this population in usual form. How would you say this number in words?
- 137.** While studying her family's history, Shikha discovers records of ancestors 12 generations back. She wonders how many ancestors she has had in the past 12 generations. She starts to make a diagram to help her figure this out. The diagram soon becomes very complex.



- Make a table and a graph showing the number of ancestors in each of the 12 generations.
- Write an equation for the number of ancestors in a given generation n .

138. About 230 billion litres of water flows through a river each day. How many litres of water flows through that river in a week? How many litres of water flows through the river in an year? Write your answer in standard notation.

139. A half-life is the amount of time that it takes for a radioactive substance to decay to one half of its original quantity.

Suppose radioactive decay causes 300 grams of a substance to decrease to 300×2^{-3} grams after 3 half-lives. Evaluate 300×2^{-3} to determine how many grams of the substance are left.

Explain why the expression 300×2^{-n} can be used to find the amount of the substance that remains after n half-lives.

140. Consider a quantity of a radioactive substance. The fraction of this quantity that remains after t half-lives can be found by using the expression 3^{-t} .

(a) What fraction of substance remains after 7 half-lives?

(b) After how many half-lives will the fraction be $\frac{1}{243}$ of the original?

141. One Fermi is equal to 10^{-15} metre. The radius of a proton is 1.3 Fermis. Write the radius of a proton in metres in standard form.

142. The paper clip below has the indicated length. What is the length in standard form.



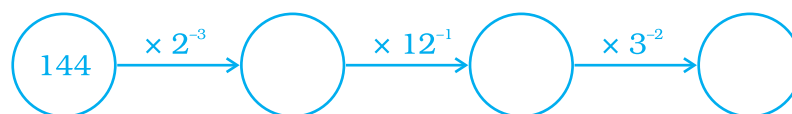
143. Use the properties of exponents to verify that each statement is true.

(a) $\frac{1}{4}(2^n) = 2^{n-2}$

(b) $4^{n-1} = \frac{1}{4}(4)^n$

(c) $25(5^{n-2}) = 5^n$

144. Fill in the blanks



145. There are 864,00 seconds in a day. How many days long is a second? Express your answer in scientific notation.

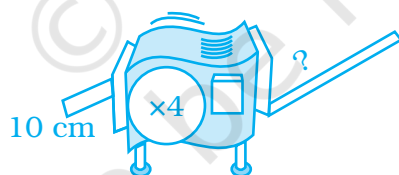
146. The given table shows the crop production of a State in the year 2008 and 2009. Observe the table given below and answer the given questions.

Crop	2008 Harvest (Hectare)	Increase/Decrease (Hectare) in 2009
Bajra	1.4×10^3	- 100
Jowar	1.7×10^6	- 440,000
Rice	3.7×10^3	- 100
Wheat	5.1×10^5	+ 190,000

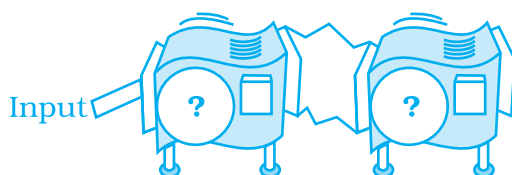
- For which crop(s) did the production decrease?
 - Write the production of all the crops in 2009 in their standard form.
 - Assuming the same decrease in rice production each year as in 2009, how many acres will be harvested in 2015? Write in standard form.
147. **Stretching Machine**

Suppose you have a stretching machine which could stretch almost anything. For example, if you put a 5 metre stick into a ($\times 4$) stretching machine (as shown below), you get a 20 metre stick.

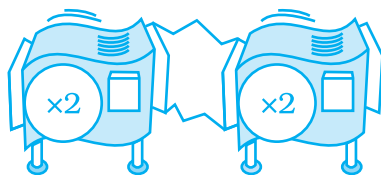
Now if you put 10 cm carrot into a ($\times 4$) machine, how long will it be when it comes out?



148. Two machines can be hooked together. When something is sent through this hook up, the output from the first machine becomes the input for the second.
- Which two machines hooked together do the same work as a ($\times 10^2$) machine does? Is there more than one arrangement of two machines that will work?

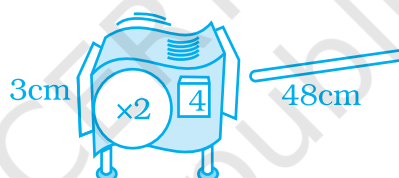


- (b) Which stretching machine does the same work as two ($\times 2$) machines hooked together?



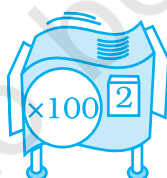
149. Repeater Machine

Similarly, repeater machine is a hypothetical machine which automatically enlarges items several times. For example, sending a piece of wire through a ($\times 2^4$) machine is the same as putting it through a ($\times 2$) machine four times. So, if you send a 3 cm piece of wire through a ($\times 2^4$) machine, its length becomes $3 \times 2 \times 2 \times 2 \times 2 = 48$ cm. It can also be written that a base (2) machine is being applied 4 times.



What will be the new length of a 4 cm strip inserted in the machine?

150. For the following repeater machines, how many times the base machine is applied and how much the total stretch is?



(a)



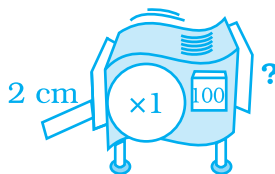
(b)



(c)

151. Find three repeater machines that will do the same work as a ($\times 64$) machine. Draw them, or describe them using exponents.

152. What will the following machine do to a 2 cm long piece of chalk?



153. In a repeater machine with 0 as an exponent, the base machine is applied 0 times.

(a) What do these machines do to a piece of chalk?



(b) What do you think the value of 6^0 is?

You have seen that a hookup of repeater machines with the same base can be replaced by a single repeater machine. Similarly, when you multiply exponential expressions with the same base, you can replace them with a single expression.

Asif Raza thought about how he could rewrite the expression $2^{20} \times 2^5$.



Asif Raza's idea is one of the *product laws of exponents*, which can be expressed like this:

Multiplying Expressions with the Same Base

$$a^b \times a^c = a^{b+c}$$

Actually, this law can be used with more than two expressions. As long as the bases are the same, to find the product you can add the exponents and use the same base. For example:

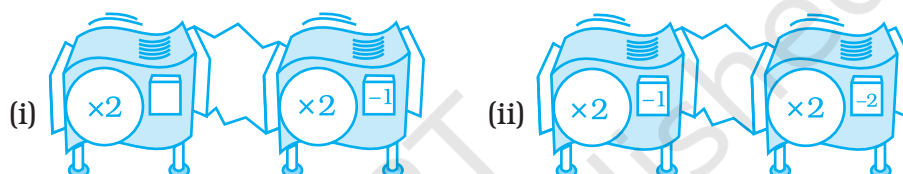
$$\begin{aligned} 3^2 \times 3^3 \times 3^{10} &= 3^{2+3+10} \\ &= 3^{15} \end{aligned}$$

154. Shrinking Machine

In a shrinking machine, a piece of stick is compressed to reduce its length. If 9 cm long sandwich is put into the shrinking machine below, how many cm long will it be when it emerges?

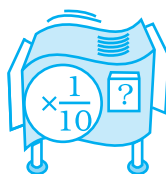


155. What happens when 1 cm worms are sent through these hook-ups?



156. Sanchay put a 1cm stick of gum through a (1×3^{-2}) machine. How long was the stick when it came out?

157. Ajay had a 1cm piece of gum. He put it through repeater machine given below and it came out $\frac{1}{100,000}$ cm long. What is the missing value?



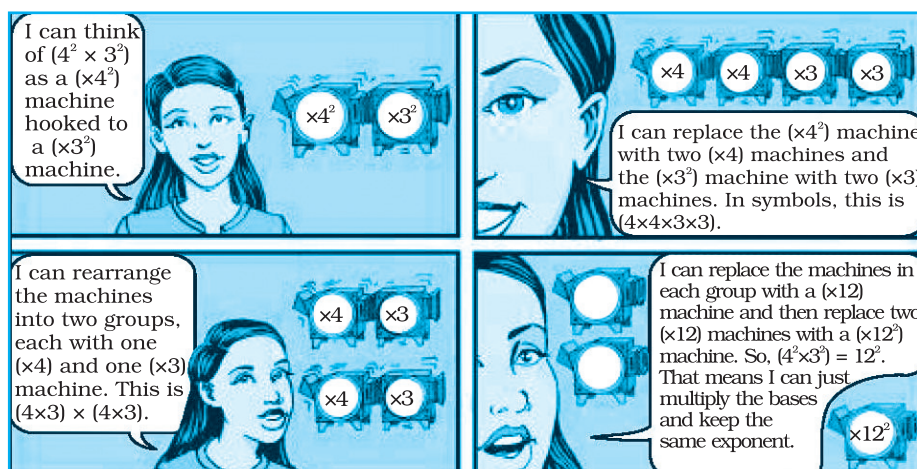
158. Find a single machine that will do the same job as the given hook-up.

(a) a $(\times 2^3)$ machine followed by $(\times 2^{-2})$ machine.

(b) a $(\times 2^4)$ machine followed by $\left(\times \left(\frac{1}{2}\right)^2\right)$ machine.

(c) a $(\times 5^{99})$ machine followed by a (5^{-100}) machine.

Maya multiplied $(4^2 \times 3^2)$ by thinking about stretching machines.



Use Maya's idea to multiply $5^3 \times 2^3$

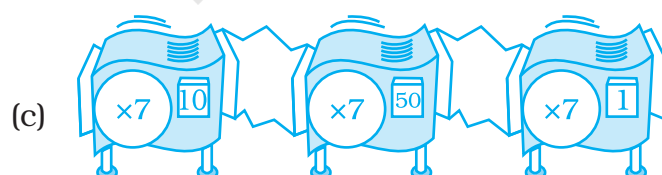
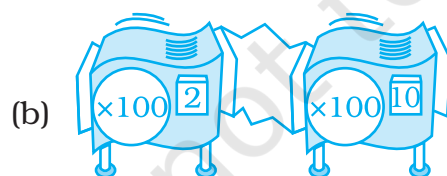
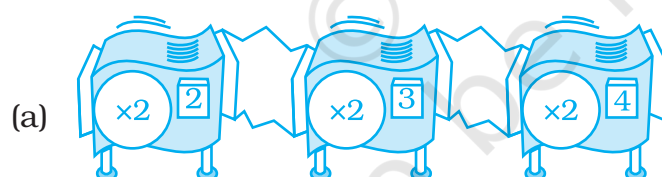
Maya's idea is another *product law of exponents*.

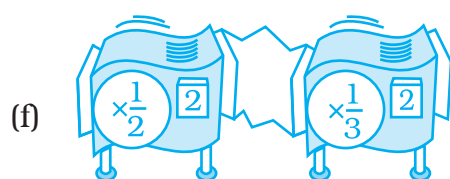
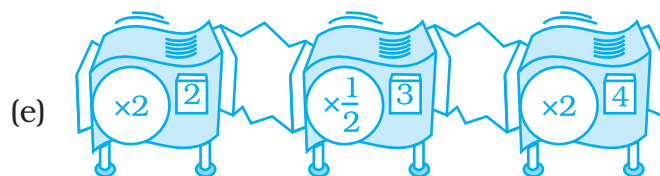
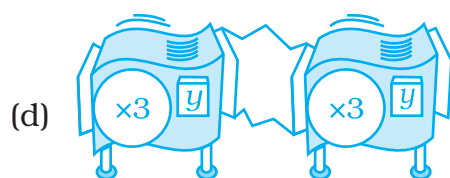
Multiplying Expressions with the Same Exponents

$$a^c \times b^c = (a \times b)^c$$

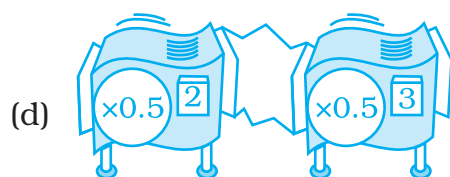
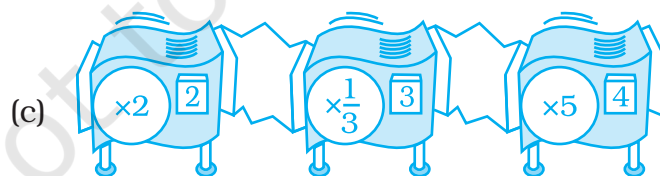
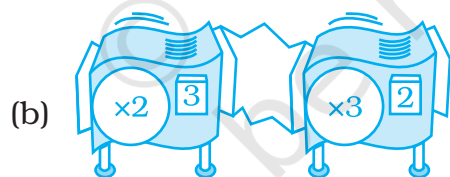
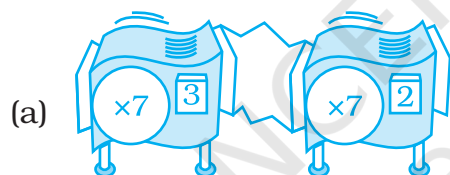
You can use this law with more than two expressions. If the exponents are the same, multiply the expressions by multiplying the bases and using the same exponent. For example, $2^8 \times 3^8 \times 7^8 = (2 \times 3 \times 7)^8 = 42^8$

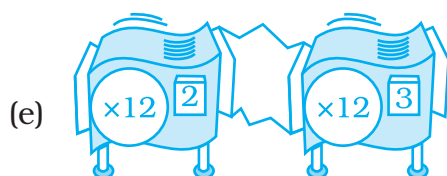
- 159.** Find a single repeater machine that will do the same work as each hook-up.





160. For each hook-up, determine whether there is a single repeater machine that will do the same work. If so, describe or draw it.

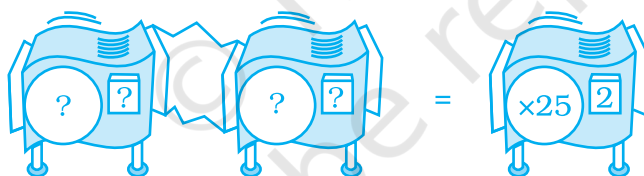




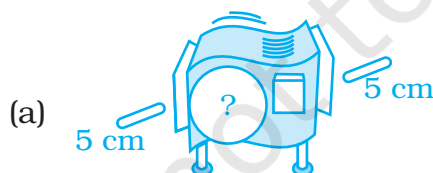
- 161.** Shikha has an order from a golf course designer to put palm trees through a $(\times 2^3)$ machine and then through a $(\times 3^3)$ machine. She thinks she can do the job with a single repeater machine. What single repeater machine should she use?

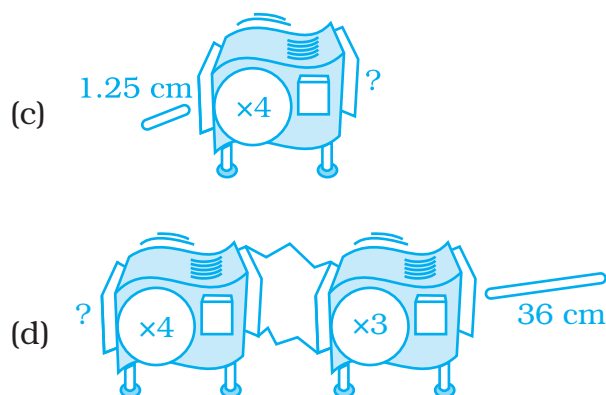


- 162.** Neha needs to stretch some sticks to 25^2 times their original lengths, but her $(\times 25)$ machine is broken. Find a hook-up of two repeater machines that will do the same work as a $(\times 25^2)$ machine. To get started, think about the hookup you could use to replace the $(\times 25)$ machine.

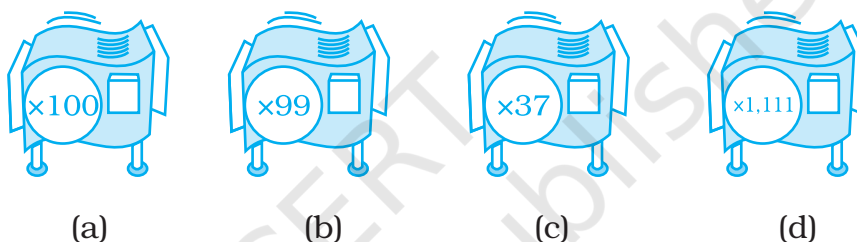


- 163.** Supply the missing information for each diagram.





- 164.** If possible, find a hook-up of prime base number machine that will do the same work as the given stretching machine. Do not use ($\times 1$) machines.



- 165.** Find two repeater machines that will do the same work as a ($\times 81$) machine.
- 166.** Find a repeater machine that will do the same work as a ($\times \frac{1}{8}$) machine.
- 167.** Find three machines that can be replaced with hook-ups of ($\times 5$) machines.
- 168.** The left column of the chart lists the lengths of input pieces of ribbon. Stretching machines are listed across the top. The other entries are the outputs for sending the input ribbon from that row through the machine from that column. Copy and complete the chart.

Input Length	Machine			
	$\times 2$			
	1	5		
3				15
	14		7	

- 169.** The left column of the chart lists the lengths of input chains of gold. Repeater machines are listed across the top. The other entries are the outputs you get when you send the input chain from that row through the repeater machine from that column. Copy and complete the chart.

Input Length	Repeater Machine		
	$\times 2^3$		
	40		125
2			
		162	

- 170.** Long back in ancient times, a farmer saved the life of a king's daughter. The king decided to reward the farmer with whatever he wished. The farmer, who was a chess champion, made an unusual request:

"I would like you to place 1 rupee on the first square of my chessboard, 2 rupees on the second square, 4 on the third square, 8 on the fourth square, and so on, until you have covered all 64 squares. Each square should have twice as many rupees as the previous square." The king thought this to be too less and asked the farmer to think of some better reward, but the farmer didn't agree.

How much money has the farmer earned?

[Hint: The following table may help you. What is the first square on which the king will place at least Rs 10 lakh?]

Position of Square on chess board	Amount (in Rs)
1st square	1
2nd square	2
3rd square	4

- 171.** The diameter of the Sun is 1.4×10^9 m and the diameter of the Earth is 1.2756×10^7 m. Compare their diameters by division.
- 172.** Mass of Mars is 6.42×10^{29} kg and mass of the Sun is 1.99×10^{30} kg. What is the total mass?

- 173.** The distance between the Sun and the Earth is 1.496×10^8 km and distance between the Earth and the Moon is 3.84×10^8 m. During solar eclipse the Moon comes in between the Earth and the Sun. What is distance between the Moon and the Sun at that particular time?
- 174.** A particular star is at a distance of about 8.1×10^{13} km from the Earth. Assuming that light travels at 3×10^8 m per second, find how long does light takes from that star to reach the Earth.
- 175.** By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?
- 176.** By what number should $(-8)^{-3}$ be multiplied so that that the product may be equal to $(-6)^{-3}$?
- 177.** Find x .

$$(1) \quad -\frac{1}{7}^{-5} \div -\frac{1}{7}^{-7} = (-7)^x$$

$$(2) \quad \frac{2}{5}^{2x+6} \times \frac{2}{5}^3 = \frac{2}{5}^{x+2}$$

$$(3) \quad 2^x + 2^x + 2^x = 192$$

$$(4) \quad \frac{-6}{7}^{x-7} = 1$$

$$(5) \quad 2^{3x} = 8^{2x+1}$$

$$(6) \quad 5^x + 5^{x-1} = 750$$

- 178.** If $a = -1$, $b = 2$, then find the value of the following:

$$(1) \quad a^b + b^a \quad (2) \quad a^b - b^a \quad (3) \quad a^b \times b^2 \quad (4) \quad a^b \div b^a$$

- 179.** Express each of the following in exponential form:

$$(1) \quad \frac{-1296}{14641} \quad (2) \quad \frac{-125}{343} \quad (3) \quad \frac{400}{3969} \quad (4) \quad \frac{-625}{10000}$$

- 180.** Simplify:

$$(1) \quad \frac{1}{2}^2 - \frac{1}{4}^3^{-1} = \times 2^{-3}$$

$$(2) \quad \frac{4}{3}^{-2} - \frac{3}{4}^{2(-2)}$$

$$(3) \quad \left(\frac{4}{13}\right)^4 \times \left(\frac{13}{7}\right)^2 \times \left(\frac{7}{4}\right)^3$$

$$(4) \quad \left(\frac{1}{5}\right)^{45} \times \left(\frac{1}{5}\right)^{-60} - \left(\frac{1}{5}\right)^{+28} \times \left(\frac{1}{5}\right)^{-43}$$

$$(5) \quad \frac{(9)^3 \times 27 \times t^4}{(3)^{-2} \times (3)^4 \times t^2}$$

$$(6) \quad \frac{(3^{-2})^2 \times (5^2)^{-3} \times (t^{-3})^2}{(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3}$$

(D) ACTIVITIES

Activity 1

To make ballot papers for the upcoming school elections, cut a sheet of paper in half. Stack the two pieces and cut them in half again. Stack the resulting four pieces and cut them in half again. Repeat this process, creating smaller sized ballot papers.



Count the ballots and record the result in the table below.

Number of Cuts	Number of Ballot papers
1	2
2	4
3	8
4	16

Now, answer the following:

- Can you predict the number of ballot papers after n number of cuts?
- Suppose you could make 40 cuts. How many ballot papers would you have?
- How many cuts would it take to make enough ballot papers for 512 students at school?
- Suppose you start with a sheet of paper having area 324 cm^2 . Copy and complete the table to show the area of each ballot paper after each of the first 10 cuts.

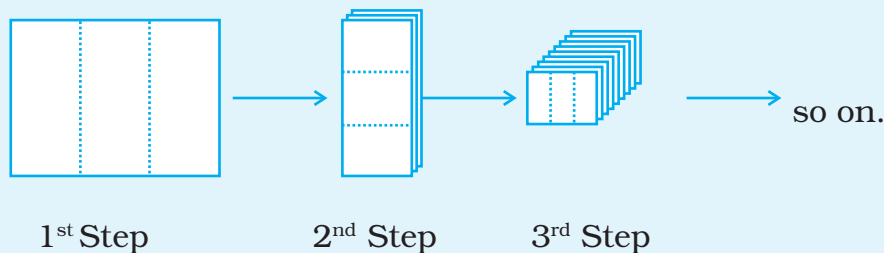
Number of Cuts	Area (cm^2)
0	324
1	162
2	81
3	4
5	
6	
7	
8	
9	
10	

Observe the table and write an expression which gives the area of ballot papers after n number of cuts on sheet of given area A .

- If you want 512 ballot papers each having area 16 cm^2 , then what will be the area of starting paper?

Activity 2

- To make ballot papers cut a sheet of paper into three. Stack the three pieces and cut the stack into three. Stack all the pieces and cut the stack into three again.



- (a) Complete the table to show the number of ballot papers after five such steps:

Number of Steps	Number of Ballot Paper
1	3
2	
3	
4	
5	

- (b) Suppose you continue this process. How many ballot papers would you have after 15 steps? How many would you have after n cuts?
- (c) How many steps would it take to make at least one lakh ballot paper?

Crossword

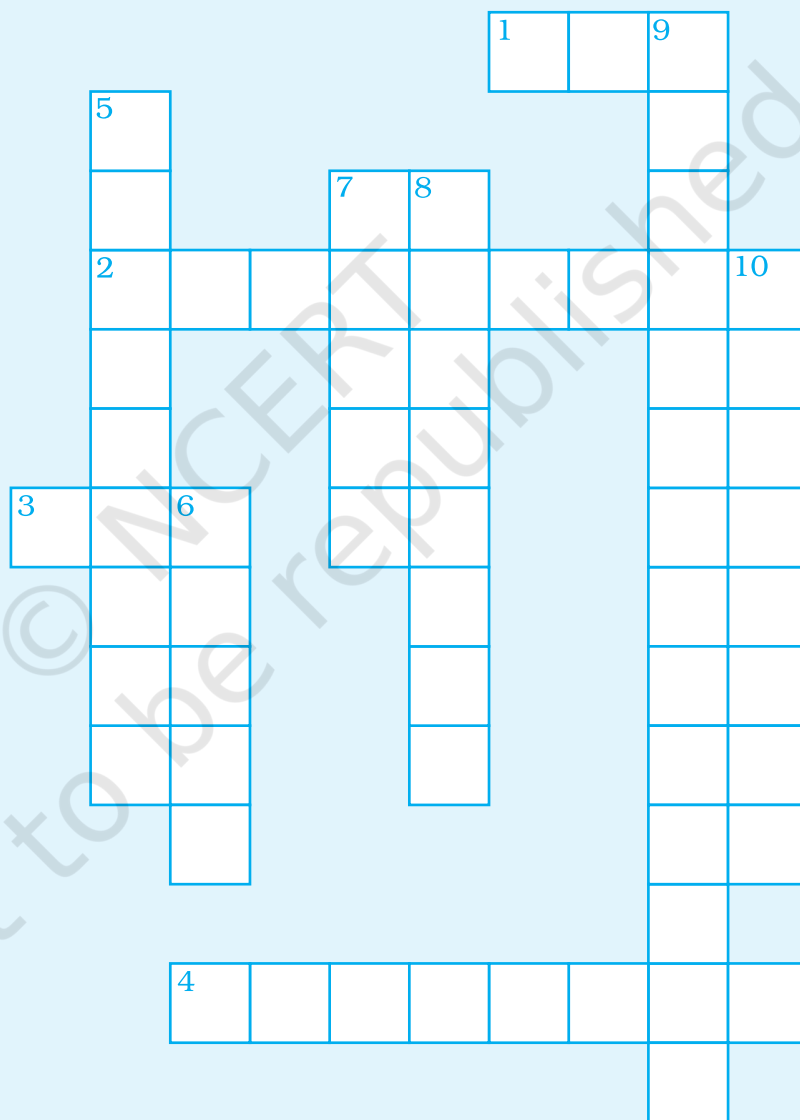
Across

- (1) In $x^m \times x^n = x^p$, p is the _____ of m and n .
- (1) -7 is the _____ of 2 exponents -5 and -2 .
- (2) Very large numbers like 6,250,000,000 can be conveniently written using _____.
- (3) The value of a^n if $n = 0$.
- (4) Very small numbers can be expressed in standard form using _____ exponents.

Down

- (5) The value of 3^{-2} .

- (6) The value of $\frac{1}{2^{-3}}$.
- (7) 5^7 is read as 5 raised to the _____ of 7.
- (8) As the exponent decreases by 1, the value becomes _____ of the previous value.
- (9) a^m is the _____ inverse of a^m .
- (10) 1.24×10^{-4} is known as the _____ form of 0.000124.



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UNIT 9

COMPARING QUANTITIES

(A) Main Concepts and Results

- Discount is a reduction given on marked price.

Discount = Marked Price – Sale Price (S.P.)

- Discount can be calculated when discount percentage is given.

Discount = Discount % of Marked Price

- Additional expenses made after buying an article are included in the cost price and are known as overhead expenses.

cost price = buying price + overhead expenses

- Sales Tax is charged on the sale of an item by the government and is added to the Bill Amount.

Sales tax = Tax% of sale amount

These days, however, the selling prices (known as MRP) include the tax known as VAT (Value Added Tax).

- The interest compounded annually is the interest calculated on the previous year's amount A, ($A = P + I$).
- The time period after which the interest is added each time to form a new principal is called the **conversion period**.
- When the interest is compounded half yearly, there are two conversion periods in a year of duration 6 months each.
- Amount when interest is compounded annually is

$$A = P \left(1 + \frac{R}{100} \right)^n$$

where

P is Principal

R is Rate of interest

n is Time Period

- Amount when interest is compounded half yearly is

$$A = P \left(1 + \frac{R}{200} \right)^{2n}$$

where $\frac{R}{200}$ is half yearly rate and $2n$ is number of half years.

(B) Solved Examples

In examples 1 to 4, there are four options out of which one is correct. Write the correct answer.

Example 1 : A shirt with marked price Rs 800 was sold at Rs 680. The rate of discount allowed on the shirt is

- (a) 10% (b) 15% (c) 20% (d) 25%

Solution : Correct answer is (b).

Example 2 : If $\frac{7}{3}\%$ of a number is 42, then the number is

- (a) 9800 (b) 8 (c) 1800 (d) 180

Solution : Correct answer is (c).

Example 3 : If the cost price of 10 shirts is equal to the selling price of 8 shirts, then which of the following is true for the transaction?

- (a) Profit of 25% (b) Loss of 25%
(c) Profit of 20% (d) Loss of 20%

Solution : Correct answer is (a).

Example 4 : Rs 1600 lent at a compound interest of 5% per annum, compounded half yearly for one year will amount to:

- (a) Rs 1640 (b) Rs 1680 (c) Rs 1681 (d) Rs 1764

Solution: Correct answer is (c).

In examples 5 to 7, fill in the blanks to make the statements true.

Example 5 : By selling 50 pens, a shopkeeper lost the amount equal to the selling price of 10 pens. His loss per cent is _____.

Solution : $\frac{50}{3}\%$.

Example 6 : The discount per cent is calculated on the _____ price of an article.

Solution : Marked Price.

Example 7 : Amna purchased a toy for Rs 660 including sales tax. If the rate of sales tax is 10%, then the selling price of the toy is _____.

Solution : Rs 600.

In examples 8 to 11, state whether the statements are true (T) or false (F).

Example 8 : When the interest is compounded half yearly, the number of conversion periods in a year is four.

Solution : False.

Example 9 : Arnav buys a book costing Rs 600. If the rate of sales tax is 7%, then the total amount payable by him is Rs 642.

Solution : True.

Example 10 : After allowing a discount of 15% on the marked price of an article, it is sold for Rs 680. The marked price of the article is Rs 800.

Solution : True.

Example 11 : Overhead charges, if any, are sometimes included in the cost price.

Solution : False.

Example 12 : A number is increased by 20% and then it is decreased by 20%. Find the net increase or decrease per cent.

Solution : Let the number be 100

Increase in the number = 20% of 100 = 20

So Increased number = 100 + 20 = 120

$$\text{Decrease in the number} = 20\% \text{ of } 120 = \frac{20}{100} \times 120 = 24$$

$$\text{So new number} = 120 - 24 = 96$$

$$\text{Net decrease} = 100 - 96 = 4$$

$$\text{Hence net decrease per cent} = \frac{4}{100} \times 100 = 4\%.$$

Example 13 : Vishakha offers a discount of 20% on all the items at her shop and still makes a profit of 12%. What is the cost price of an article marked at Rs 280?

Solution : Marked Price = Rs 280

$$\text{Discount} = 20\% \text{ of Rs 280}$$

$$= \frac{20}{100} \times 280 = \text{Rs } 56$$

$$\text{So selling price} = \text{Rs } (280 - 56)$$

$$= \text{Rs } 224$$

Let the cost price be Rs 100

$$\text{Profit} = 12\% \text{ of Rs 100}$$

$$= \text{Rs } 12$$

$$\text{So selling price} = \text{Rs } (100 + 12) = \text{Rs } 112$$

If the selling price is Rs 112, cost price = Rs 100

$$\text{If the selling price is Rs 224, cost price} = \text{Rs } \left(\frac{100}{112} \times 224 \right)$$

$$= \text{Rs } 200.$$

Example 14 : Find the compound interest on Rs 48,000 for one year at 8% per annum when compounded half yearly.

Solution : Principal (P) = Rs 48,000

$$\text{Rate (R)} = 8\% \text{ p.a.}$$

$$\text{Time (n)} = 1 \text{ year}$$

Interest is compounded half yearly

$$\therefore A = P \left(1 + \frac{R}{200} \right)^{2n}$$

$$= 48,000 \left(1 + \frac{8}{200}\right)^2$$

$$= 48,000 \times \frac{26}{25} \times \frac{26}{25}$$

$$= 76.8 \times 26 \times 26$$

$$= \text{Rs } 51,916.80$$

Therefore Compound Interest = A – P

$$= \text{Rs } (51,916.80 - 48,000)$$

$$= \text{Rs } 3,916.80$$

Application on Problem Solving Strategy



Example 15 :

Lemons were bought at Rs 60 a dozen and sold at the rate of Rs 40 per 10. Find the gain or loss percent.



Understand and Explore the problem

- What do you know?

Cost of lemons per dozen : Rs 60

and S.P. of 10 lemons : Rs 40

Which other information is needed to solve the given problem?

Gain/Loss



Plan a Strategy

- Change: either convert C.P. of 1 dozen into C.P. of 10 lemons or convert S.P. of 10 lemons into S.P. of 1 dozen
- If C.P. > S.P. find Loss = C.P. – S.P.
If S.P. > C.P. find gain = S.P. – C.P.
If S.P. = C.P. No gain, No loss
- If gain, find gain %
If loss, find loss %
Gain/Loss are always calculated on the basis of C.P.



Solve

- C.P. of 1 dozen lemons = Rs 60

C.P. of 1 lemon = Rs 5

C.P. of 10 lemons = Rs 50

- S.P. of 10 lemons Rs 40

- As S.P. < C.P. = Loss

$$\begin{aligned}\text{Loss} &= \text{C.P.} - \text{S.P.} \\ &= \text{Rs } 50 - \text{Rs } 40 \\ &= \text{Rs } 10\end{aligned}$$

$$\begin{aligned}\text{Loss} &= \frac{\text{Loss}}{\text{C.P.}} \times 100\% \\ &= \frac{10}{50} \times 100\% \\ &= 20\%\end{aligned}$$



Revise

- Verify your answer by adopting Backward method

i.e. Loss % = 20%

Loss = 20% of 60

$$= \frac{20}{100} \times 60 = \text{Rs } 12$$

C.P. of 1 dozen = Rs 60

Loss on 1 dozen = Rs 12

S.P. = C.P. - Loss = 60 - 12 = Rs 48

S.P. of 1 dozen = Rs 48

S.P. of 1 lemon = Rs $\frac{48}{12}$ = Rs 4

S.P. of 10 lemons = Rs 40

Hence verified.

Think and Discuss

- (a) What would happen if C.P. of 1 dozen is Rs 48?
 (b) What would happen if S.P. of 10 lemons is Rs 60?

(C) Exercise

In questions 1 to 20, there are four options out of which one is correct.

Write the correct answer.

- Suppose for the principal P , rate $R\%$ and time T , the simple interest is S and compound interest is C . Consider the possibilities.
 (i) $C > S$ (ii) $C = S$ (iii) $C < S$
 Then
 (a) only (i) is correct.
 (b) either (i) or (ii) is correct.
 (c) either (ii) or (iii) is correct.
 (d) only (iii) is correct.
- Suppose a certain sum doubles in 2 years at $r\%$ rate of simple interest per annum or at $R\%$ rate of interest per annum compounded annually. We have
 (a) $r < R$ (b) $R < r$ (c) $R = r$ (d) can't be decided
- The compound interest on Rs 50,000 at 4% per annum for 2 years compounded annually is
 (a) Rs 4,000 (b) Rs 4,080 (c) Rs 4,280 (d) Rs 4,050
- If marked price of an article is Rs 1,200 and the discount is 12% then the selling price of the article is
 (a) Rs 1,056 (b) Rs 1,344 (c) Rs 1,212 (d) Rs 1,188
- If 90% of x is 315 km, then the value of x is
 (a) 325 km (b) 350 km (c) 350 m (d) 325 m

6. To gain 25% after allowing a discount of 10%, the shopkeeper must mark the price of the article which costs him Rs 360 as
 (a) Rs 500 (b) Rs 450 (c) Rs 460 (d) Rs 486
7. If $a\%$ is the discount per cent on a marked price x , then discount is
 (a) $\frac{x}{a} \times 100$ (b) $\frac{a}{x} \times 100$ (c) $x \times \frac{a}{100}$ (d) $\frac{100}{x \times a}$
8. Ashima took a loan of Rs 1,00,000 at 12% p.a. compounded half-yearly. She paid Rs 1,12,360. If $(1.06)^2$ is equal to 1.1236, then the period for which she took the loan is
 (a) 2 years (b) 1 year (c) 6 months (d) $1\frac{1}{2}$ years
9. For calculation of interest compounded half yearly, keeping the principal same, which one of the following is true.
 (a) Double the given annual rate and half the given number of years.
 (b) Double the given annual rate as well as the given number of years.
 (c) Half the given annual rate as well as the given number of years.
 (d) Half the given annual rate and double the given number of years.
10. Shyama purchases a scooter costing Rs 36,450 and the rate of sales tax is 9%, then the total amount paid by her is
 (a) Rs 36,490.50 (b) Rs 39,730.50
 (c) Rs 36,454.50 (d) Rs 33,169.50
11. The marked price of an article is Rs 80 and it is sold at Rs 76, then the discount rate is
 (a) 5% (b) 95% (c) 10% (d) appx. 11%
12. A bought a tape recorder for Rs 8,000 and sold it to B. B in turn sold it to C, each earning a profit of 20%. Which of the following is true:
 (a) A and B earn the same profit.

- The discount is the amount by which the regular price is reduced.
- The sale price is the regular price minus the discount.

- (b) A earns more profit than B.
(c) A earns less profit than B.
(d) Cannot be decided.
- 13.** Latika bought a teapot for Rs 120 and a set of cups for Rs 400. She sold teapot at a profit of 5% and cups at a loss of 5%. The amount received by her is
(a) Rs 494 (b) Rs 546 (c) Rs 506 (d) Rs 534
- 14.** A jacket was sold for Rs 1,120 after allowing a discount of 20%. The marked price of the jacket is
(a) Rs 1440 (b) Rs 1400 (c) Rs 960 (d) Rs 866.66
- 15.** A sum is taken for two years at 16% p.a. If interest is compounded after every three months, the number of times for which interest is charged in 2 years is
(a) 8 (b) 4 (c) 6 (d) 9
- 16.** The original price of a washing machine which was bought for Rs 13,500 inclusive of 8% VAT is
(a) Rs 12,420 (b) Rs 14,580 (c) Rs 12,500 (d) Rs 13,492
- 17.** Avinash bought an electric iron for Rs 900 and sold it at a gain of 10%. He sold another electric iron at 5% loss which was bought Rs 1200. On the transaction he has a
(a) Profit of Rs 75 (b) Loss of Rs 75
(c) Profit of Rs 30 (d) Loss of Rs 30
- 18.** A TV set was bought for Rs 26,250 including 5% VAT. The original price of the TV set is
(a) Rs 27,562.50 (b) Rs 25,000 (c) Rs 24,937.50 (d) Rs 26,245
- 19.** 40% of $[100 - 20\% \text{ of } 300]$ is equal to
(a) 20 (b) 16 (c) 140 (d) 64
- 20.** Radhika bought a car for Rs 2,50,000. Next year its price decreased by 10% and further next year it decreased by 12%. In the two years overall decrease per cent in the price of the car is
(a) 3.2% (b) 22% (c) 20.8% (d) 8%

In questions 21 to 45 fill in the blanks to make the statements true.

21. _____ is a reduction on the marked price of the article.
22. Increase of a number from 150 to 162 is equal to increase of _____ per cent.
23. 15% increase in price of an article, which is Rs 1,620, is the increase of Rs _____.
24. Discount = _____ - _____.
25. Discount = Discount % of _____.
26. _____ is charged on the sale of an item by the government and is added to the bill amount.
27. Amount when interest is compounded annually is given by the formula _____.
28. Sales tax = tax % of _____.
29. The time period after which the interest is added each time to form a new principal is called the _____.
30. _____ expenses are the additional expenses incurred by a buyer for an item over and above its cost of purchase.
31. The discount on an item for sale is calculated on the _____.
32. When principal P is compounded semi-annually at $r\%$ per annum for t years, then
Amount = _____.
33. Percentages are _____ to fractions with _____ equal to 100.
34. The marked price of an article when it is sold for Rs 880 after a discount of 12% is _____.
35. The compound interest on Rs 8,000 for one year at 16% p.a. compounded half yearly is _____, given that $(1.08)^2 = 1.1664$.
36. In the first year on an investment of Rs 6,00,000 the loss is 5% and in the second year the gain is 10%, the net result is _____.

- The amount of money that is earning interest or that you are borrowing is called the **principal**.
- The amount due is equal to the principal plus the accrued interest.

Simple interest $I = prt$

37. If amount on the principal of Rs 6,000 is written as $6000 \left(1 + \frac{5}{100}\right)^3$ and compound interest payable half yearly, then rate of interest p.a. is _____ and time in years is _____.
38. By selling an article for Rs 1,12,000 a girl gains 40%. The cost price of the article was _____.
39. The loss per cent on selling 140 geometry boxes at the loss of S.P. of 10 geometry boxes is equal to _____.
40. The cost price of 10 tables is equal to the sale price of 5 tables. The profit per cent in this transaction is _____.
41. Abida bought 100 pens at the rate of Rs 3.50 per pen and pays a sales tax of 4%. The total amount paid by Abida is _____.
42. The cost of a tape-recorder is Rs 10,800 inclusive of sales tax charged at 8%. The price of the tape-recorder before sales tax was charged is _____.
43. 2500 is greater than 500 by _____%.
44. Four times a number is a _____% increase in the number.
45. 5% sales tax is charged on an article marked Rs 200 after allowing a discount of 5%, then the amount payable is _____.

In questions 46 to 65 state whether the statements are true (T) or false (F).

46. To calculate the growth of a bacteria if the rate of growth is known, the formula for calculation of amount in compound interest can be used.
47. Additional expenses made after buying an article are included in the cost price and are known as Value Added Tax.
48. Discount is a reduction given on cost price of an article.
49. Compound interest is the interest calculated on the previous year's amount.
50. $C.P. = M.P. - \text{Discount}$.
51. A man purchased a bicycle for Rs 1,040 and sold it for Rs 800. His gain per cent is 30%.

- 52.** Three times a number is 200% increase in the number, then one-third of the same number is 200% decrease in the number.
- 53.** Simple interest on a given amount is always less than or equal to the compound interest on the same amount for the same time period and at the same rate of interest per annum.
- 54.** The cost of a sewing machine is Rs 7,000. Its value depreciates at 8% p.a. Then the value of the machine after 2 years is Rs 5,924.80.
- 55.** If the discount of Rs y is available on the marked price of Rs x , then the discount percent is $\frac{x}{y} \times 100\%$.
- 56.** Number of students appearing for class X CBSE examination increases from 91,422 in 1999–2000 to 11,6054 in 2008–09. Increase in the number of students appeared is approximately 27%.
- 57.** Selling price of 9 articles is equal to the cost price of 15 articles. In this transaction there is profit of $66\frac{2}{3}\%$.
- 58.** The compound interest on a sum of Rs P for T years at $R\%$ per annum compounded annually is given by the formula $P \left(1 + \frac{R}{100}\right)^T$.
- 59.** In case of gain, $S.P. = \frac{(100 + \text{gain}\%) \times C.P.}{100}$.
- 60.** In case of loss, $C.P. = \frac{100 \times S.P.}{100 + \text{Loss}\%}$
- 61.** The value of a car, bought for Rs 4,40,000 depreciates each year by 10% of its value at the beginning of that year. So its value becomes Rs 3,08,000 after three years.

Problem Solving Tip

Another way to find the total cost of an item is to add the sales tax per cent to 100%. Then multiply this new per cent by the regular price.

For example, if the regular price is Rs 35.98 and the sales tax is 6%, add 6% to 100%. Then find 106% of Rs 35.98.

$$1.06 \times 35.98 = 38.14.$$

The total cost is Rs 38.14.

- 62.** The cost of a book marked at Rs 190 after paying a sales tax of 2% is Rs 192.
- 63.** The buying price of 5 kg of flour with the rate Rs 20 per kg, when 5% ST is added on the purchase is Rs 21.
- 64.** The original price of a shampoo bottle bought for Rs 324 if 8% VAT is included in the price is Rs 300.
- 65.** Sales tax is always calculated on the cost price of an item and is added to the value of the bill.

Solve the following :

- 66.** In a factory, women are 35% of all the workers, the rest of the workers being men. The number of men exceeds that of women by 252. Find the total number of workers in the factory.
- 67.** Three bags contain 64.2 kg of sugar. The second bag contains $\frac{4}{5}$ of the contents of the first and the third contains $45\frac{1}{2}\%$ of what there is in the second bag. How much sugar is there in each bag?
- 68.** Find the S.P. if
- (a) M.P. = Rs 5450 and discount = 5%
- (b) M.P. = Rs 1300 and discount = 1.5%
- 69.** Find the M.P. if
- (a) S.P. = Rs 495 and discount = 1%
- (b) S.P. = Rs 9,250 and discount = $7\frac{1}{2}\%$
- 70.** Find discount in per cent when
- (a) M.P. = Rs 625 and S.P. = Rs 562.50
- (b) M.P. = Rs 900 and S.P. = Rs 873
- 71.** The marked price of an article is Rs 500. The shopkeeper gives a discount of 5% and still makes a profit of 25%. Find the cost price of the article.
- 72.** In 2007 – 08, the number of students appeared for Class X examination was 1,05,332 and in 2008–09, the number was 1,16,054. If 88,151 students pass the examination in 2007–08 and 103804 students in 2008–09. What is the increase or decrease in pass % in Class X result?

- 73.** A watch worth Rs 5400 is offered for sale at Rs 4,500. What per cent discount is offered during the sale?
- 74.** In the year 2001, the number of malaria patients admitted in the hospitals of a state was 4,375. Every year this number decreases by 8%. Find the number of patients in 2003.
- 75.** Jyotsana bought a product for Rs 3,155 including 4.5% sales tax. Find the price before tax was added.
- 76.** An average urban Indian uses about 150 litres of water every day.

Activity	Litres per person per day
Drinking	3
Cooking	4
Bathing	20
Sanitation	40
Washing clothes	40
Washing utensils	20
Gardening	23
Total	150

- (a) What per cent of water is used for bathing and sanitation together per day?
- (b) How much less per cent of water is used for cooking in comparison to that used for bathing?
- (c) What per cent of water is used for drinking, cooking and gardening together?

Problem Solving Tip

Another method to find the sale price of an item is to subtract the per cent of discount from 100. Then multiply this number to the regular price.

- 77.** In 1975, the consumption of water for human use was about 3850 cu.km/year. It increased to about 6000 cu.km/year in the year 2000. Find the per cent increase in the consumption of water from 1975 to 2000. Also, find the annual per cent increase in consumption (assuming water consumption increases uniformly).
- 78.** Harshna gave her car for service at service station on 27-05-2009 and was charged as follows:
- (a) 3.10 litres engine oil @ Rs 178.75 per litre and VAT @ 20%.
 - (b) Rs 1,105.12 for all other services and VAT @ 12.5%.
 - (c) Rs 2,095.80 as labour charges and service tax @10%.
 - (d) 3% cess on service Tax.
- Find the bill amount.
- 79.** Given the principal = Rs 40,000, rate of interest = 8% p.a. compounded annually. Find
- (a) Interest if period is one year.
 - (b) Principal for 2nd year.
 - (c) Interest for 2nd year.
 - (d) Amount if period is 2 years.
- 80.** In Delhi University, in the year 2009 – 10, 49,000 seats were available for admission to various courses at graduation level. Out of these 28,200 seats were for the students of General Category while 7,400 seats were reserved for SC and 3,700 seats for ST. Find the per centage of seats available for
- (i) Students of General Category.
 - (ii) Students of SC Category and ST Category taken together.
- 81.** Prachi bought medicines from a medical store as prescribed by her doctor for Rs 36.40 including 4% VAT. Find the price before VAT was added.
- 82.** Kritika ordered one pizza and one garlic bread from a pizza store and paid Rs 387 inclusive of taxes of Rs 43. Find the tax%.

83. Arunima bought household items whose marked price and discount % is as follows:

Item	Quantity	Rate	Amount	Discount%
(a) Atta	1 packet	200	200	16%
(b) Detergent	1 packet	371	371	22.10%
(c) Namkeen	1 packet	153	153	18.30%

Find the total amount of the bill she has to pay.

84. Devangi's phone subscription charges for the period 17-02-09 to 16-03-09 were as follows :

Period	Amount (in Rs)	Service Tax %
17-02-09 to 23-02-09	199.75	12
24-02-09 to 16-03-09	599.25	10

Find the final bill amount if 3% education cess was also charged on service tax.

85. If principal = Rs 1,00,000. rate of interest = 10% compounded half yearly. Find
- Interest for 6 months.
 - Amount after 6 months.
 - Interest for next 6 months.
 - Amount after one year.
86. Babita bought 160 kg of mangoes at Rs 48 per kg. She sold 70% of the mangoes at Rs 70 per kg and the remaining mangoes at Rs 40 per kg. Find Babita's gain or loss per cent on the whole dealing.
87. A shopkeeper was selling all his items at 25% discount. During the off season, he offered 30% discount over and above the existing discount. If Pragya bought a skirt which was marked for Rs 1,200, how much did she pay for it?

- People employed in India must pay an income tax based on their income. The net pay, or take-home pay, is the amount of money that a person is paid.
- Homeowners pay property taxes based on the value of their house and property.

88. Ayesha announced a festival discount of 25% on all the items in her mobile phone shop. Ramandeep bought a mobile phone for himself. He got a discount of Rs 1,960. What was the marked price of the mobile phone?
89. Find the difference between Compound Interest and Simple Interest on Rs 45,000 at 12% per annum for 5 years.
90. A new computer costs Rs 1,00,000. The depreciation of computers is very high as new models with better technological advantages are coming into the market. The depreciation is as high as 50% every year. How much will the cost of computer be after two years?
91. The population of a town was decreasing every year due to migration, poverty and unemployment. The present population of the town is 6,31,680. Last year the migration was 4% and the year before last, it was 6%. What was the population two years ago?
92. Lemons were bought at Rs 48 per dozen and sold at the rate of Rs 40 per 10. Find the gain or loss per cent.
93. If the price of petrol, diesel and LPG is slashed as follows:

Fuel	Old prices/ litre (in Rs)	New price/ litre (in Rs)	% Decrease
Petrol / L	45.62	40.62	_____
Diesel / L	32.86	30.86	_____
LPG/14.2kg	304.70	279.70	_____

Complete the above table.

94. What is the percentage increase or decrease in the number of seats won by A, B, C and D in the general elections of 2009 as compared to the results of 2004?

Political party	Number of seats won in 2004	Number of seats won in 2009
A	206	145
B	116	138
C	4	24
D	11	12

95. How much more per cent seats were won by X as compared to Y in Assembly Election in the state based on the data given below.

Party	Won (out of 294)
X	158
Y	105
Z	18
W	13

96. Ashima sold two coolers for Rs 3,990 each. On selling one cooler she gained 5% and on selling the the other she suffered a loss of 5%. Find her overall gain or loss % in whole transaction.
97. A lady buys some pencils for Rs 3 and an equal number for Rs 6. She sells them for Rs 7. Find her gain or loss%.
98. On selling a chair for Rs 736, a shopkeeper suffers a loss of 8%. At what price should he sell it so as to gain 8%?
99. A dining table is purchased for Rs 3,200 and sold at a gain of 6%. If a customer pays sales tax at the rate of 5%. How much does the customer pay in all for the table?
100. Achal bought a second-hand car for Rs 2,25,000 and spend Rs 25,000 for repairing. If he sold it for Rs 3,25,000, what is his profit per cent?
101. A lady bought an air-conditioner for Rs 15,200 and spent Rs 300 and Rs 500 on its transportation and repair respectively. At what price should she sell it to make a gain of 15%?
102. What price should a shopkeeper mark on an article that costs him Rs 600 to gain 20%, after allowing a discount of 10%

Problem Solving Tip

When an item is on sale, you can save money buying it at less than regular price. The discount is the amount that the regular price is reduced. The sale price is the regular price minus the discount.

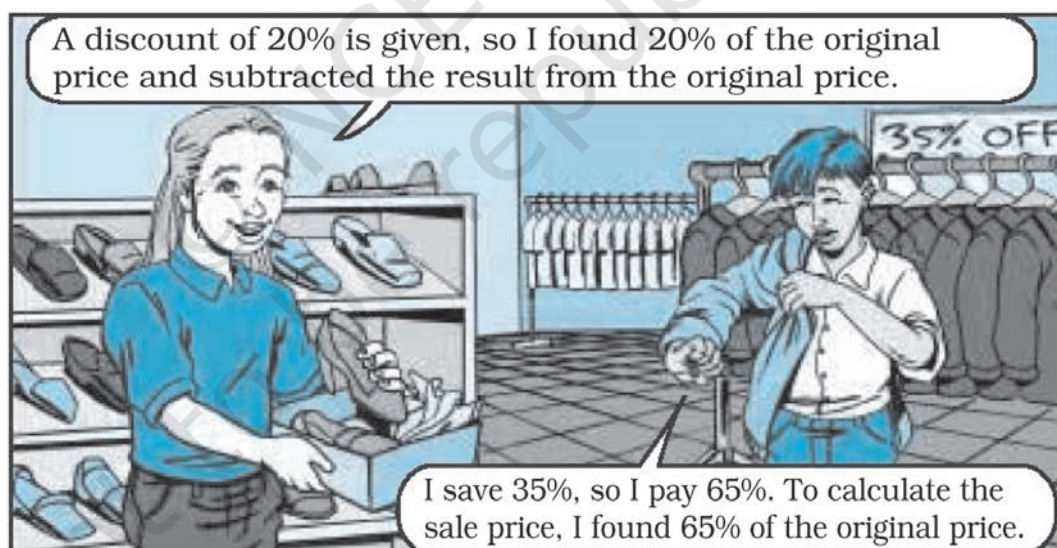
You can use a proportion or an equation to solve problems involving discount and sale price.

- 103.** Brinda purchased 18 coats at the rate of Rs 1,500 each and sold them at a profit of 6%. If customer is to pay sales tax at the rate of 4%, how much will one coat cost to the customer and what will be the total profit earned by Brinda after selling all coats?
- 104.** Rahim borrowed Rs 10,24,000 from a bank for one year. If the bank charges interest of 5% per annum, compounded half-yearly, what amount will he have to pay after the given time period. Also, find the interest paid by him.
- 105.** The following items are purchased from showroom:
T-Shirt worth Rs 1200.
Jeans worth Rs 1000.
2 Skirts worth Rs 1350 each.
What will these items cost to Shikha if the sales tax is 7%?
- 106.** The food labels given below give information about 2 types of soup: cream of tomato and sweet corn. Use these labels to answer the given questions. (All the servings are based on a 2000 calorie diet.)

Sweet Corn		Cream of Tomato	
Nutrition Facts		Nutrition Facts	
Serving Size 1 cup (240ml)		Serving Size 1 cup (240ml)	
About 2 serving per Container		About 2 serving per Container	
Amount Per Serving		Amount Per Serving	
Calories 90	Calories from Fat 9	Calories 100	Calories from Fat 20
% Daily Value*		% Daily Value	
Total Fat 2g	2%	Total Fat 2g	3%
Saturated Fat-0g	0%	Saturated Fat-1.5g	6%
Cholesterol 0mg	0%	Cholesterol 10mg	3%
Sodium 540mg	22%	Sodium 690mg	29%
Total Carbohydrate 17g	6%	Total Carbohydrate 17g	6%
Dietary Fibre 3 gram	14%	Dietary Fibre 4 gram	18%
Sugar 5g		Sugar 11g	
Protein 3g		Protein 2g	
Vitamin A 30%	Vitamin C 10%	Vitamin A 20%	Vitamin C 20%
Calcium 2%	Iron 6%	Calcium 0%	Iron 8%
*Per cent Daily Values are based on a 2,000 calorie diet.		*Per cent Daily Values are based on a 2,000 calorie diet.	

- (a) Which can be measured more accurately : the total amount of fat in cream of tomato soup or the total amount of fat in sweet corn soup? Explain.
 - (b) One serving of cream of tomato soup contains 29% of the recommended daily value of sodium for a 2000 calorie diet. What is the recommended daily value of sodium in milligrams? Express the answer upto 2 decimal places.
 - (c) Find the increase per cent of sugar consumed if cream of tomato soup is chosen over sweet corn soup.
 - (d) Calculate ratio of calories from fat in sweet corn soup to the calories from fat in cream of tomato soup.
- 107.** Music CD originally priced at Rs 120 is on sale for 25% off. What is the S.P.?

Sonia and Rahul have different ways of calculating the sale price for the items they bought.



As you work on the next problem, try both of these methods to see which you prefer.

- 108.** Store A and Store B both charge Rs 750 for a video game. This week the video game is on sale for Rs 600 at Store B and for 25% off at Store A. At which store is the game less expensive?

- 109.** At a toy shop price of all the toys is reduced to 66% of the original price.
- What is the sale price of a toy that originally costs Rs 90?
 - How much money would you save on a toy costing Rs 90?
- 110.** A store is having a 25% discount sale. Sheela has a Rs 50 gift voucher and wants to use it to buy a board game marked for Rs 320. She is not sure how to calculate the concession she will get. The sales clerk has suggested two ways to calculate the amount payable.
- **Method 1:** Subtract Rs 50 from the price and take 25% off the resulting price.
 - **Method 2:** Take 25% off the original price and then subtract Rs 50.
- Do you think both the methods will give the same result? If not, predict which method will be beneficial for her.
 - For each method, calculate the amount Sheela would have to pay. Show your work.
 - Which method do you think stores actually use? Why?
- 111. Living on your own:** Sanjay is looking for one-bedroom apartment on rent. At Neelgiri apartments, rent for the first two months is 20% off. The one bedroom rate at Neelgiri is Rs 6,000 per month. At Savana apartments, the first month is 50% off. The one bedroom rate at Savana apartments is Rs 7000 per month. Which apartment will be cheaper for the first two months? By how much?
- 112.** For an amount, explain why, a 20% increase followed by a 20% decrease is less than the original amount.

Problem Solving Tip

There are various taxes that people must pay. Some examples are sales tax, property tax and income tax. A tax is charge, usually a percentage, that is imposed by the government.

You can set up a proportion or equation to solve problems involving taxes.

- 113.** Sunscreens block harmful ultraviolet (UV) rays produced by the sun. Each sunscreen has a Sun Protection Factor (SPF) that tells you how many minutes you can stay in the sun before you receive one

minute of burning UV rays. For example, if you apply sunscreen with SPF 15, you get 1 minute of UV rays for every 15 minutes you stay in the sun.

1. A sunscreen with SPF 15 allows only $\frac{1}{15}$ of the sun's UV rays. What per cent of UV rays does the sunscreen block?
2. Suppose a sunscreen allows 25% of the sun's UV rays.
 - a. What fraction of UV rays does this sunscreen block? Give your answer in lowest terms.
 - b. Use your answer from Part (a) to calculate this sunscreen's SPF. Explain how you found your answer.
3. A label on a sunscreen with SPF 30 claims that the sunscreen blocks about 97% of harmful UV rays. Assuming the SPF factor is accurate, is this claim true? Explain.
- 114.** A real estate agent receives Rs 50,000 as commission, which is 4% of the selling price. At what price does the agent sell the property?
- 115.** With the decrease in prices of tea by 15% Tonu, the chaiwallah, was able to buy 2 kg more of tea with the same Rs 45 that he spent each month on buying tea leaves for his chai shop. What was the reduced price of tea? What was the original price of tea?



The per cent of decrease tells what per cent the amount of decrease is of the original number.

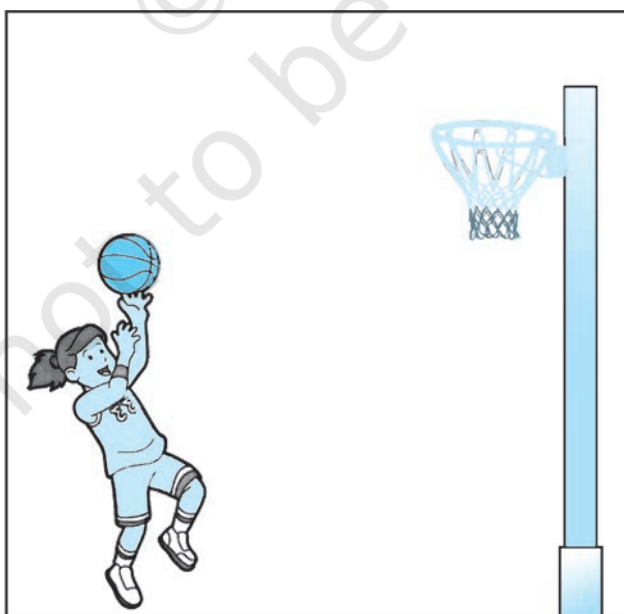
To find the per cent of decrease express a ratio of the amount of decrease to the original number as per cent.

$$\text{Per cent of Decrease} = \frac{\text{amount of decrease}}{\text{original number}} \times 100$$

- 116.** Below is the Report Card of Vidit Atrey. Vidit's teacher left the last column blank. Vidit is not able to make out, in which subject he performed better and in which he needs improvement. Complete the table to help Vidit know his comparative performance.

Assessment Report for 2009-2010				
Class : 9B	Name : Vidit Atrey		Date : 31 March 2010	
Subject	Internal assessment	Examination	Total	Final%
1. English Literature	20/25	82/100	102/125	
2. English Language	22/25	91/100	113/125	
3. Hindi Literature	18/25	67/75	85/100	
4. Hindi Language	16/25	68/75	84/100	
5. Mathematics	42/50	88/100	130/150	
6. Sanskrit	14/20	75/100	99/120	
7. Physics	45/50	90/100	135/150	
8. Chemistry	41/50	82/100	123/150	
9. Biology	43/50	87/100	130/150	
10. History and Civics	19/25	68/75	87/100	
11. Geography	17/20	71.5/80	88.5/100	

- 117.** Sita is practicing basket ball. She has managed to score 32 baskets in 35 attempts. What is her success rate in per centage?



118. During school hours, Neha finished 73% of her homework and Minakshi completed $\frac{5}{8}$ of her homework. Who must finish a greater per cent of homework?
119. Rain forests are home to 90,000 of the 2,50,000 identified plant species in the world. What per cent of the world's identified plant species are found in rain forests?
120. Madhu's room measures $6\text{m} \times 3\text{m}$. Her carpet covers 8m^2 . What per cent of floor is covered by the carpet?
121. The human body is made up mostly of water. In fact, about 67% of a person's total body weight is water. If Jyoti weights 56 kg, how much of her weight is water?
122. The per cent of pure gold in 14 carat gold is about 58.3%. A 14 carat gold ring weighs 7.6 grams. How many grams of pure gold are in the ring?
123. A student used the proportion $\frac{n}{100} = \frac{5}{32}$ to find 5% of 32. What did the student do wrong?
124. The table shows the cost of sunscreen of two brands with and without sales tax. Which brand has a greater sales tax rate? Give the sales tax rate of each brand.

	Cost (in Rs)	Cost+Tax (in Rs)
1. X (100 gm)	70	75
2. Y (100 gm)	62	65

(D) Applications, Gamess and Puzzles

1. Complete the grid using the clues given below.

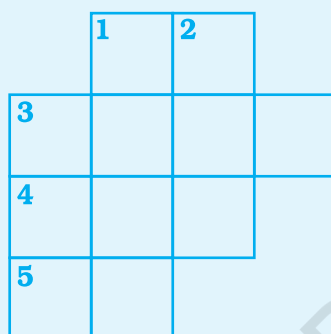
Clues

Down

- (1) The total cost price of a TV set whose cost price is Rs 6,900 and money spent on repairs is Rs 300.
- (2) The price of a cycle is Rs 800. It is now increased by 20%. The new sale price is Rs _____.
- (3) The list price of a bag is Rs 220. A discount of 15% is offered to make the sales price as Rs _____.

Across

- (1) Discount on an item marked at Rs 800 and sold for Rs 721.
- (3) The selling price of a fan costing Rs 1,200 if a profit of Rs 5% is to be made.
- (4) The cost price of an item sold at Rs 1,600 at a 100% profit.
- (5) The profit per cent of an item marked at Rs 800 and sold at Rs. 1,360.

**2.**

1. Children go to school for their overall personality development. Co-curricular activities (CCA) play a major role in this. Fill the following table to know, how the students of your class did in the I and II term.

S. No.	Names	Number of CCA activities (I term)	Number of CCA activities (II term)	Difference	Increase/ Decrease Per cent

Now, answer the following questions.

- (a) Who is the most active participant in CCA activities in I term? In II term?
- (b) What is the percentage of your participation in I term? In II term?

- 3.** Discuss with your parents and create a checkbook to keep a record of monthly expenses of your family as shown below (You may change the entries as per your requirements).

Sr. No.	Date	Description of Transaction	Payment Debt. (-)	Payment Credit (+)	Balance
1.		Payment Deposit			
2.		Rent			
3.		School expenses			
4.		Travelling expenses			
5.		Groceries			
6.		Gas and Electricity			
7.		Spending money on outings			
8.		Water			
9.		Phone			
10.		Credit card payment			
11.		Others			

Now, answer the following questions.

- What per cent of the total income is spend on school expenses?
- Are the expenditure on gas and electricity more than the expenditure on travelling expense?
- What is your family's savings for a month?
- What per cent of family income is saved?
- How can you increase your family's savings?

4.

Observe your daily schedule and note down the following:

Activities	Hours per day
Sleeping	
At School	
Socialising	
Watching TV	
Doing Homework	
Phone chatting	
Net chatting	
Eating	
Bathing / Growing	
Other activities	

- Find the ratio of number of hours spent at school to number of hours spent while sleeping.
- Find percentage of hours spent in
 - studying at home
 - watching TV.
- Find the ratio of total time spent on chatting to studying.

5 School has arranged for an excursion. Students are thrilled and plan to decide their menu. You being the class representative have the responsibility to survey different shops to make the least expenditure. The following table may help you.

Item	Name of Item	Shop	Marked Price	Discount (if any)	Cost Price
1.	Juice	A			
		B			
2.	Cookies	A			
		B			
3.					
4.					
5.					

Have a discussion for the above table to make an economical purchase. Enjoy your excursion.

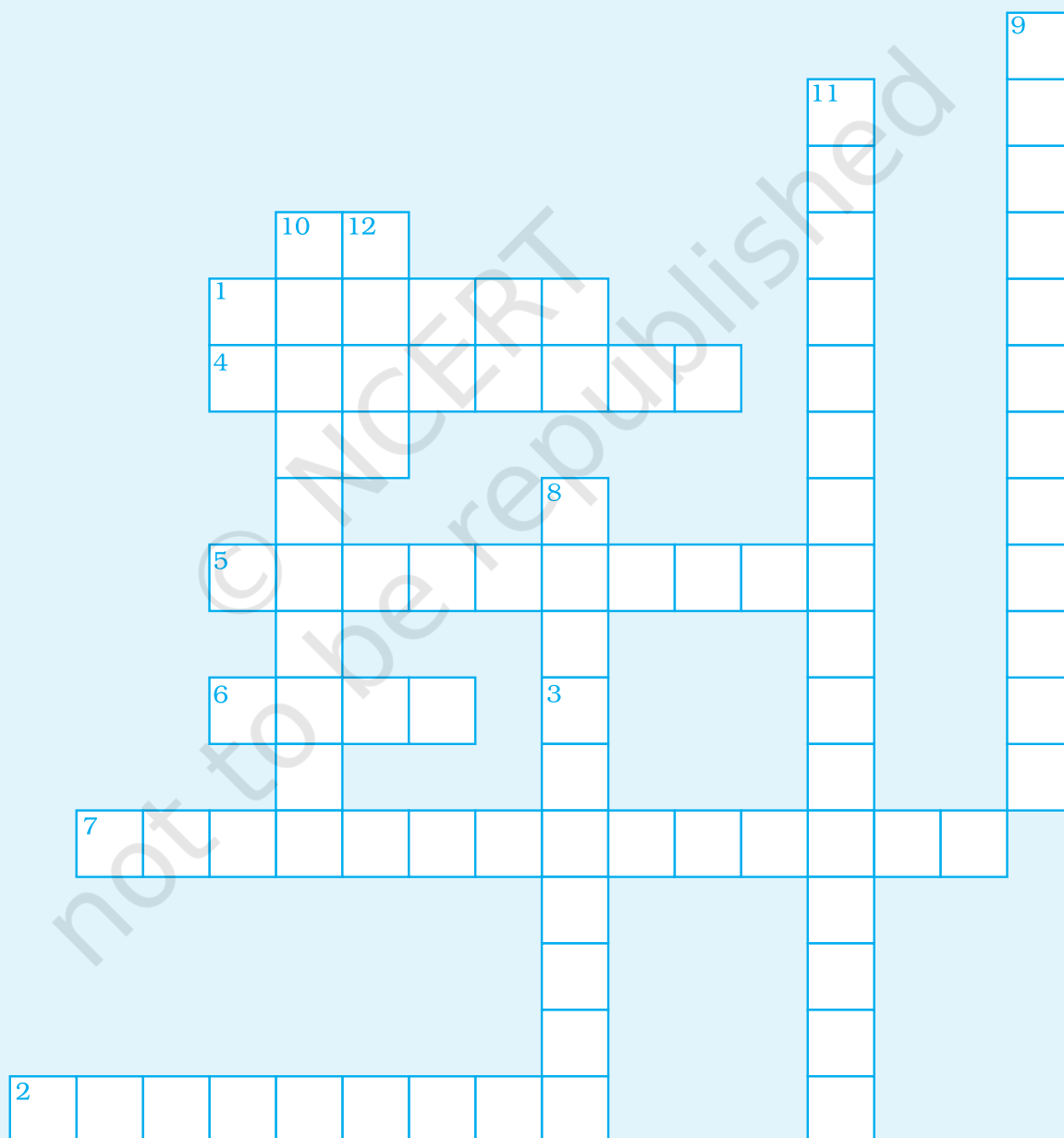
6. Crossword

Across

- Shopkeeper earns it if $S.P. > C.P.$
- The price at which the article is purchased.
- List price of an article.
- It is a reduction given on M.P.
- Duration for the sum is borrowed.
- This becomes half if the interest is compounded half yearly.
- Interest computed on the original principal.

Down

8. This comes from Latin word per centum.
9. The price at which article is sold.
10. The original sum deposited or borrowed.
11. The interest is calculated on the amount of the previous year..
12. Shopkeeper bears it if $S.P. < C.P.$



UNIT 10

DIRECT AND INVERSE PROPORTIONS

(A) Main Concepts and Results

- Two quantities x and y are said to be in direct proportion if they increase or decrease together in such a manner that the ratio of their corresponding values remains constant. That is, $\frac{x}{y} = k$ where k is a positive number, if x and y are in direct proportion or vary directly. In case of direct proportion, if y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively, then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$.
- Two quantities x and y are said to be in inverse proportion if an increase in x causes a proportional decrease in y and vice-versa, in such a manner that the product of their corresponding values remains constant. That is, $xy = k$ where k is a positive number, if x and y are in inverse proportion. In this case, if y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively, then $x_1 y_1 = x_2 y_2$ or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.
- Quantities increasing or decreasing together need not always be in direct proportion, same in the case of inverse proportion.
- When two quantities x and y are in direct proportion (or vary directly), they are written as $x \propto y$. Symbol “ $\propto$ ” stands for ‘is proportional to’.

- When two quantities x and y are in inverse proportion (or vary inversely) they are written as $x \propto \frac{1}{y}$.

(B) Solved Examples

In examples 1 to 3, there are four options out of which one is correct. Choose the correct answer,

Example 1 : If x and y are directly proportional and when $x = 13$, $y = 39$, which of the following is not a possible pair of corresponding values of x and y ?

- (a) 1 and 3 (b) 17 and 51
(c) 30 and 10 (d) 6 and 18

Solution : The correct answer is (c).

Example 2 : A car covers a distance in 40 minutes with an average speed of 60 km per hour. The average speed to cover the same distance in 30 minutes is

- (a) 80 km/h (b) $\frac{45}{2}$ km/h
(c) 70 km/h (d) 45 km/h

Solution : The correct answer is (a).

Example 3 : Which of the following is in direct proportion?

- One side of a cuboid and its volume.
- Speed of a vehicle and the distance travelled in a fixed time interval.
- Change in weight and height among individuals.
- Number of pipes to fill a tank and the time required to fill the same tank.

Solution : The correct answer is (b).

[Because, in a fixed time interval, as the speed of a vehicle increases, the distance travelled by it also increases in the same ratio.]

In examples 4 to 6, fill in the blanks to make the statements true.

Example 4 : Amrita takes 18 hours to travel 720 kilometres. Time taken by her to travel 360 kilometres is _____.

Solution : 9 hours.

Example 5 : If x and y are inversely proportional then _____ = k where k is positive constant.

Solution : xy .

Example 6 : Side of a rhombus and its perimeter are in _____ proportion.

Solution : Direct.

In examples 7 to 9, state whether the statements are true (T) or false (F):

Example 7 : When two quantities x and y are in inverse proportion, then $\frac{x}{y}$ is a constant.

Solution : False.

Example 8 : If the cost of 10 pencils is Rs 90, then the cost of 19 pencils is Rs 171.

Solution : True.

Example 9 : If 5 persons can finish a job in 10 days then one person will finish it in 2 days.

Solution : False.

Example 10 : In a scout camp, there is food provision for 300 cadets for 42 days. If 50 more persons join the camp, for how many days will the provision last?

Solution : More the persons, the sooner would be the provision exhausted. So, this is a case of inverse proportion.

Let the required number of days be x .

$$\text{Hence, } 300 \times 42 = (300 + 50) \times x$$

$$300 \times 42 = 350 \times x$$

$$\frac{300 \times 42}{350} = x$$

$$x = 36$$

Example 11 : If two cardboard boxes occupy 500 cubic centimetres space, then how much space is required to keep 200 such boxes?

Solution : As the number of boxes increases, the space required to keep them also increases.

So, this is a case of direct proportion.

Number of boxes	2	200
Space occupied (in cubic centimetres)	500	x

$$\text{So } \therefore \frac{2}{500} = \frac{200}{x}$$

$$2x = 500 \times 200$$

$$x = \frac{500 \times 200}{2}$$

$$x = 50,000$$

Thus, the required space is 50,000 cubic centimetres.

Example 12 : Under the condition that the temperature remains constant, the volume of gas is inversely proportional to its pressure. If the volume of gas is 630 cubic centimetres at a pressure of 360 mm of mercury, then what will be the pressure of the gas if its volume is 720 cubic centimetres at the same temperature?

Solution : Given that, at constant temperature pressure and volume of a gas are inversely proportional. Let the required pressure be x .

Volume of gas (in cubic centimetres)	630	720
Pressure of gas (in mm)	360	x

$$\text{Then, } 630 \times 360 = 720 \times x$$

$$\frac{630 \times 360}{720} = x$$

$$x = 315$$

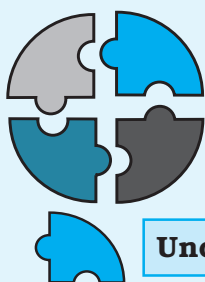
Therefore, the required pressure is 315 mm of mercury.

The time t required to download a 4-megabyte music file from an internet music seller is inversely proportional to the rate r at which data is transferred to the receiving computer.



- How long will it take to download a 4-megabyte file if the transmission occurs at a rate of 2.5 megabytes per minute? How long if the transmission rate is 0.8 megabytes per minute?
- How can the relationship of t and r be expressed in symbolic form?
- How does the value of t change as the value of r increases steadily? How is this pattern of change related to the constant of proportionality?

Application on Problem Solving Strategy



Example 13:

Lemons were bought at Rs 60 a dozen and sold at the rate of Rs 40 per 10. Find the gain or loss per cent.

Understand and Explore the problem

- Rewrite given equation in your own words.
30 men can reap a field in 17 days. If the field is to be reaped in 10 days then how many men will be required? How many extra men are to be employed?
- What do you know?
30 men can reap a field in 17 days.

Plan a Strategy

- Think that 30 men are reaping the field in 17 days, so to reap the field in 10 days, i.e. in less number of days, men required will be more or less?
- No. of days has reduced so men to be employed will increase. Therefore we will use indirect variation.
- Find difference between no. of men required and the number 30.

Solve

- Let number of men required to finish the job in 10 days be x

No. of days	17 decrease 10
No. of men	30 increase x

- No. of days has decreased so number of men will increase. Hence we will use inverse variation.

$$\text{Hence } 30 \times 17 = x \times 10$$

$$x = \frac{30 \times 17}{10} = 51$$

51 men will be required.

$$\text{Extra number of men required} = 51 - 30 = 21 \text{ men.}$$



Revise

- Verify your answer by adopting some other method e.g. Here instead of using variation we can use unitary method.

e.g.	No. of days	No. of men
	17	30
	1	?
	10	x

No. of men required to complete the job in 17 days = 30

No. of men required to complete the job in 1 day = 30×17

No. of men required to complete the job in 10 days =

$$\frac{30 \times 17}{10} = 51$$

No. of extra men required = $51 - 30 = 21$ men.

Hence verified.

Think and Discuss

- What will happen if question is:

If 30 men can reap a field in 17 days, then 10 men reap the field in how many days?

- In the questions of men and work we always use indirect variation. Now think of some situation related to men where direct variation will be used, e.g. If maximum 15 men can travel by three cars, then find minimum maximum number of cars required for (a) 25 men (b) 38 men.

(C) Exercises

In questions 1 to 16, there are four options out of which one is correct.

Write the correct answer.

- Both u and v vary directly with each other. When u is 10, v is 15, which of the following is not a possible pair of corresponding values of u and v ?
(a) 2 and 3 (b) 8 and 12 (c) 15 and 20 (d) 25 and 37.5
- Both x and y vary inversely with each other. When x is 10, y is 6, which of the following is not a possible pair of corresponding values of x and y ?
(a) 12 and 5 (b) 15 and 4 (c) 25 and 2.4 (d) 45 and 1.3
- Assuming land to be uniformly fertile, the area of land and the yield on it vary
(a) directly with each other.
(b) inversely with each other.
(c) neither directly nor inversely with each other.
(d) sometimes directly and sometimes inversely with each other.
- The number of teeth and the age of a person vary
(a) directly with each other.
(b) inversely with each other.
(c) neither directly nor inversely with each other.
(d) sometimes directly and sometimes inversely with each other.

Direct Variation: If the relationship of variables y and x can be expressed in the form:

$$y = kx \text{ for some constant } k,$$

then we say that y varies directly with x or that y is directly proportional to x . The number k is called the constant of proportionality for the relationship.

The close connection between multiplication and division of numbers

implies that if y is directly proportional to x , then $\frac{y}{x} = k$. The symbolic

form $\frac{y}{x} = k$ shows that the ratio of y to x is constant, for any corresponding values of y and x .

5. A truck needs 54 litres of diesel for covering a distance of 297 km. The diesel required by the truck to cover a distance of 550 km is
 (a) 100 litres (b) 50 litres (c) 25.16 litres (d) 25 litres
6. By travelling at a speed of 48 kilometres per hour, a car can finish a certain journey in 10 hours. To cover the same distance in 8 hours, the speed of the car should be
 (a) 60 km/h (b) 80 km/h (c) 30 km/h (d) 40 km/h
7. In which of the following case, do the quantities vary directly with each other?

(a)

x	0.5	2	8	32
y	2	8	32	128

(b)

p	1^2	2^2	3^2	4^2
q	1^3	2^3	3^3	4^3

(c)

r	2	5	10	25	50
s	25	10	5	2	0.5

(d)

u	2	4	6	9	12
v	18	9	6	4	3

8. Which quantities in the previous question vary inversely with each other?
 (a) x and y (b) p and q (c) r and s (d) u and v
9. Which of the following vary inversely with each other?
 (a) speed and distance covered.
 (b) distance covered and taxi fare.
 (c) distance travelled and time taken.
 (d) speed and time taken.
10. Both x and y are in direct proportion, then $\frac{1}{x}$ and $\frac{1}{y}$ are
 (a) in indirect proportion.
 (b) in inverse proportion.
 (c) neither in direct nor in inverse proportion.
 (d) sometimes in direct and sometimes in inverse proportion.

11. Meenakshee cycles to her school at an average speed of 12 km/h and takes 20 minutes to reach her school. If she wants to reach her school in 12 minutes, her average speed should be
- (a) $\frac{20}{3}$ km/h (b) 16 km/h
(c) 20 km/h (d) 15 km/h
12. 100 persons had food provision for 24 days. If 20 persons left the place, the provision will last for
- (a) 30 days (b) $\frac{96}{5}$ days
(c) 120 days (d) 40 days
13. If two quantities x and y vary directly with each other, then
- (a) $\frac{x}{y}$ remains constant. (b) $x - y$ remains constant.
(c) $x + y$ remains constant. (d) $x \times y$ remains constant.
14. If two quantities p and q vary inversely with each other, then
- (a) $\frac{p}{q}$ remains constant. (b) $p + q$ remains constant.
(c) $p \times q$ remains constant. (d) $p - q$ remains constant.
15. If the distance travelled by a rickshaw in one hour is 10 km, then the distance travelled by the same rickshaw with the same speed in one minute is
- (a) $\frac{250}{9}$ m (b) $\frac{500}{9}$ m (c) 1000 m (d) $\frac{500}{3}$ m

Inverse Variation: If the relationship of variables y and x can be expressed in the form.

$$y = \frac{k}{x} \text{ for some constant } k,$$

then we say that y varies directly with x or that y is inversely proportional to x . The number k is called the constant of proportionality for the relationship.

Once again, the close connection between multiplication and division of numbers implies that if y is inversely proportional to x , then $xy = k$. The symbolic form $xy = k$ shows that the product of y and x is constant, for any corresponding values of x and y .

- 16.** Both x and y vary directly with each other and when x is 10, y is 14, which of the following is not a possible pair of corresponding values of x and y ?
- (a) 25 and 35 (b) 35 and 25 (c) 35 and 49 (d) 15 and 21

In questions 17 to 42, fill in the blanks to make the statements true:

- 17.** If $x = 5y$, then x and y vary _____ with each other.
- 18.** If $xy = 10$, then x and y vary _____ with each other.
- 19.** When two quantities x and y are in _____ proportion or vary _____ they are written as $x \propto y$.
- 20.** When two quantities x and y are in _____ proportion or vary _____ they are written as $x \propto \frac{1}{y}$.
- 21.** Both x and y are said to vary _____ with each other if for some positive number k , $xy = k$.
- 22.** x and y are said to vary directly with each other if for some positive number k , _____ = k .
- 23.** Two quantities are said to vary _____ with each other if they increase (decrease) together in such a manner that the ratio of their corresponding values remains constant.
- 24.** Two quantities are said to vary _____ with each other if an increase in one causes a decrease in the other in such a manner that the product of their corresponding values remains constant.
- 25.** If 12 pumps can empty a reservoir in 20 hours, then time required by 45 such pumps to empty the same reservoir is _____ hours.



Understand the problem

- If you write a problem in your own words, you may understand it better. Before writing a problem in your own words, you may need to read it over several times – perhaps aloud, so that you can hear yourself say the words.
- Once you have written the problem in your own words, you may want to make sure you have included all of the necessary information to solve the problem.

26. If x varies inversely as y , then

x	—	60
y	2	10

27. If x varies directly as y , then

x	12	6
y	48	—

28. When the speed remains constant, the distance travelled is _____ proportional to the time.
29. On increasing a , b increases in such a manner that $\frac{a}{b}$ remains _____ and positive, then a and b are said to vary directly with each other.
30. If on increasing a , b decreases in such a manner that _____ remains _____ and positive, then a and b are said to vary inversely with each other.
31. If two quantities x and y vary directly with each other, then _____ of their corresponding values remains constant.
32. If two quantities p and q vary inversely with each other then _____ of their corresponding values remains constant.
33. The perimeter of a circle and its diameter vary _____ with each other.
34. A car is travelling 48 km in one hour. The distance travelled by the car in 12 minutes is _____.
35. An auto rickshaw takes 3 hours to cover a distance of 36 km. If its speed is increased by 4 km/h, the time taken by it to cover the same distance is _____.
36. If the thickness of a pile of 12 cardboard sheets is 45 mm, then the thickness of a pile of 240 sheets is _____ cm.
37. If x varies inversely as y and $x = 4$ when $y = 6$, then when $x = 3$ the value of y is _____.
38. In direct proportion, $\frac{a_1}{b_1}$ _____ $\frac{a_2}{b_2}$

39. In case of inverse proportion, $\frac{a_2}{a_1} = \frac{b_1}{b_2}$
40. If the area occupied by 15 postal stamps is 60 cm², then the area occupied by 120 such postal stamps will be _____.
41. If 45 persons can complete a work in 20 days, then the time taken by 75 persons will be _____ hours.
42. Devangi travels 50 m distance in 75 steps, then the distance travelled in 375 steps is _____ km.

In questions from 43 to 59, state whether the statements are true (T) or false (F).

43. Two quantities x and y are said to vary directly with each other if for some rational number k , $xy = k$.
44. When the speed is kept fixed, time and distance vary inversely with each other.
45. When the distance is kept fixed, speed and time vary directly with each other.
46. Length of a side of a square and its area vary directly with each other.
47. Length of a side of an equilateral triangle and its perimeter vary inversely with each other.



Plan a strategy

- Concept maps are visual tools for organising information. A concept map shows how key concepts are related and can help you summarise and analyse information in lessons or chapters.

Create a Concept Map

- Give your concept map a title;
- Identify the main idea of your concept map;
- List the key concepts;
- Link the concepts to show the relationships between the concepts and the main idea.

48. If d varies directly as t^2 , then we can write $dt^2 = k$, where k is some constant.
49. If a tree 24 m high casts a shadow of 15 m, then the height of a pole that casts a shadow of 6 m under similar conditions is 9.6 m.
50. If x and y are in direct proportion, then $(x - 1)$ and $(y - 1)$ are also in direct proportion.
51. If x and y are in inverse proportion, then $(x + 1)$ and $(y + 1)$ are also in inverse proportion.
52. If p and q are in inverse variation then $(p + 2)$ and $(q - 2)$ are also in inverse proportion.
53. If one angle of a triangle is kept fixed then the measure of the remaining two angles vary inversely with each other.
54. When two quantities are related in such a manner that, if one increases, the other also increases, then they always vary directly.
55. When two quantities are related in such a manner that if one increases and the other decreases, then they always vary inversely.
56. If x varies inversely as y and when $x = 6$, $y = 8$, then for $x = 8$ the value of y is 10.
57. The number of workers and the time to complete a job is a case of direct proportion.
58. For fixed time period and rate of interest, the simple interest is directly proportional to the principal.
59. The area of cultivated land and the crop harvested is a case of direct proportion.

In questions 60 to 62, which of the following vary directly and which vary inversely with each other and which are neither of the two?

60. (i) The time taken by a train to cover a fixed distance and the speed of the train.
- (ii) The distance travelled by CNG bus and the amount of CNG used.
- (iii) The number of people working and the time to complete a given work.

- (iv) Income tax and the income.
- (v) Distance travelled by an auto-rickshaw and time taken.
- 61. (i) Number of students in a hostel and consumption of food.
- (ii) Area of the walls of a room and the cost of white washing the walls.
- (iii) The number of people working and the quantity of work.
- (iv) Simple interest on a given sum and the rate of interest.
- (v) Compound interest on a given sum and the sum invested.
- 62. (i) The quantity of rice and its cost.
- (ii) The height of a tree and the number of years.
- (iii) Increase in cost and number of shirts that can be purchased if the budget remains the same.
- (iv) Area of land and its cost.
- (v) Sales Tax and the amount of the bill.

Solve the following :

- 63. If x varies inversely as y and $x = 20$ when $y = 600$, find y when $x = 400$.
- 64. The variable x varies directly as y and $x = 80$ when y is 160. What is y when x is 64?
- 65. l varies directly as m and l is equal to 5, when $m = \frac{2}{3}$. Find l when $m = \frac{16}{3}$.
- 66. If x varies inversely as y and $y = 60$ when $x = 1.5$. Find x when $y = 4.5$.
- 67. In a camp, there is enough flour for 300 persons for 42 days. How long will the flour last if 20 more persons join the camp?
- 68. A contractor undertook a contract to complete a part of a stadium in 9 months with a team of 560 persons. Later on, it was required to complete the job in 5 months. How many extra persons should he employ to complete the work?
- 69. Sobi types 108 words in 6 minutes. How many words would she type in half an hour?

- 70.** A car covers a distance in 40 minutes with an average speed of 60 km/h. What should be the average speed to cover the same distance in 25 minutes?
- 71.** It is given that l varies directly as m .
- Write an equation which relates l and m .
 - Find the constant of proportion (k), when l is 6 then m is 18.
 - Find l , when m is 33.
 - Find m when l is 8.
- 72.** If a deposit of Rs 2,000 earns an interest of Rs 500 in 3 years, how much interest would a deposit of Rs 36,000 earn in 3 years with the same rate of simple interest?
- 73.** The mass of an aluminium rod varies directly with its length. If a 16 cm long rod has a mass of 192 g, find the length of the rod whose mass is 105 g.
- 74.** Find the values of x and y if a and b are in inverse proportion:
- 12×8
 - $30 \times y$
- 75.** If Naresh walks 250 steps to cover a distance of 200 metres, find the distance travelled in 350 steps.
- 76.** A car travels a distance of 225 km in 25 litres of petrol. How many litres of petrol will be required to cover a distance of 540 kilometres by this car?
- 77.** From the following table, determine if x and y are in direct proportion or not.

(i)

x	3	6	15	20	30
y	12	24	45	60	120

(ii)

x	4	7	10	16
y	24	42	60	96

(iii)

x	1	4	9	20
y	1.5	6	13.5	30

- 78.** If a and b vary inversely to each other, then find the values of $p, q, r ; x, y, z$ and l, m, n .

(i)

a	6	8	q	25
b	18	p	39	r

(ii)

a	2	y	6	10
b	x	12.5	15	z

(iii)

a	l	9	n	6
b	5	m	25	10

- 79.** If 25 metres of cloth costs Rs 337.50, then

- What will be the cost of 40 metres of the same type of cloth?
- What will be the length of the cloth bought for Rs 810?

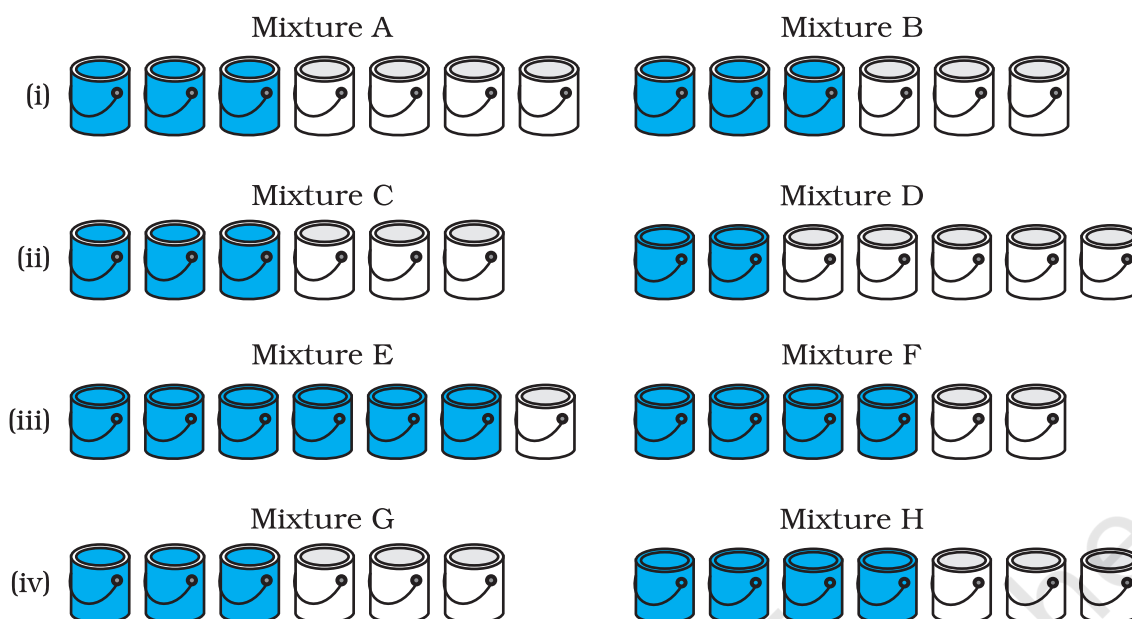
- 80.** A swimming pool can be filled in 4 hours by 8 pumps of the same type. How many such pumps are required if the pool is to be filled in $2\frac{2}{3}$ hours?

- 81.** The cost of 27 kg of iron is Rs 1,080, what will be the cost of 120 kg of iron of the same quality?

- 82.** At a particular time, the length of the shadow of Qutub Minar whose height is 72 m is 80 m. What will be the height of an electric pole, the length of whose shadow at the same time is 1000 cm?

- 83.** In a hostel of 50 girls, there are food provisions for 40 days. If 30 more girls join the hostel, how long will these provisions last?

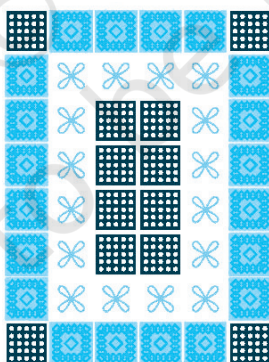
- 84.** Campus and Welfare Committee of school is planning to develop a blue shade for painting the entire school building. For this purpose various shades are tried by mixing containers of blue paint and white paint. In each of the following mixtures, decide which is a lighter shade of blue and also find the lightest blue shade among all of them.



If one container has one litre paint and the building requires 105 litres for painting, how many container of each type is required to paint the building by darkest blue shade?

85. Posing a question

Work with a partner to write at least five ratio statements about this quilt, which has white, blue, and purple squares.



How many squares of each colour will be there in 12 such quilts?

86. A packet of sweets was distributed among 10 children and each of them received 4 sweets. If it is distributed among 8 children, how many sweets will each child get?

87. 44 cows can graze a field in 9 days. How many less/more cows will graze the same field in 12 days?
88. 30 persons can reap a field in 17 days. How many more persons should be engaged to reap the same field in 10 days?
89. Shabnam takes 20 minutes to reach her school if she goes at a speed of 6 km/h. If she wants to reach school in 24 minutes, what should be her speed?
90. Ravi starts for his school at 8:20 a.m. on his bicycle. If he travels at a speed of 10km/h, then he reaches his school late by 8 minutes but on travelling at 16 km/h he reaches the school 10 minutes early. At what time does the school start?

91. Match each of the entries in Column I with the appropriate entry in Column II

Column I

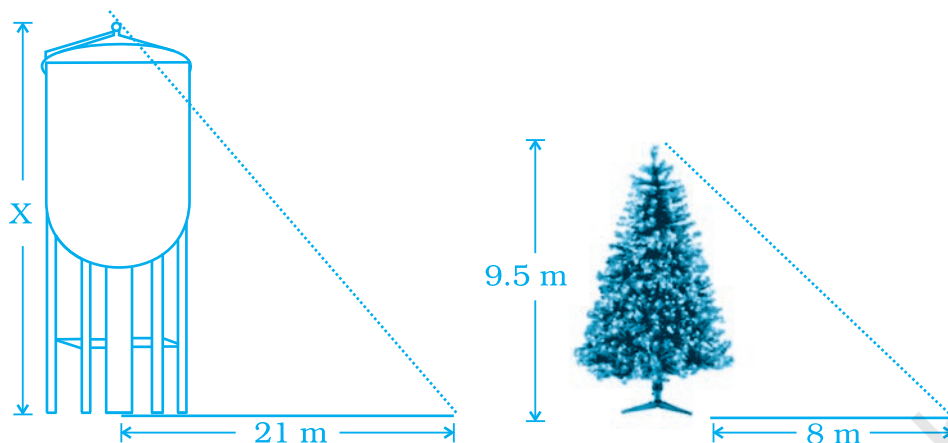
1. x and y vary inversely to each other
2. Mathematical representation of inverse variation of quantities p and q
3. Mathematical representation of direct variation of quantities m and n
4. When $x = 5$, $y = 2.5$ and when $y = 5$, $x = 10$
5. When $x = 10$, $y = 5$ and when $x = 20$, $y = 2.5$
6. x and y vary directly with each other
7. If x and y vary inversely then on decreasing x
8. If x and y vary directly then on decreasing x

Column II

- A. $\frac{x}{y} = \text{Constant}$
- B. y will increase in proportion
- C. $xy = \text{Constant}$
- D. $p \propto \frac{1}{q}$
- E. y will decrease in proportion
- F. x and y are directly proportional
- G. $m \propto n$
- H. x and y vary inversely
- I. $p \propto q$
- J. $m \propto \frac{1}{n}$

92. There are 20 grams of protein in 75 grams of sauted fish. How many grams of protein is in 225 gm of that fish?
93. Ms. Anita has to drive from Jhareda to Ganwari. She measures a distance of 3.5 cm between these villages on the map. What is the actual distance between the villages if the map scale is 1 cm = 10 km?

- 94.** A water tank casts a shadow 21 m long. A tree of height 9.5 m casts a shadow 8 m long at the same time. The lengths of the shadows are directly proportional to their heights. Find the height of the tank.

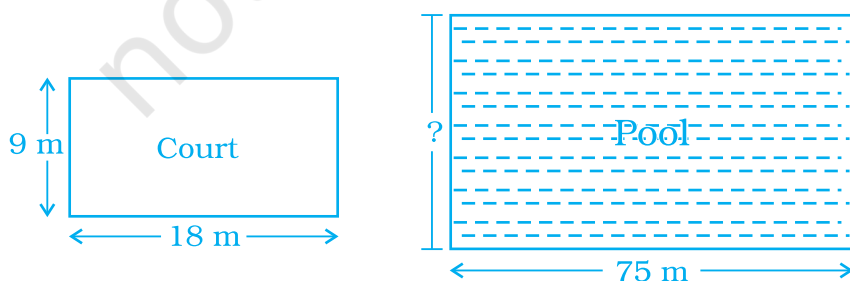


- 95.** The table shows the time four elevators take to travel various distances. Find which elevator is fastest and which is slowest.

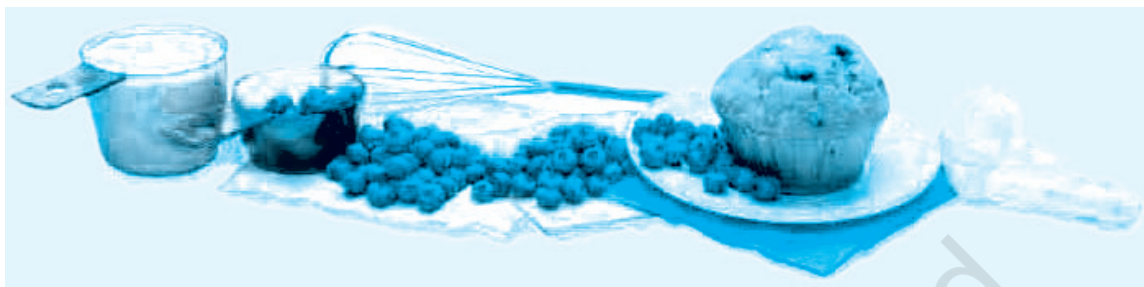
	Distance (m)	Time (sec.)
Elevator- A	435	29
Elevator- B	448	28
Elevator- C	130	10
Elevator- D	85	5

How much distance will be travelled by elevators B and C separately in 140 sec? Who travelled more and by how much?

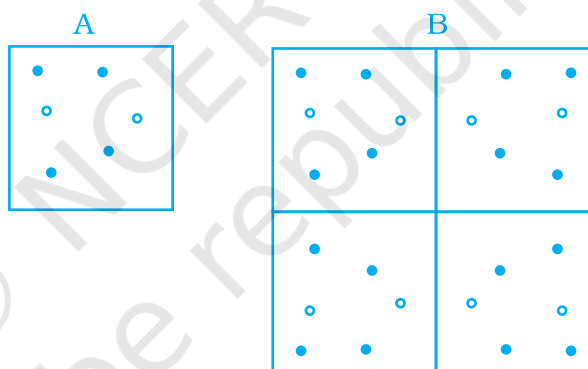
- 96.** A volleyball court is in a rectangular shape and its dimensions are directly proportional to the dimensions of the swimming pool given below. Find the width of the pool.



- 97.** A recipe for a particular type of muffins requires 1 cup of milk and 1.5 cups of chocolates. Riya has 7.5 cups of chocolates. If she is using the recipe as a guide, how many cups of milk will she need to prepare muffins?



- 98.** Pattern B consists of four tiles like pattern A. Write a proportion involving red dots and blue dots in pattern A and B. Are they in direct proportion? If yes, write the constant of proportion.



- 99.** A bowler throws a cricket ball at a speed of 120 km/h. How long does this ball take to travel a distance of 20 metres to reach the batsman?
- 100.** The variable x is inversely proportional to y . If x increases by $p\%$, then by what per cent will y decrease?
- 101.** Here is a key board of a harmonium:
- (a) Find the ratio of white keys to black keys on the keyboard.



- (b) What is the ratio of black keys to all keys on the given keyboard.
- (c) This pattern of keys is repeated on larger keyboard. How many black keys would you expect to find on a keyboard with 14 such patterns.

- 102.** The following table shows the distance travelled by one of the new eco-friendly energy-efficient cars travelled on gas.

Litres of gas	1	0.5	2	2.5	3	5
Distance (km)	15	7.5	30	37.5	45	75

Which type of properties are indicated by the table? How much distance will be covered by the car in 8 litres of gas?

- 103.** Kritika is following this recipe for bread. She realises her sister used most of sugar syrup for her breakfast. Kritika has only $\frac{1}{6}$ cup of syrup, so she decides to make a small size of bread. How much of each ingredient shall she use?

Bread recipe

1 cup quick cooking oats

2 cups bread flour

$\frac{1}{3}$ cup sugar syrup

1 tablespoon cooking oil

$1\frac{1}{3}$ cups water

3 tablespoons yeast

1 teaspoon salt.

- 104.** Many schools have a recommended students-teacher ratio as 35:1. Next year, school expects an increase in enrolment by 280 students. How many new teachers will they have to appoint to maintain the students-teacher ratio?
- 105.** Kusum always forgets how to convert miles to kilometres and back again. However she remembers that her car's speedometer shows both miles and kilometres. She knows that travelling 50 miles per hour is same as travelling 80 kilometres per hour. To cover a distance of 200 km, how many miles Kusum would have to go?

- 106.** The students of Anju's class sold posters to raise money. Anju wanted to create a ratio for finding the amount of money her class would make for different numbers of posters sold. She knew they could raise Rs 250 for every 60 posters sold.
- How much money would Anju's class make for selling 102 posters?
 - Could Anju's class raise exactly Rs 2,000? If so, how many posters would they need to sell? If not, why?

(D) Application, Games and Puzzles

1.
$$\text{Speed} = \left(\frac{\text{Distance Travelled}}{\text{Time Taken}} \right)$$

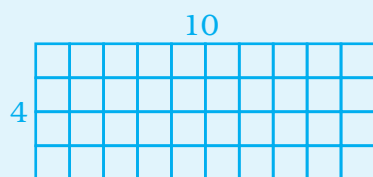
Calculate the speed for at least 10 students of your class by giving them a certain distance to walk. Measure the distance each student has walked and record the time taken by each to cover the distance. Then, complete the table given below:

Name of the student	Distance walked (in metres)	Time taken (in minutes)	Rate of speed (in m/min)
1			
2			

Which student ran the fastest?

- 2.** Figures that have the same shape but not necessarily the same size are called similar figures. We can make rectangles with similar figures by increasing or decreasing its dimensions in the same ratio. Let us make similar figures using square tiles.

What is the length of a similar rectangle where width is made up of 12 tiles? Let us consider a rectangle having 10 square tiles along the length and 4 along the breadth as shown in the figure.



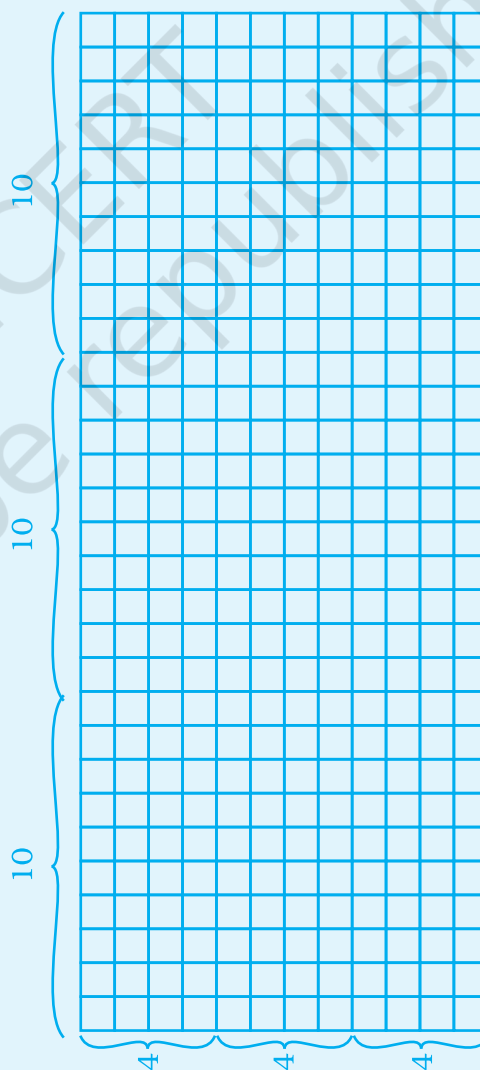
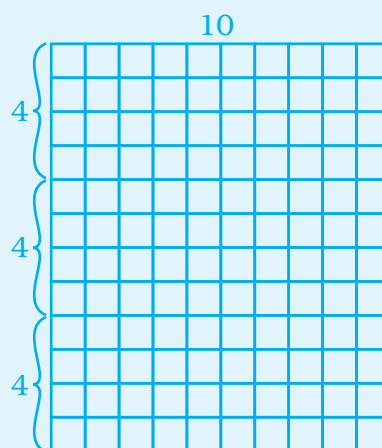
Use tiles to make a 10×4 rectangles.

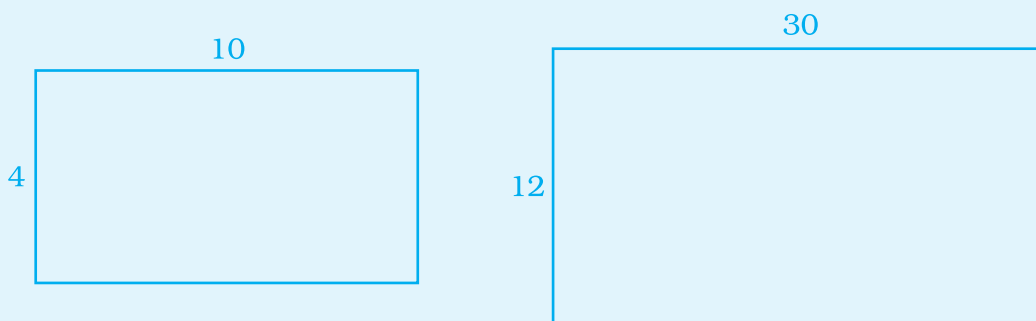
Add tiles to increase width of the rectangle to 12 tiles.

The width of the new rectangle is three times the width of the original rectangle. To keep the ratios of the lengths of two rectangles proportional, the length of this new rectangle must also be three times the length of the original rectangle.

Add tiles to increase the length of the rectangle to 30 tiles.

To check our answers, we can use the idea of direct proportion.





$$\frac{4}{10} = \frac{12}{30}$$

$$\text{or, } \frac{2}{5} = \frac{2}{5}$$

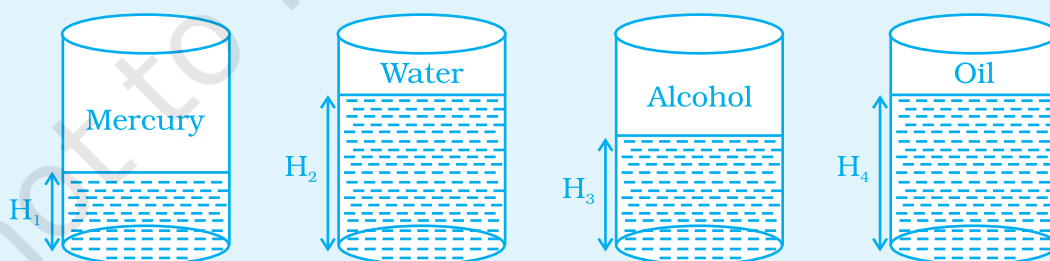
Do yourself

Use square tiles to make similar rectangles with given dimensions and find x .

- The original rectangle is 8 tiles wide and 6 tiles long. The similar rectangle is 16 tiles wide and x tiles long.
- The original rectangle is 3 tiles wide and 7 tiles long. The similar rectangle is 9 tiles wide and x tiles long.

3 Inverse Variation

Take four cylindrical containers of the same size each of radius 5 cm. Fill the containers with different types of liquids with same mass (different density) like Mercury, Water, Alcohol, Oil.



Note the height in each case at which the level of liquid stands. Tabulate this information in the following table and show that this is case of inverse proportion.

Density	Mercury	Water	Alcohol	Oil
(g/cm ³)	13.6 (D ₁)	.99 (D ₂)	.78 (D ₃)	.96 (D ₄)
Height (cm)	H ₁	H ₂	H ₃	H ₄

Density $\times$ Height = Constant.

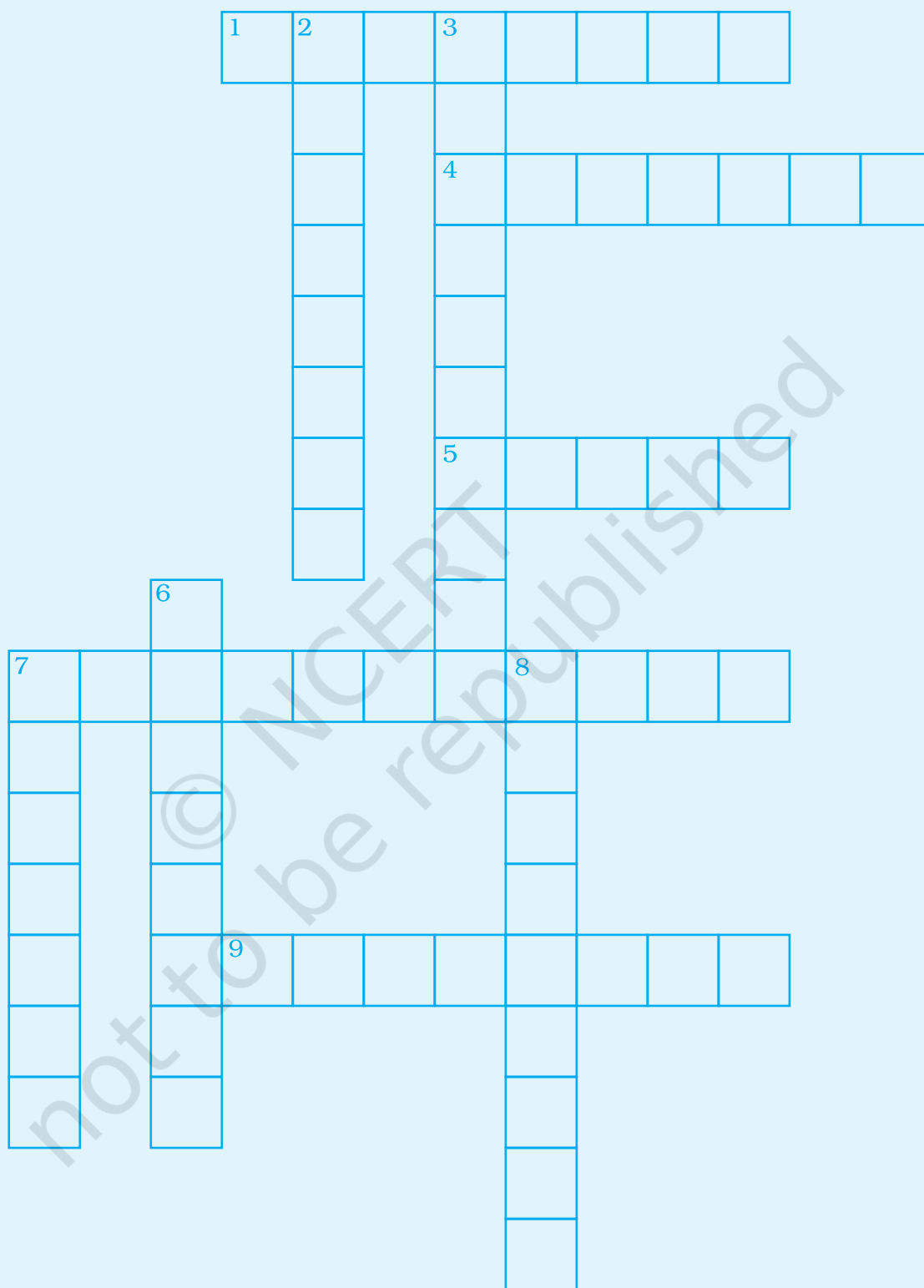
4. Crossword

Across

- Two things are said to be varying _____ if they increase (decrease) together such that ratio of their corresponding values remains constant.
- Problems based on direct proportion can be solved using _____ method.
- More the number of workers, _____ the number of days to finish a job.
- Indirect _____.
- Two quantities are said to be in inverse proportion if an increase in one quantity causes a proportional _____ in other.

Down

- Speed and time are in _____ proportion to each other if distance remains the same.
- It is used to compare two ratios or make _____ fractions.
- 'k' is called _____ of variation.
- In inverse proportion, _____ of corresponding values remains constant.
- With an increase in quantity of milk, cost of milk also _____.



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UNIT 11

MENSURATION

(A) Main Concepts and Results

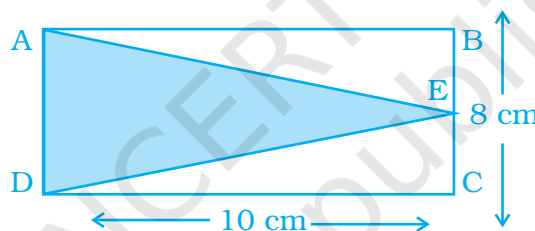
- Length of boundary of a simple closed figure is known as **perimeter**.
- Area is the measure of region enclosed in a simple closed curve.
- Perimeter of a rectangle = 2 (length + breadth).
- Area of a rectangle = length $\times$ breadth.
- Perimeter of a square = 4 $\times$ side.
- Area of a square = side $\times$ side.
- Area of a triangle = $\frac{1}{2} \times$ Base $\times$ Corresponding Height.
- Area of a parallelogram = Base $\times$ Corresponding Height.
- Area of a circle = πr^2 , where r is the radius.
- Area of a trapezium = $\frac{1}{2} \times$ (Sum of parallel sides) $\times$ Height.
- Area of a rhombus = $\frac{1}{2} \times$ Product of diagonals.
- Lateral surface area of a cube = 4 (side)².
- Total surface area of a cube = 6 (side)².
- Lateral surface area of a cuboid = 2 $\times$ height $\times$ (length + breadth).
- Total surface area of a cuboid = 2(lb + bh + hl).
- Lateral (curved) surface area of a cylinder = $2\pi rh$.
- Total surface area of a cylinder = $2\pi r (r + h)$, where r is the radius and h is the height.

- Amount of space occupied by a solid is called its **volume**.
- Volume of a cube = (side)³.
- Volume of a cuboid = length × breadth × height.
- Volume of a cylinder = $\pi r^2 h$.
- $1\text{ cm}^3 = 1\text{ ml}$
- $1\text{ L} = 1000\text{ cm}^3$.
- $1\text{ m}^3 = 10,00,000\text{ cm}^3 = 1,000\text{ L}$.

(B) Solved Examples

In examples 1 and 2, there are four options out of which one is correct. Write the correct answer.

Example 1 : What is the area of the triangle ADE in the following figure?



- (a) 45 cm^2 (b) 50 cm^2 (c) 55 cm^2 (d) 40 cm^2

Solution : The correct answer is (d).

Example 2 : What will be the change in the volume of a cube when its side becomes 10 times the original side?

- (a) Volume becomes 1000 times.
 (b) Volume becomes 10 times.
 (c) Volume becomes 100 times.
 (d) Volume becomes $\frac{1}{1000}$ times.

Solution : The correct answer is (a).

In examples 3 and 4, fill in the blanks to make the statements true.

Example 3 : Area of a rhombus is equal to _____ of its diagonals.

Solution : Half the product.

Example 4 : If the area of a face of a cube is 10 cm^2 , then the total surface area of the cube is _____.

Solution : 60 cm^2 .

In examples 5 and 6, state whether the statements are true (T) or false (F).

Example 5 : $1\text{L} = 1000 \text{ cm}^3$

Solution : True.

Example 6 : Amount of region occupied by a solid is called its surface area.

Solution : False.

Example 7 : 160 m^3 of water is to be used to irrigate a rectangular field whose area is 800 m^2 . What will be the height of the water level in the field?

Solution : Volume of water = 160 m^3

Area of rectangular field = 800 m^2

Let h be the height of water level in the field.

Now, volume of water = volume of cuboid formed on the field by water.

$160 = \text{Area of base} \times \text{height}$

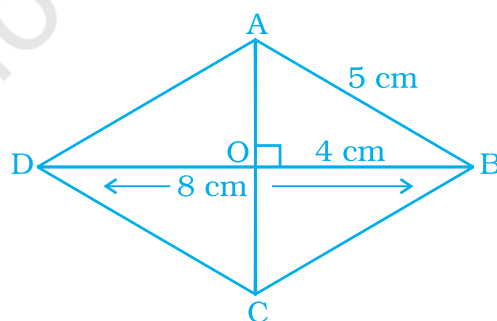
$= 800 \times h$

$$h = \frac{160}{800} = 0.2$$

So, required height = 0.2 m

Example 8 : Find the area of a rhombus whose one side measures 5 cm and one diagonal as 8 cm .

Solution : Let ABCD be the rhombus as shown below.



DO = OB = 4cm, since diagonals of a rhombus are perpendicular bisectors of each other. Therefore, using Pythagoras theorem in $\triangle AOB$,

$$AO^2 + OB^2 = AB^2$$

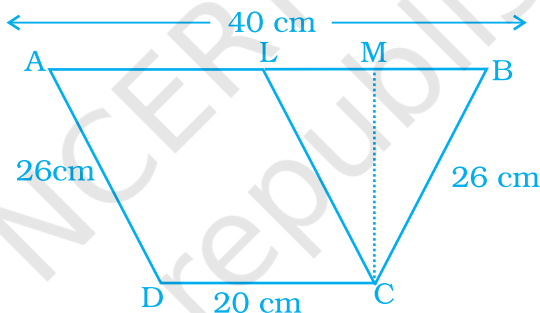
$$AO = \sqrt{AB^2 - OB^2} = \sqrt{5^2 - 4^2} = 3 \text{ cm}$$

$$\text{So, } AC = 2 \times 3 = 6 \text{ cm}$$

$$\text{Thus, the area of the rhombus} = \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2.$$

Example 9 : The parallel sides of a trapezium are 40 cm and 20 cm. If its non-parallel sides are both equal, each being 26 cm, find the area of the trapezium.

Solution : Let ABCD be the trapezium such that AB = 40 cm and CD = 20 cm and AD = BC = 26 cm.



Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following:

1. The square root of a number is one of the two equal factors of the number. For example, 3 is a square root because $3 \times 3 = 9$. How might picturing plant roots help you remember the meaning of square root?
2. The word 'perimeter' comes from the Greek roots *peri*, meaning 'all around,' and *metron*, meaning 'measure.' What do the Greek roots tell you about the perimeter of a geometric figure?
3. To square a number means 'to multiply the number by itself,' as in 2×2 . Keeping this idea of square in mind, what do you think a perfect square might be?
4. The word 'circumference' comes from the Latin word *circumferre*, meaning to "carry around". How does the Latin meaning help you define the circumference of a circle?

Now, draw $CL \parallel AD$

Then $ALCD$ is a parallelogram

So $AL = CD = 20$ cm and $CL = AD = 26$ cm.

In $\triangle CLB$, we have

$CL = CB = 26$ cm

Therefore, $\triangle CLB$ is an isosceles triangle.

Draw altitude CM of $\triangle CLB$.

Since $\triangle CLB$ is an isosceles triangle. So, CM is also the median.

Then $LM = MB = \frac{1}{2}BL = \frac{1}{2} \times 20$ cm = 10 cm

[as $BL = AB - AL = (40 - 20)$ cm = 20 cm].

Applying Pythagoras theorem in $\triangle CLM$, we have

$$CL^2 = CM^2 + LM^2$$

$$26^2 = CM^2 + 10^2$$

$$CM^2 = 26^2 - 10^2 = (26 - 10)(26 + 10) = 16 \times 36 = 576$$

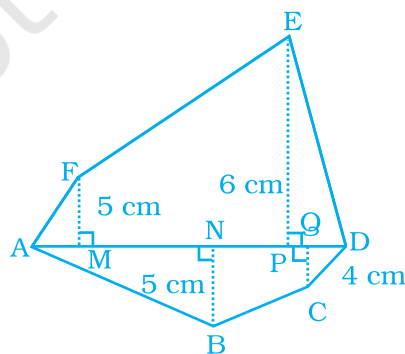
$$CM = \sqrt{576} = 24$$
 cm

Hence, the area of the trapezium = $\frac{1}{2}$ (sum of parallel

sides) $\times$ Height = $\frac{1}{2} (20 + 40) \times 24 = 30 \times 24 = 720$ cm².

Example 10 : Find the area of polygon $ABCDEF$, if $AD = 18$ cm, $AQ = 14$ cm, $AP = 12$ cm, $AN = 8$ cm, $AM = 4$ cm, and FM , EP , QC and BN are perpendiculars to diagonal AD .

Solution :



In the figure

$$MP = AP - AM = (12 - 4) \text{ cm} = 8 \text{ cm}$$

$$PD = AD - AP = (18 - 12) \text{ cm} = 6 \text{ cm}$$

$$NQ = AQ - AN = (14 - 8) \text{ cm} = 6 \text{ cm}$$

$$QD = AD - AQ = (18 - 14) \text{ cm} = 4 \text{ cm}$$

Area of the polygon ABCDEF

$$= \text{area of } \triangle AFM + \text{area of trapezium FMPE} + \text{area of } \triangle EPD \\ + \text{area of } \triangle ANB + \text{area of trapezium NBCQ} + \text{area of } \triangle QCD.$$

$$= \frac{1}{2} \times AM \times FM + \frac{1}{2} (FM + EP) \times MP + \frac{1}{2} PD \times EP + \frac{1}{2} \times$$

$$AN \times NB + \frac{1}{2} (NB + CQ) \times NQ + \frac{1}{2} QD \times CQ$$

$$= \frac{1}{2} \times 4 \times 5 + \frac{1}{2} (5 + 6) \times 8 + \frac{1}{2} \times 6 \times 6 + \frac{1}{2} \times 8 \times 5 + \frac{1}{2} \times$$

$$(5 + 4) \times 6 + \frac{1}{2} \times 4 \times 4.$$

$$= 10 + 44 + 18 + 20 + 27 + 8 = 127 \text{ cm}^2$$

Application on Problem Solving Strategy



Example 11 :

Horse stable is in the form of a cuboid, whose external dimensions are 70 m × 35 m × 40 m, surrounded by a cylinder halved vertically through diameter 35 m and it is open from one rectangular face 70 m × 40 m. Find the cost of painting the exterior of the stable at the rate of Rs 2/m².



Understand and Explore the problem

- What do you know?

Here you know dimensions of cuboid, $L = 70 \text{ m}$, $B = 35 \text{ m}$, $H = 40 \text{ m}$, diameter of cylinder 35 m and cost of painting Rs. 2 per m².

- What fact do you need to solve the question and is not given? Height of cylinder.



Plan a Strategy

- Begin by visualising the shape of the stable and draw it (open from shaded part).
- Think of area to be painted in cuboidal part as well as in cylindrical part.
- Add the two areas calculated in step 2.
- Find cost.



Solve

- Area of cylindrical top be painted

$$\begin{aligned}
 &= \frac{1}{2} [\text{T.S.A}] \\
 &= \frac{1}{2} [2\pi R (R + H)] \\
 &= \frac{1}{2} \left[2 \times \frac{22}{7} \times \frac{35}{2} \left(\frac{35}{2} + 70 \right) \right]
 \end{aligned}$$

$$= 4812.5 \text{ m}^2$$

- Area of cuboid to be painted = area of three walls
 $= lh + 2bh$
 $= 70 \times 40 + 2 \times 40 \times 35$
 $= 2800 + 2800$
 $= 5600 \text{ m}^2$

- Total area to be painted
 $= 4812.5 + 5600$
 $= 10412.5 \text{ m}^2$

- Cost of painting per $\text{m}^2 = \text{Rs } 2$
 Cost of painting $10412.5 \text{ m}^2 = \text{Rs } 10412.5 \times 2$
 $= \text{Rs } 20825$



Revise

- Verify your answer by adopting some other plan, i.e. here in this problem instead of taking area in two steps, let's find in one step.

Area to be painted

= Area of three walls + Area of cylindrical part.

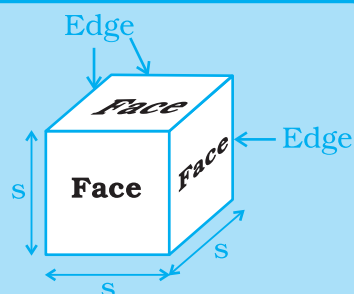
$$\begin{aligned}
 &= 2bh + lh + \frac{1}{2} [2\pi RH + 2\pi R^2] \\
 &= h [2b + l] + [\pi R (R + H)] \\
 &= 40 [2 \times 35 + 70] + \frac{22}{7} \times \frac{35}{2} \left(\frac{35}{2} + 70 \right) \\
 &= 40 [140] + 55 \times 87.5 \\
 &= 5600 + 4812.5 \\
 &\text{Final cost (same as in previous method)} \\
 &\text{Hence verified.}
 \end{aligned}$$

Think and Discuss

- (a) What would be the cost of painting if cylindrical root is not to be painted?
 (b) What would be the cost if one face is not included.
 Is there any difference in the cost?

A cube is a three-dimensional solid with six square faces.

Its **surface area** is the total area of all Six of its faces. As each face is a square, the formula for surface area of a cube is **$A = 6s^2$**



(C) Exercise

In questions 1 to 28, there are four options out of which one is correct. Write the correct answer.

1. A cube of side 5 cm is painted on all its faces. If it is sliced into 1 cubic centimetre cubes, how many 1 cubic centimetre cubes will have exactly one of their faces painted?
- (a) 27 (b) 42 (c) 54 (d) 142

2. A cube of side 4 cm is cut into 1 cm cubes. What is the ratio of the surface areas of the original cubes and cut-out cubes?
 (a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 6
3. A circle of maximum possible size is cut from a square sheet of board. Subsequently, a square of maximum possible size is cut from the resultant circle. What will be the area of the final square?
 (a) $\frac{3}{4}$ of original square. (b) $\frac{1}{2}$ of original square.
 (c) $\frac{1}{4}$ of original square. (d) $\frac{2}{3}$ of original square.
4. What is the area of the largest triangle that can be fitted into a rectangle of length l units and width w units?
 (a) $lw/2$ (b) $lw/3$ (c) $lw/6$ (d) $lw/4$
5. If the height of a cylinder becomes $\frac{1}{4}$ of the original height and the radius is doubled, then which of the following will be true?
 (a) Volume of the cylinder will be doubled.
 (b) Volume of the cylinder will remain unchanged.
 (c) Volume of the cylinder will be halved.
 (d) Volume of the cylinder will be $\frac{1}{4}$ of the original volume.

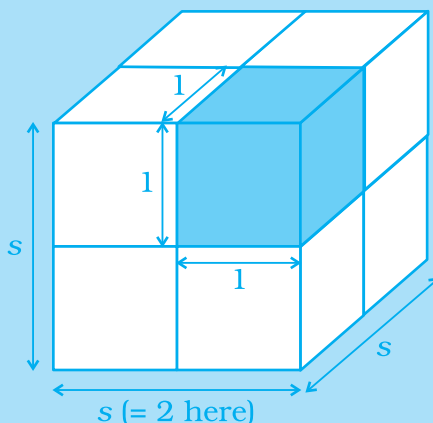
Volume is a measurement of the amount of space inside a three-dimensional object.

It's measured in cubic units and equals the number of unit cubes (cubes whose edges have length 1) that fit inside the object.

In the diagram on the right, each side has a length of 2 units, so two unit cubes fit along each side. (One unit cube is shaded blue.)

You can calculate the volume of cube using the formula.

$$V = S \times S \times S \quad \text{Or} \quad V = S^3$$

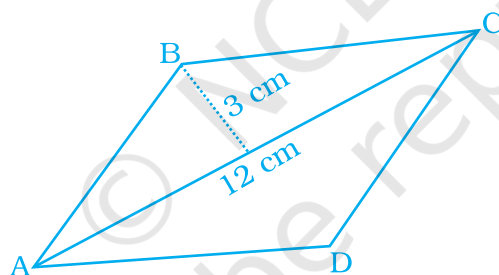


6. If the height of a cylinder becomes $\frac{1}{4}$ of the original height and the radius is doubled, then which of the following will be true?
 - (a) Curved surface area of the cylinder will be doubled.
 - (b) Curved surface area of the cylinder will remain unchanged.
 - (c) Curved surface area of the cylinder will be halved.
 - (d) Curved surface area will be $\frac{1}{4}$ of the original curved surface.
7. If the height of a cylinder becomes $\frac{1}{4}$ of the original height and the radius is doubled, then which of the following will be true?
 - (a) Total surface area of the cylinder will be doubled.
 - (b) Total surface area of the cylinder will remain unchanged.
 - (c) Total surface of the cylinder will be halved.
 - (d) None of the above.
8. The surface area of the three coterminal faces of a cuboid are 6, 15 and 10 cm² respectively. The volume of the cuboid is
 - (a) 30 cm³
 - (b) 40 cm³
 - (c) 20 cm³
 - (d) 35 cm³
9. A regular hexagon is inscribed in a circle of radius r . The perimeter of the regular hexagon is
 - (a) $3r$
 - (b) $6r$
 - (c) $9r$
 - (d) $12r$
10. The dimensions of a godown are 40 m, 25 m and 10 m. If it is filled with cuboidal boxes each of dimensions 2 m $\times$ 1.25 m $\times$ 1 m, then the number of boxes will be
 - (a) 1800
 - (b) 2000
 - (c) 4000
 - (d) 8000

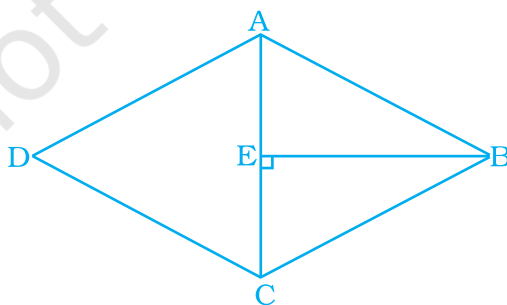
Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Explain how to find the total area of all the faces of a rectangular box.
2. Explain how to find the number of identical cubes it will take to fill a rectangular box.
3. Suppose several different nets are made for a given box. What do all of the nets have in common? What might be different?

11. The volume of a cube is 64 cm^3 . Its surface area is
 (a) 16 cm^2 (b) 64 cm^2 (c) 96 cm^2 (d) 128 cm^2
12. If the radius of a cylinder is tripled but its curved surface area is unchanged, then its height will be
 (a) tripled (b) constant (c) one sixth (d) one third
13. How many small cubes with edge of 20 cm each can be just accommodated in a cubical box of 2 m edge?
 (a) 10 (b) 100 (c) 1000 (d) 10000
14. The volume of a cylinder whose radius r is equal to its height is
 (a) $\frac{1}{4} \pi r^3$ (b) $\frac{\pi r^3}{32}$ (c) πr^3 (d) $\frac{r^3}{8}$
15. The volume of a cube whose edge is $3x$ is
 (a) $27x^3$ (b) $9x^3$ (c) $6x^3$ (d) $3x^3$
16. The figure ABCD is a quadrilateral in which $AB = CD$ and $BC = AD$. Its area is



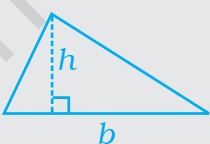
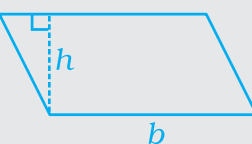
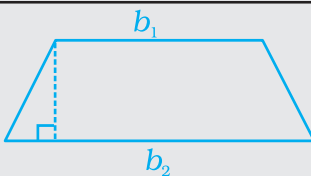
- (a) 72 cm^2 (b) 36 cm^2 (c) 24 cm^2 (d) 18 cm^2
17. What is the area of the rhombus ABCD below if $AC = 6 \text{ cm}$, and $BE = 4 \text{ cm}$?



- (a) 36 cm^2 (b) 16 cm^2 (c) 24 cm^2 (d) 13 cm^2

18. The area of a parallelogram is 60 cm^2 and one of its altitude is 5 cm. The length of its corresponding side is
 (a) 12 cm (b) 6 cm (c) 4 cm (d) 2 cm
19. The perimeter of a trapezium is 52 cm and its each non-parallel side is equal to 10 cm with its height 8 cm. Its area is
 (a) 124 cm^2 (b) 118 cm^2 (c) 128 cm^2 (d) 112 cm^2
20. Area of a quadrilateral ABCD is 20 cm^2 and perpendiculars on BD from opposite vertices are 1 cm and 1.5 cm. The length of BD is
 (a) 4 cm (b) 15 cm (c) 16 cm (d) 18 cm
21. A metal sheet 27 cm long, 8 cm broad and 1 cm thick is melted into a cube. The side of the cube is
 (a) 6 cm (b) 8 cm (c) 12 cm (d) 24 cm
22. Three cubes of metal whose edges are 6 cm, 8 cm and 10 cm respectively are melted to form a single cube. The edge of the new cube is
 (a) 12 cm (b) 24 cm (c) 18 cm (d) 20 cm
23. A covered wooden box has the inner measures as 115 cm, 75 cm and 35 cm and thickness of wood as 2.5 cm. The volume of the wood is
 (a) $85,000 \text{ cm}^3$ (b) $80,000 \text{ cm}^3$ (c) $82,125 \text{ cm}^3$ (d) $84,000 \text{ cm}^3$
24. The ratio of radii of two cylinders is 1: 2 and heights are in the ratio 2:3. The ratio of their volumes is
 (a) 1:6 (b) 1:9 (c) 1:3 (d) 2:9

Finding Areas of Plane Figures

Triangle	Parallelogram	Trapezoid	Circle
 $A = \frac{1}{2}bh$	 $A = bh$	 $A = \frac{1}{2}(b_1 + b_2)h$	 $A = \pi r^2$

25. Two cubes have volumes in the ratio 1:64. The ratio of the area of a face of first cube to that of the other is

- (a) 1:4 (b) 1:8 (c) 1:16 (d) 1:32

26. The surface areas of the six faces of a rectangular solid are 16, 16, 32, 32, 72 and 72 square centimetres. The volume of the solid, in cubic centimetres, is

- (a) 192 (b) 384 (c) 480 (d) 2592

27. Ramesh has three containers.

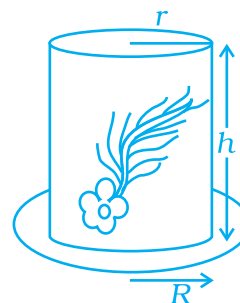
- (a) Cylindrical container A having radius r and height h ,
 (b) Cylindrical container B having radius $2r$ and height $1/2 h$, and
 (c) Cuboidal container C having dimensions $r \times r \times h$

The arrangement of the containers in the increasing order of their volumes is

- (a) A, B, C
 (b) B, C, A
 (c) C, A, B
 (d) cannot be arranged

28. If R is the radius of the base of the hat, then the total outer surface area of the hat is

- (a) $\pi r (2h + R)$ (b) $2\pi r (h + R)$
 (c) $2\pi rh + \pi R^2$ (d) None of these



In questions 29 to 52, fill in the blanks to make the statements true.

29. A cube of side 4 cm is painted on all its sides. If it is sliced in 1 cubic cm cubes, then number of such cubes that will have exactly two of their faces painted is _____.

30. A cube of side 5 cm is cut into 1 cm cubes. The percentage increase in volume after such cutting is _____.

31. The surface area of a cuboid formed by joining two cubes of side a face to face is _____.

32. If the diagonals of a rhombus get doubled, then the area of the rhombus becomes _____ its original area.
33. If a cube fits exactly in a cylinder with height h , then the volume of the cube is _____ and surface area of the cube is _____.
34. The volume of a cylinder becomes _____ the original volume if its radius becomes half of the original radius.
35. The curved surface area of a cylinder is reduced by _____ per cent if the height is half of the original height.
36. The volume of a cylinder which exactly fits in a cube of side a is _____.
37. The surface area of a cylinder which exactly fits in a cube of side b is _____.
38. If the diagonal d of a quadrilateral is doubled and the heights h_1 and h_2 falling on d are halved, then the area of quadrilateral is _____.
39. The perimeter of a rectangle becomes _____ times its original perimeter, if its length and breadth are doubled.
40. A trapezium with 3 equal sides and one side double the equal side can be divided into _____ equilateral triangles of _____ area.
41. All six faces of a cuboid are _____ in shape and of _____ area.
42. Opposite faces of a cuboid are _____ in area.
43. Curved surface area of a cylinder of radius h and height r is _____.
44. Total surface area of a cylinder of radius h and height r is _____.
45. Volume of a cylinder with radius h and height r is _____.



Revise

- Is your answer reasonable?

After you solve a word problem, ask yourself if your answer makes sense. You can round the numbers in the problem and estimate to find a reasonable answer. It may also help to write your answer in sentence form.

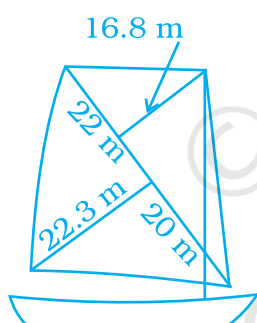
46. Area of a rhombus = $\frac{1}{2}$ product of _____.
47. Two cylinders A and B are formed by folding a rectangular sheet of dimensions $20 \text{ cm} \times 10 \text{ cm}$ along its length and also along its breadth respectively. Then volume of A is _____ of volume of B.
48. In the above question, curved surface area of A is _____ curved surface area of B.
49. _____ of a solid is the measurement of the space occupied by it.
50. _____ surface area of room = area of 4 walls.
51. Two cylinders of equal volume have heights in the ratio 1:9. The ratio of their radii is _____.
52. Two cylinders of same volume have their radii in the ratio 1:6, then ratio of their heights is _____.

In question 53 to 61, state whether the statements are true (T) or false (F).

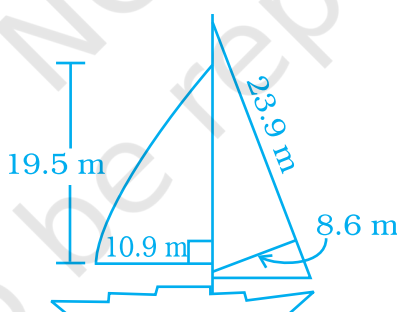
53. The areas of any two faces of a cube are equal.
54. The areas of any two faces of a cuboid are equal.
55. The surface area of a cuboid formed by joining face to face 3 cubes of side x is 3 times the surface area of a cube of side x .
56. Two cuboids with equal volumes will always have equal surface areas.
57. The area of a trapezium become 4 times if its height gets doubled.
58. A cube of side 3 cm painted on all its faces, when sliced into 1 cubic centimetre cubes, will have exactly 1 cube with none of its faces painted.
59. Two cylinders with equal volume will always have equal surface areas.
60. The surface area of a cube formed by cutting a cuboid of dimensions $2 \times 1 \times 1$ in 2 equal parts is 2 sq. units.
61. Ratio of area of a circle to the area of a square whose side equals radius of circle is $1 : \pi$.

Solve the following:

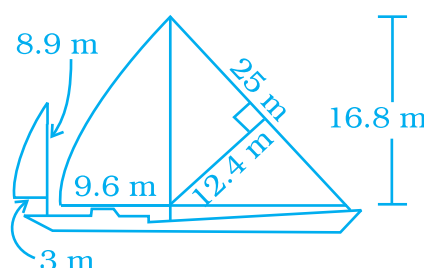
62. The area of a rectangular field is 48 m^2 and one of its sides is 6m. How long will a lady take to cross the field diagonally at the rate of 20 m/minute?
63. The circumference of the front wheel of a cart is 3 m long and that of the back wheel is 4 m long. What is the distance travelled by the cart, when the front wheel makes five more revolutions than the rear wheel?
64. Four horses are tethered with equal ropes at 4 corners of a square field of side 70 metres so that they just can reach one another. Find the area left ungrazed by the horses.
65. The walls and ceiling of a room are to be plastered. The length, breadth and height of the room are 4.5 m, 3 m, and 350 cm respectively. Find the cost of plastering at the rate of Rs 8 per m^2 .
66. Most of the sailboats have two sails, the jib and the mainsail. Assume that the sails are triangles. Find the total area of each sail of the sail boats to the nearest tenth.



(i)



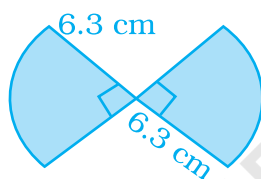
(ii)



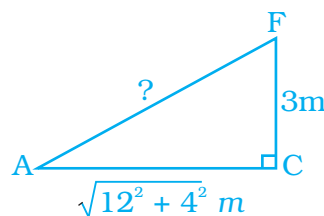
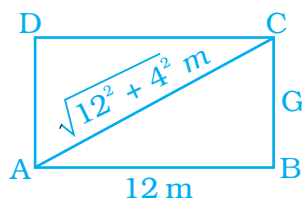
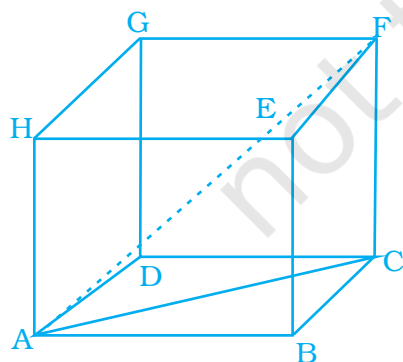
(iii)

67. The area of a trapezium with equal non-parallel sides is 168 m^2 . If the lengths of the parallel sides are 36 m and 20 m, find the length of the non-parallel sides.
68. Mukesh walks around a circular track of radius 14 m with a speed of 4 km/hr. If he takes 20 rounds of the track, for how long does he walk?

69. The areas of two circles are in the ratio 49:64. Find the ratio of their circumferences.
70. There is a circular pond and a footpath runs along its boundary. A person walks around it, exactly once keeping close to the edge. If his step is 66 cm long and he takes exactly 400 steps to go around the pond, find the diameter of the pond.
71. A running track has 2 semicircular ends of radius 63 m and two straight lengths. The perimeter of the track is 1000 m. Find each straight length.
72. Find the perimeter of the given figure.

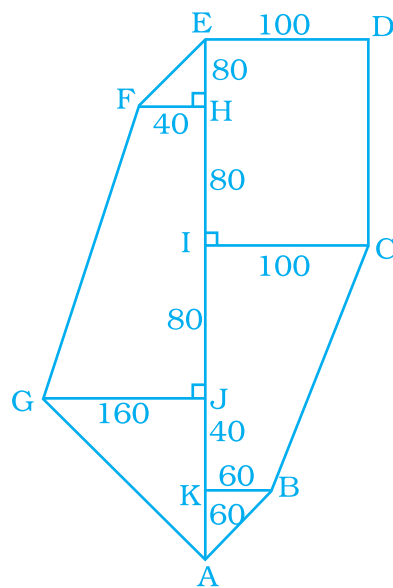


73. A bicycle wheel makes 500 revolutions in moving 1 km. Find the diameter of the wheel.
74. A boy is cycling such that the wheels of the cycle are making 140 revolutions per hour. If the diameter of the wheel is 60 cm, calculate the speed in km/h with which the boy is cycling.
75. Find the length of the largest pole that can be placed in a room of dimensions $12\text{ m} \times 4\text{ m} \times 3\text{ m}$.

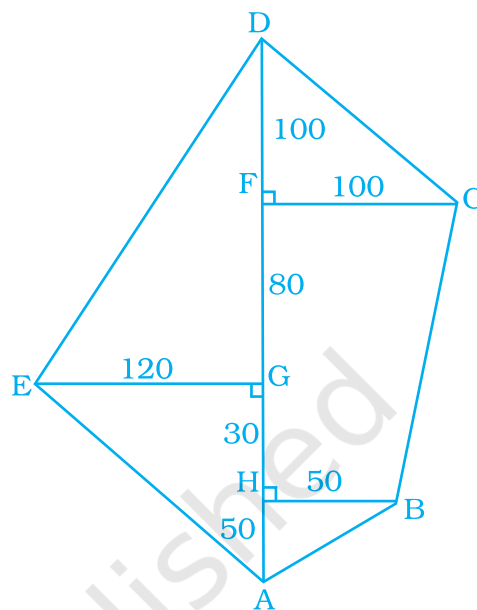


Find the area of the following fields. All dimensions are in metres.

76.

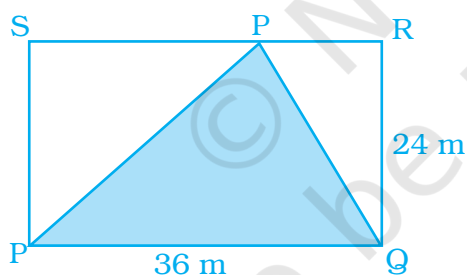


77.

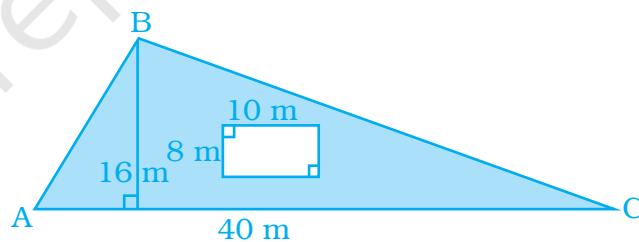


Find the area of the shaded portion in the following figures.

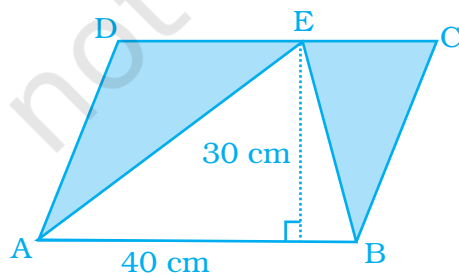
78.



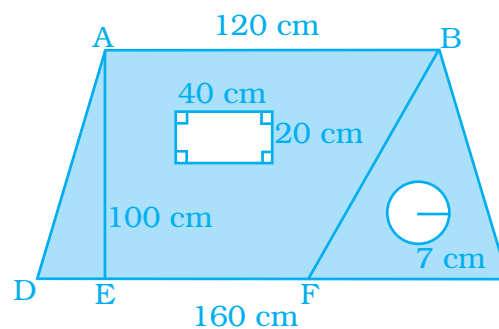
79.



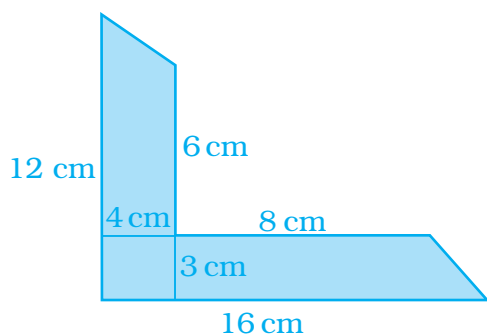
80.



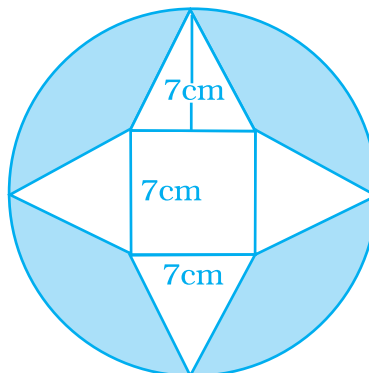
81.



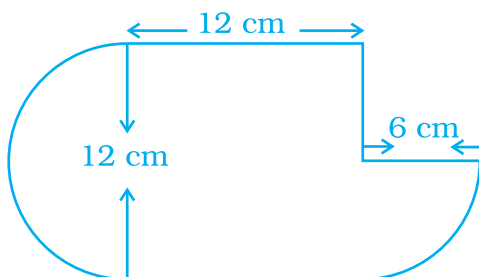
82.



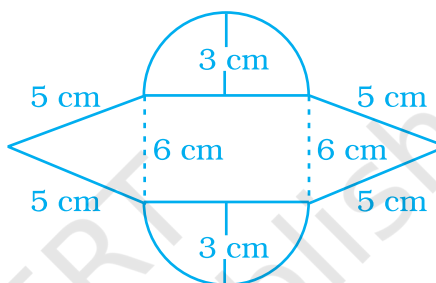
83.



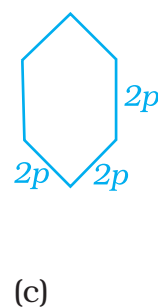
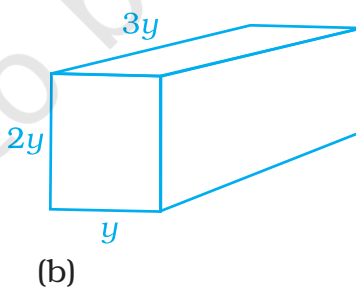
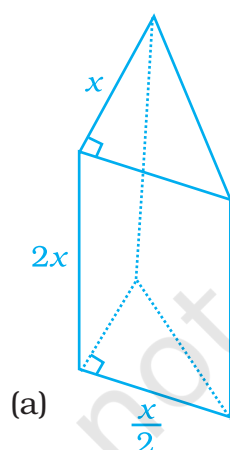
84.



85.



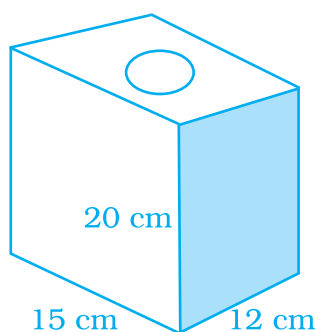
86. Find the volume of each of the given figure if volume = base area $\times$ height.



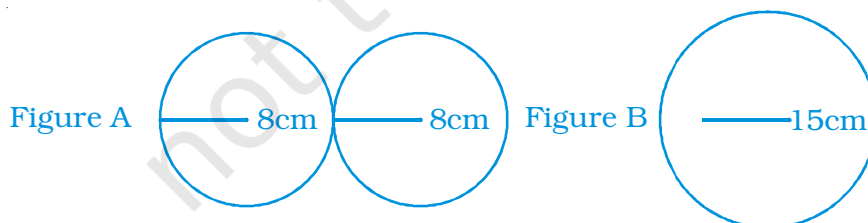
87. A cube of side 5 cm is cut into as many 1 cm cubes as possible. What is the ratio of the surface area of the original cube to that of the sum of the surface areas of the smaller cubes?

- 88.** A square sheet of paper is converted into a cylinder by rolling it along its side. What is the ratio of the base radius to the side of the square?
- 89.** How many cubic metres of earth must be dug to construct a well 7 m deep and of diameter 2.8 m?
- 90.** The radius and height of a cylinder are in the ratio 3:2 and its volume is $19,404 \text{ cm}^3$. Find its radius and height.
- 91.** The thickness of a hollow metallic cylinder is 2 cm. It is 70 cm long with outer radius of 14 cm. Find the volume of the metal used in making the cylinder, assuming that it is open at both the ends. Also find its weight if the metal weighs 8 g per cm^3 .
- 92.** Radius of a cylinder is r and the height is h . Find the change in the volume if the
 - (a) height is doubled.
 - (b) height is doubled and the radius is halved.
 - (c) height remains same and the radius is halved.
- 93.** If the length of each edge of a cube is tripled, what will be the change in its volume?
- 94.** A carpenter makes a box which has a volume of $13,400 \text{ cm}^3$. The base has an area of 670 cm^2 . What is the height of the box?
- 95.** A cuboidal tin box opened at the top has dimensions $20 \text{ cm} \times 16 \text{ cm} \times 14 \text{ cm}$. What is the total area of metal sheet required to make 10 such boxes?
- 96.** Find the capacity of water tank, in litres, whose dimensions are 4.2 m, 3 m and 1.8 m?
- 97.** How many cubes each of side 0.5 cm are required to build a cube of volume 8 cm^3 ?
- 98.** A wooden box (including the lid) has external dimensions 40 cm by 34 cm by 30 cm. If the wood is 1 cm thick, how many cm^3 of wood is used in it?
- 99.** A river 2 m deep and 45 m wide is flowing at the rate of 3 km per hour. Find the amount of water in cubic metres that runs into the sea per minute.

- 100.** Find the area to be painted in the following block with a cylindrical hole. Given that length is 15 cm, width 12 cm, height 20 cm and radius of the hole 2.8 cm.



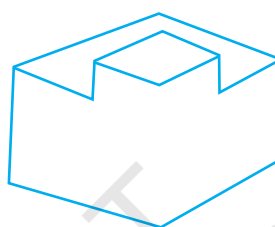
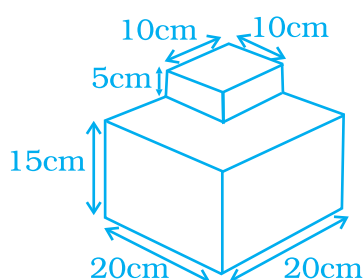
- 101.** A truck carrying 7.8 m^3 concrete arrives at a job site. A platform of width 5 m and height 2 m is being constructed at the site. Find the length of the platform, constructed from the amount of concrete on the truck?
- 102.** A hollow garden roller of 42 cm diameter and length 152 cm is made of cast iron 2 cm thick. Find the volume of iron used in the roller.
- 103.** Three cubes each of side 10 cm are joined end to end. Find the surface area of the resultant figure.
- 104.** Below are the drawings of cross sections of two different pipes used to fill swimming pools. Figure A is a combination of 2 pipes each having a radius of 8 cm. Figure B is a pipe having a radius of 15 cm. If the force of the flow of water coming out of the pipes is the same in both the cases, which will fill the swimming pool faster?



- 105.** A swimming pool is 200 m by 50 m and has an average depth of 2 m. By the end of a summer day, the water level drops by 2 cm. How many cubic metres of water is lost on the day?

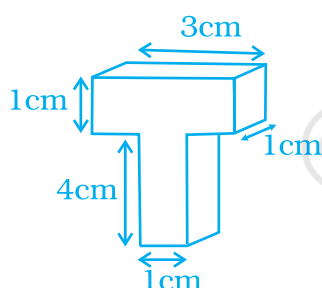
- 106.** A housing society consisting of 5,500 people needs 100 L of water per person per day. The cylindrical supply tank is 7 m high and has a diameter 10 m. For how many days will the water in the tank last for the society?
- 107.** Metallic discs of radius 0.75 cm and thickness 0.2 cm are melted to obtain 508.68 cm^3 of metal. Find the number of discs melted (use $\pi = 3.14$).
- 108.** The ratio of the radius and height of a cylinder is 2:3. If its volume is $12,936 \text{ cm}^3$, find the total surface area of the cylinder.
- 109.** External dimensions of a closed wooden box are in the ratio 5:4:3. If the cost of painting its outer surface at the rate of Rs 5 per dm^2 is Rs 11,750, find the dimensions of the box.
- 110.** The capacity of a closed cylindrical vessel of height 1 m is 15.4 L. How many square metres of metal sheet would be needed to make it?
- 111.** What will happen to the volume of the cube, if its edge is (a) tripled (b) reduced to one-fourth?
- 112.** A rectangular sheet of dimensions $25 \text{ cm} \times 7 \text{ cm}$ is rotated about its longer side. Find the volume and the whole surface area of the solid thus generated.
- 113.** From a pipe of inner radius 0.75 cm, water flows at the rate of 7 m per second. Find the volume in litres of water delivered by the pipe in 1 hour.
- 114.** Four times the area of the curved surface of a cylinder is equal to 6 times the sum of the areas of its bases. If its height is 12 cm, find its curved surface area.
- 115.** A cylindrical tank has a radius of 154 cm. It is filled with water to a height of 3 m. If water to a height of 4.5 m is poured into it, what will be the increase in the volume of water in kl?
- 116.** The length, breadth and height of a cuboidal reservoir is 7 m, 6 m and 15 m respectively. 8400 L of water is pumped out from the reservoir. Find the fall in the water level in the reservoir.
- 117.** How many bricks of size $22 \text{ cm} \times 10 \text{ cm} \times 7 \text{ cm}$ are required to construct a wall 11 m long, 3.5 m high and 40 cm thick, if the cement and sand used in the construction occupy $(1/10)^{\text{th}}$ part of the wall?

- 118.** A rectangular examination hall having seats for 500 candidates has to be built so as to allow 4 cubic metres of air and 0.5 square metres of floor area per candidate. If the length of hall be 25 m, find the height and breadth of the hall.
- 119.** The ratio between the curved surface area and the total surface area of a right circular cylinder is 1:2. Find the ratio between the height and radius of the cylinder.
- 120.** A birthday cake has two tiers as shown in the figure below. Find the volume of the cake.

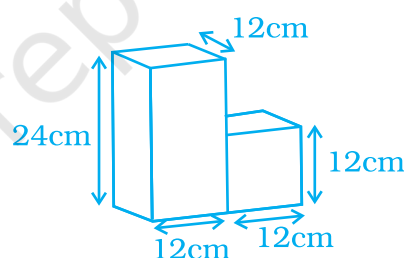


Work out the surface area of following shapes in questions 121 to 124 (use $\pi = 3.14$).

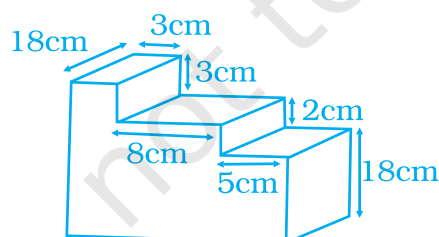
121.



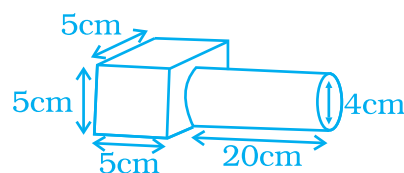
122.



123.



124.

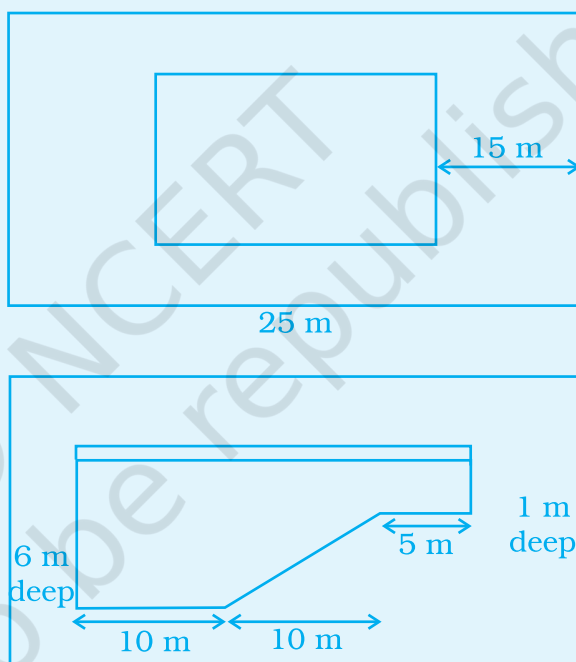


- 125.** Water flows from a tank with a rectangular base measuring 80 cm by 70 cm into another tank with a square base of side 60 cm. If the water in the first tank is 45 cm deep, how deep will it be in the second tank?

- 126.** A rectangular sheet of paper is rolled in two different ways to form two different cylinders. Find the volume of cylinders in each case if the sheet measures $44 \text{ cm} \times 33 \text{ cm}$.

(D) Applications, Games and Puzzles

- 1.** Rashid has decided to build a swimming pool as shown in the figure on an empty plot 25 metres long and 15 metres wide. He is discussing with his son Majid about his plan to build the pool, put tiles on the bottom of the pool and other requirements of the pool. Can you help Majid to answer the following questions which his father has asked in the discussion?

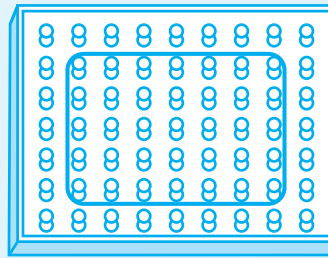


- What is the surface area of the pool?
- If Rashid plans to cover the bottom and sides of the pool with square tiles having side 25 cm, how many such tiles will be required?
- If each tile costs Rs 40, how much will be the total cost?
- A local digging company charges at the rate of Rs 150/- per cubic metre. How much Rashid has to pay for digging the swimming pool?
[Hint : Volume = base area $\times$ height]

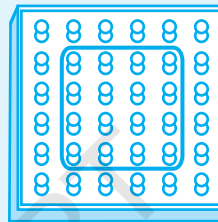
- 5. How long will it take for the swimming pool to be filled completely, if a pipe is pouring water into the pool at the rate of 40 litres per minute?
 - 6. What is the area of the wall at the shallow end of the swimming pool?
 - 7. What is the area of the wall at the deep end of the swimming pool?
 - 8. How much Rashid has to pay in total for making the swimming pool operational, considering cost of digging the pool and fixing tiles?
2. The following table shows the dimensions of cuboids such that, their volumes remain the same. Extend the table with as many more dimensions such that all the cuboids thus formed have the same volume. Complete the table and write your conclusion on surface area and volume of each cuboid.

Dimensions of cuboid (in units)	Surface Area (sq. unit)	Volume (cube unit)
15, 10, 8		1200
6, 10, 20		1200
--	--	--
--	--	--
--	--	--
--	--	--

3. The figure shown is a geoboard in which a rectangle has been outlined using a rubberband.
- 1. What is the area of the rectangle?
 - 2. Draw a similar figure whose area is 50% larger than this figure.
 - 3. Draw a similar figure whose area is 25% larger than this figure.
 - 4. Suppose that the figure shown is 75% of another figure. What would the other figure look like?

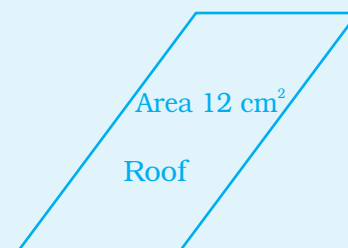
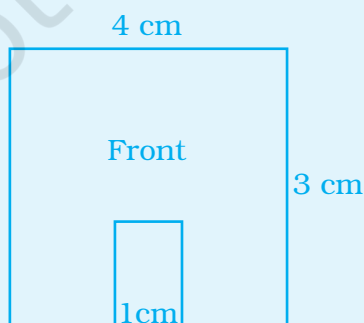
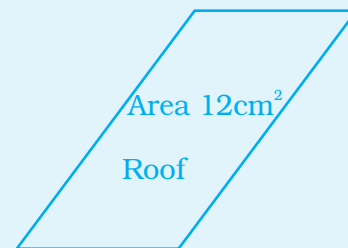
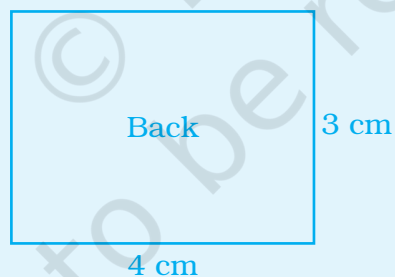


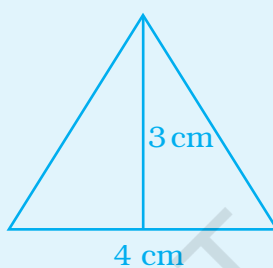
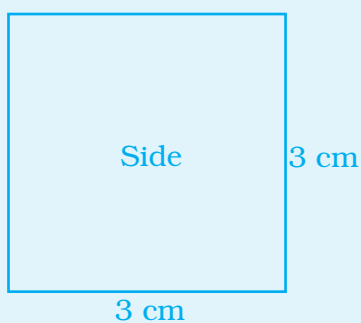
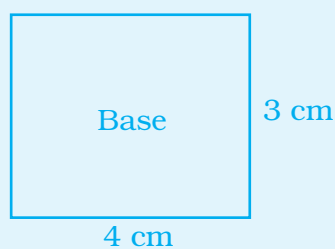
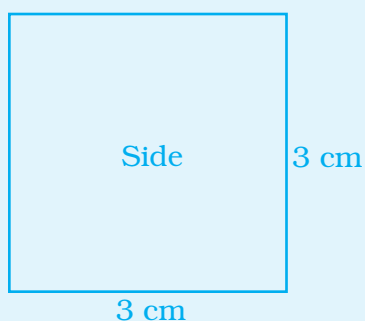
5. The enclosed area represents 75% of another area on the geoboard. Use a geoboard or draw a diagram of a geoboard to represent 100% of the area.



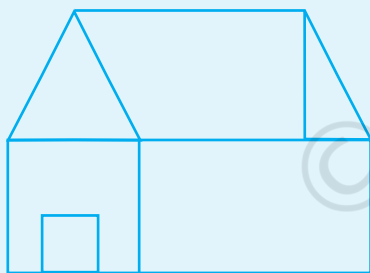
4

- i. Given here are sketches of front, back, sides and roof of a kennel. The drawings are as per the scale. 1 cm = 10 cm.

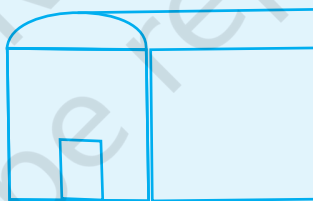




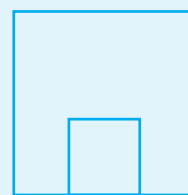
(ii) Sketches given above belong to which kennel?



(a)



(b)



(c)

(iii) Draw the net of the correct choice on the graph paper.

(iv) Take a waste piece of cardboard. Trace the net you have drawn above on the cardboard and fold it to make the kennel.

(v) If you had to pay Rs 2 for each square cm of surface area, how much would it cost you to paint the kennel?

5.

Word Maze

r	h	r	h	o	m	b	u	s	z
a	t	h	a	m	o	b	s	u	q
b	t	r	a	p	e	z	i	u	m
c	y	l	i	n	d	e	r	b	c
t	z	w	v	a	m	q	r	e	u
i	j	l	t	q	n	g	b	a	b
k	b	d	f	v	s	g	t	r	o
s	z	q	c	i	r	c	l	e	i
a	w	h	m	a	n	k	p	e	d

Find the names of the solids from the given word maze whose areas or volumes are given below by colouring the boxes using the given colour code.

Area/Volume

Colour Code

1. $\frac{1}{2} d_1 \times d_2$ red
2. $l b h$ blue
3. $\pi r^2 h$ yellow
4. πr^2 green
5. $\frac{1}{2} b h$ orange
6. $\frac{1}{2} (a + b) \times h$ pink

UNIT 12

INTRODUCTION TO GRAPHS

(A) Main Concepts and Results

- Graphical representation of data is easier to understand.
- A **bar graph**, a **pie chart** and **histogram** are graphical representations of data.
- A **line graph** displays data that changes continuously over periods of time.
- A line graph in which all the line segments form a part of a single line is called a **linear graph**.
- For fixing a point on the graph sheet, we need two mutually perpendicular lines (in which horizontal line is called x axis and the vertical line as y axis) alongwith, x coordinate (abscissa) and y coordinate (ordinate) of the point. The process of fixing a point with the help of the coordinates is known as **plotting a point in the plane**.
- The relation between a **dependent variable** and an **independent variable** is shown through a graph.

(B) Solved Examples

In examples 1 and 2, there are four options out of which one is correct. Write the correct answer.

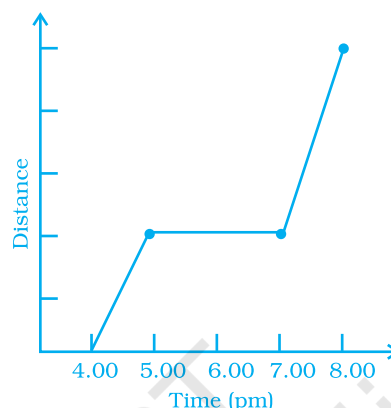
Example 1 : Every point on the x axis is of the form.

- (a) $(0, y)$ (b) $(x, 0)$ (c) (x, y) (d) $(x, 1)$

Solution : The correct answer is (b).

Example 2 : The given graph shows Nisha's trip to a mall by a car. Observe the graph carefully and find what was she doing between 5 pm and 7 pm?

- (a) Driving to the mall. (b) Driving back home.
(c) Was not driving. (d) Not enough data to answer.



Solution : The correct answer is (c).

In examples 3 and 4, fill in the blanks to make the statements true.

Example 3 : In a _____ graph, all the points on the graph lie on the same straight line.

Solution : Linear.

Example 4 : The coordinates of the origin are _____

Solution : (0, 0).

In examples 5 and 6, state whether the statements are true (T) or false (F).

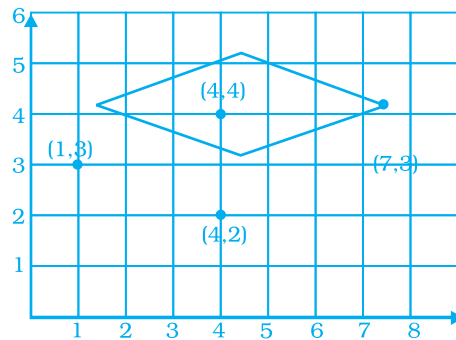
Example 5 : Points (3, 4) and (4, 3) represent the same point on the graph.

Solution : False.

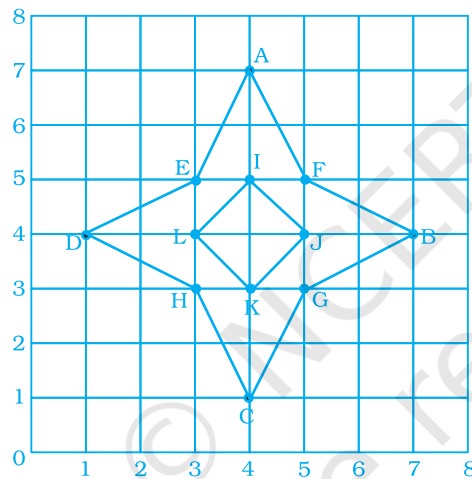
Example 6 : The y coordinate of any point lying on the x axis will be 0.

Solution : True.

Example 7 : Plot the points (4, 4), (1, 3), (4, 2) and (7, 3) on a graph paper and connect them with line segments. Name the shape formed by these points.

Solution :

The shape formed is a rhombus.

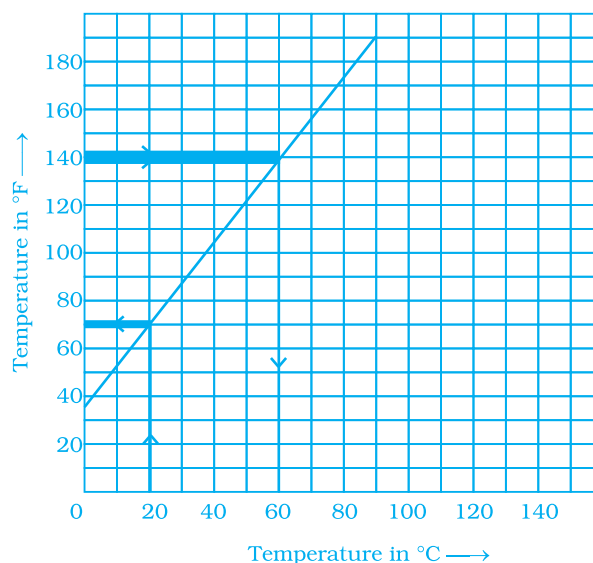
Example 8 : Write the coordinates of all the points in the given graph.

Solution : (A) (4, 7) (E) (3, 5) (I) (4, 5)
 (B) (7, 4) (F) (5, 5) (J) (5, 4)
 (C) (4, 1) (G) (5, 3) (K) (4, 3)
 (D) (1, 4) (H) (3, 3) (L) (3, 4)

Example 9 : The following is a conversion graph of temperature in $^{\circ}\text{C}$ and $^{\circ}\text{F}$.

Use the graph to answer the following questions.

- Convert 140°F to $^{\circ}\text{C}$.
- Convert 20°C to $^{\circ}\text{F}$



Solution : (a) $140^{\circ}\text{F} = 60^{\circ}\text{C}$.

(b) $20^{\circ}\text{C} = 68^{\circ}\text{F}$

Example 10 : Following graph shows a comparison of the approximate sale of items manufactured by a company for the first two years of its operation.

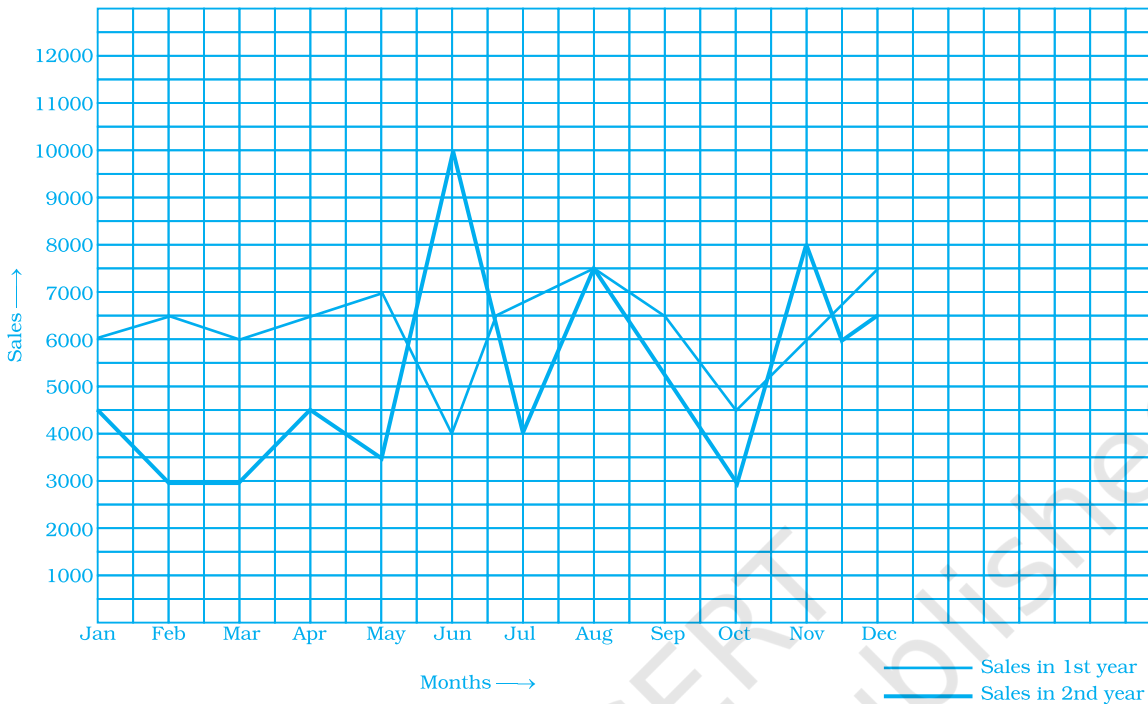
- In which months there was maximum difference in the sale of items of two years?
- In which year was there more stability in the sale of items?
- In which month the sale remains the same in both the years?

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following.

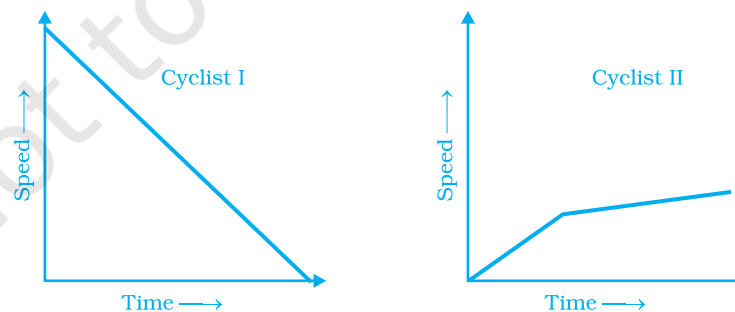
- The word origin means 'beginning.' How do you think this might apply to graphing?
- The root of the word 'quadrant' is 'quad,' which means 'four.' What do you think a quadrant of a graph might be?
- The word 'ordered' means 'arranged according to a rule.' Do you think it refers to a rule? Do you think it matters which number comes first in an ordered pair? Explain.

- (d) In which month was the sales of first year less than that of second year?



- Solution** : (a) The maximum difference was in June.
 (b) There was more stability in sales in the first year.
 (c) The sales remained the same in August.
 (d) June and November.

Example 11 : The given graphs show the progress of two different cyclists during a ride. For each graph, describe the rider's progress over the period of time.



- Solution** : (a) As time passes, the speed of cyclist I decreases steadily.
 (b) Speed of cyclist II increases for a short time period, and then increases very slowly.

- Example 12 :**
- A double bar graph is useful for the _____ of two sets of data.
 - Data represented in a circular form is called a _____ chart.
 - The graph of a linear equation is always a _____ line.
 - The cartesian system used two axes which are _____ to each other.

Solution : Comparison, Pie, Straight, Perpendicular.

Think and Discuss

1. **Describe** the kind of data that is best represented by a bar graph.
2. **Give** a situation in which you would use a line graph to display data.

Application on Problem Solving Strategy



Example 11 :

Complete the given table and draw a graph for it.

x	0	1	2	3	4
$y = 2x$					



Understand and Explore the problem

- What information does the question give?
The x -coordinates and the equation for finding the y -coordinate
- What are you trying to find?
 - (1) The y -coordinates.
 - (2) The coordinates of all 5 points.
 - (3) Plotting the graph of these 5 points.



Plan a Strategy

- You have learnt to solve linear equations. Use the concept to find the y -coordinates by putting $x = 0, 1, 2, 3, 4$ in the equation $y = 2x$.
- Take a graph sheet and draw the 2 axes and locate the points on it. Join the points to get a graph.

**Solve**

- Given $y = 2x$

If $x = 0$, $y = 2(0) = 0$

If $x = 1$, $y = 2(1) = 2$

If $x = 2$, $y = 2(2) = 4$

If $x = 3$, $y = 2(3) = 6$

If $x = 4$, $y = 2(4) = 8$

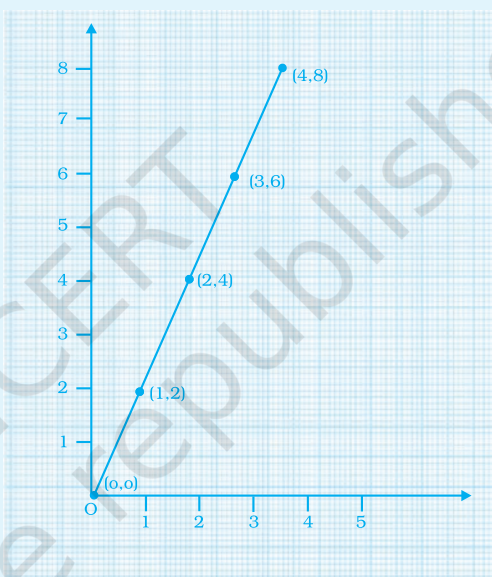
So the completed table will be

x	0	1	2	3	4
$y = 2x$	0	2	4	6	8

The coordinates of the 5 points are $(0,0)$, $(1, 2)$, $(2, 4)$, $(3, 6)$, $(4, 8)$

Take a graph sheet and plot the coordinates of these 5 points on it. Join the points to get a graph.

The graph sheet will look like this.

**Revise**

- Substitute the values of x and y from each coordinate in the given equation $y = 2x$ and see if the coordinates satisfy the equation.

For coordinate $(0, 0)$

L.H.S. = 0

R.H.S. = $2(0) = 0$ LHS = RHS

Hence satisfied.

For coordinate $(1, 2)$

L H S = 2

R H S = $2(1) = 2$ = LHS = RHS

Hence satisfied.

Similarly, you can verify for other coordinates to see if the coordinates found were correct.

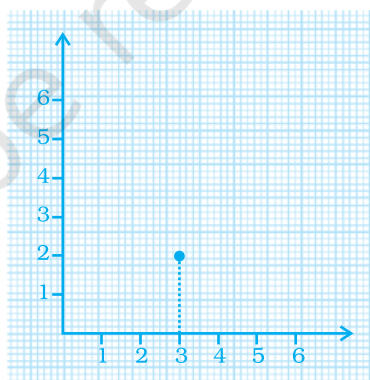
Think and Discuss

- (a) Can you predict from the graph, the value of y when $x = 7$?
 (b) How would the graph change when the equation changes to $y = 3x$?

(C) Exercises

In questions 1 to 10, there are four options out of which one is correct. Write the correct answer.

- Comparison of parts of a whole may be done by a
 (a) bar graph (b) pie chart (c) linear graph (d) line graph
- A graph that displays data that changes continuously over periods of time is
 (a) bar graph (b) pie chart (c) histogram (d) line graph
- In the given graph the coordinates of point x are
 (a) (0, 2) (b) (2, 3) (c) (3, 2) (d) (3, 0)



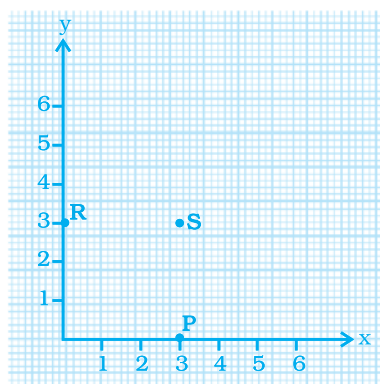
Find Some Solutions to Plot a Graph

To graph a linear equation, you need to find some ordered pairs to plot that are solutions to the linear equation.

You do this by putting some x -values into the equation and finding their corresponding y -values.

4. In the given graph the letter that indicates the point $(0, 3)$ is

- (a) P (b) Q (c) R (d) S



5. The point $(3, 4)$ is at a distance of

- (a) 3 from both the axis (b) 4 from both the axis
(c) 4 from the x axis and (d) 3 from x axis and
3 from y axis from y axis

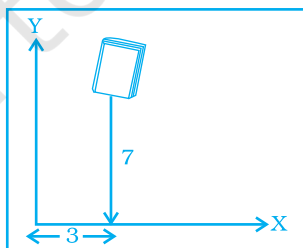
6. A point which lies on both the axis is _____

- (a) $(0, 0)$ (b) $(0, 1)$ (c) $(1, 0)$ (d) $(1, 1)$

7. The coordinates of a point at a distance of 3 units from the x axis and 6 units from the y axis is

- (a) $(0, 3)$ (b) $(6, 0)$ (c) $(3, 6)$ (d) $(6, 3)$

8. In the given figure the position of the book on the table may be given by

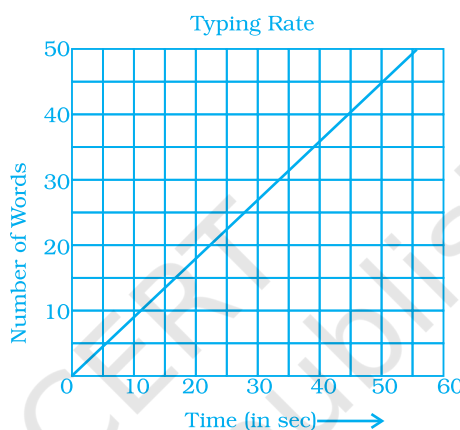


- (a) $(7, 3)$ (b) $(3, 7)$
(c) $(3, 3)$ (d) $(7, 7)$

Think and Discuss

1. Give the coordinates of a point on the x -axis and a point on the y -axis.
2. Give the missing y -coordinates for the solutions to $y = 5x + 2$; $(1, y)$, $(3, y)$, $(10, y)$.

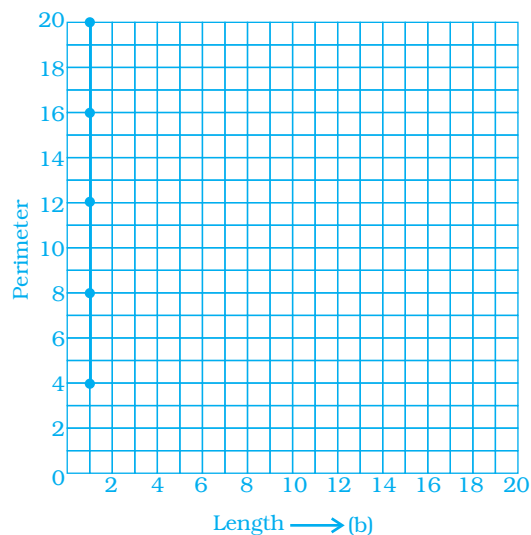
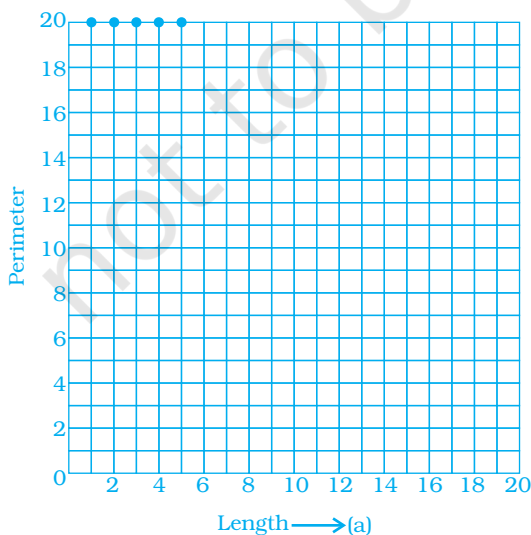
9. Data was collected on a student's typing rate and graph was drawn as shown below. Approximately how many words had this student typed in 30 seconds?

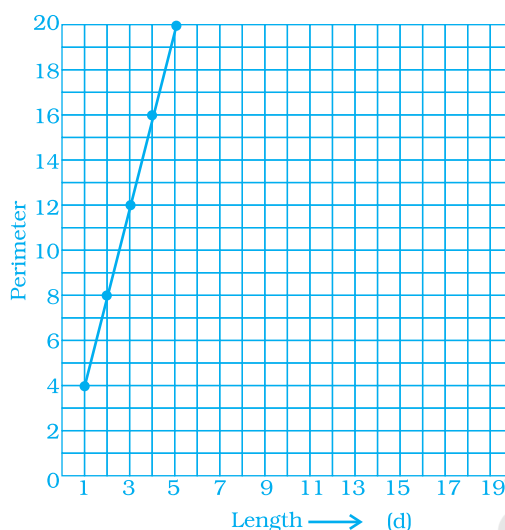
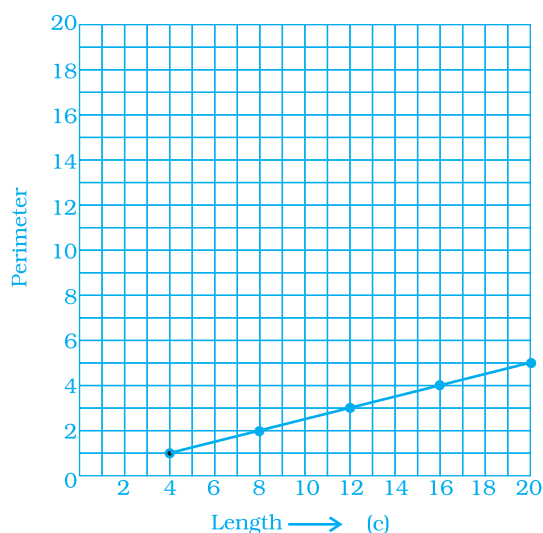


- (a) 20 (b) 24 (c) 28 (d) 34

10. Which graphs of the following represent the table below?

Length of Side of a Square	1	2	3	4	5
Perimeter	4	8	12	16	20





In questions 11 to 25, fill in the blanks to make the statements true.

11. _____ displays data that changes continuously over periods of time.
12. The relation between dependent and independent variables is shown through a _____.
13. We need _____ coordinates for representing a point on the graph sheet.
14. A point in which the x -coordinate is zero and y -coordinate is non-zero will lie on the _____.
15. The horizontal and vertical line in a line graph are usually called _____ and _____.
16. The process of fixing a point with the help of the coordinates is known as _____ of the point.
17. The distance of any point from the y -axis is the _____ coordinate.
18. All points with y -coordinate as zero lie on the _____.
19. For the point (5, 2), the distance from the x -axis is _____ units.
20. The x -coordinate of any point lying on the y -axis will be _____.
21. The y -coordinate of the point (2, 4) is _____.
22. In the point (4, 7), 4 denotes the _____.

- 23.** A point has 5 as its x -coordinate and 4 as its y -coordinate. Then the coordinates of the point are given by _____.
- 24.** In the coordinates of a point, the second number denotes the _____.
- 25.** The point where the two axes intersect is called the _____.

In the questions 26 to 34, state whether the statements are true (T) or false (F).

- 26.** For fixing a point on the graph sheet we need two coordinates.

Distribution of Primes

Remember that a prime number is only divisible by 1 and itself. There are infinitely many prime numbers, but there is no algebraic formula to find them. The largest known prime number, discovered on November 14, 2001, is $2^{13,466,917} - 1$. In standard form, this number would have 4,053,948 digits.

Sieve of Eratosthenes

One way to find prime numbers is called the Sieve of Eratosthenes. Use a list of whole numbers in order. Cross off 1. The next number 2 is prime. Circle it. Then cross off all multiples of 2, because they are not prime. Circle the next number on the list. Cross off all of its multiples. Repeat this step until all of the numbers are circled or crossed off. The circled numbers will all be primes.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~
11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19	~~20~~
21	~~22~~	23	~~24~~	25	~~26~~	27	~~28~~	29	~~30~~
31	~~32~~	33	~~34~~	35	~~36~~	37	~~38~~	39	~~40~~
41	~~42~~	43	~~44~~	45	~~46~~	47	~~48~~	49	~~50~~

- Use the Sieve of Eratosthenes to find all prime number less than 50.
- On graph paper plot the first 15 prime number. Use the prime number as the x -coordinates and their positions in the sequence as the y -coordinates; 2 is the 1st prime, 3 is the 2nd prime, and so on.

Prime Number	2	3	5	7											
Position in Sequence	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

- Estimate the line of best fit and use it to estimate the number of primes under 100. Use the Sieve of Eratosthenes to check your estimate.

27. A line graph can also be a whole unbroken line.
28. The distance of any point from the x -axis is called the x -coordinate.
29. The distance of the point (3, 5) from the y -axis is 5.
30. The ordinate of a point is its distance from the y -axis.
31. In the point (2, 3), 3 denotes the y -coordinate.
32. The coordinates of the origin are (0, 0).
33. The points (3, 5) and (5, 3) represent the same point.
34. The y -coordinate of any point lying on the x -axis will be zero.
35. Match the coordinates given in **Column A** with the items mentioned in **Column B**.

Column A**Column B**

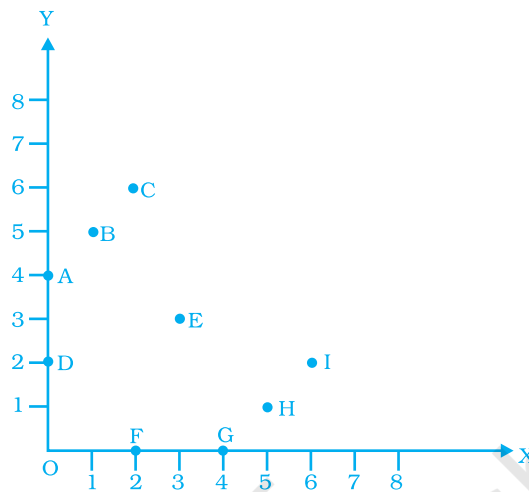
(1) (0, 5)	(a) y coordinate is $2 \times x$ - coordinate + 1.
(2) (2, 3)	(b) Coordinates of origin.
(3) (4, 8)	(c) Only y -coordinate is zero.
(4) (3, 7)	(d) The distance from x -axis is 5.
(5) (0, 0)	(e) y coordinate is double of x -coordinate.
(6) (5, 0)	(f) The distance from y -axis is 2.

36. Match the ordinates of the points given in **Column A** with the items mentioned in **Column B**.

Column A**Column B**

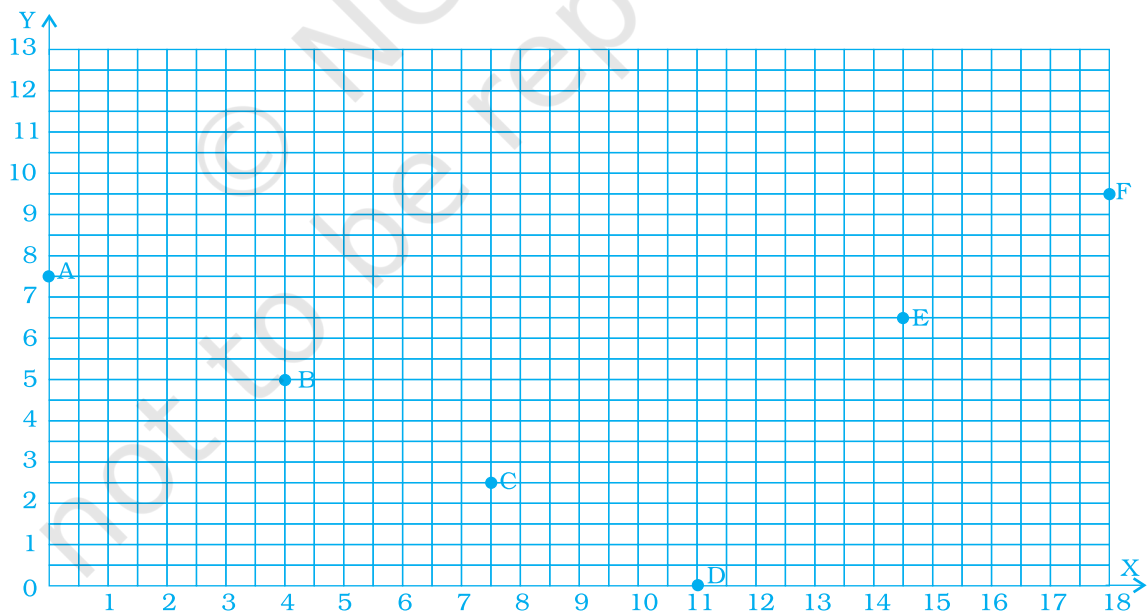
(a) (7, 0)	(i) The ordinate is double the abscissa.
(b) (11, 11)	(ii) The ordinate is zero.
(c) (4, 8)	(iii) The ordinate is equal to the abscissa.
(d) (6, 2)	(iv) The abscissa is double the ordinate.
(e) (0, 9)	(v) The abscissa is triple the ordinate.
(f) (6, 3)	(vi) The abscissa is zero.

- 37.** From the given graph, choose the letters that indicate the location of the points given below.



- (a) $(2, 0)$ (b) $(0, 4)$ (c) $(5, 1)$ (d) $(2, 6)$ (e) $(3, 3)$

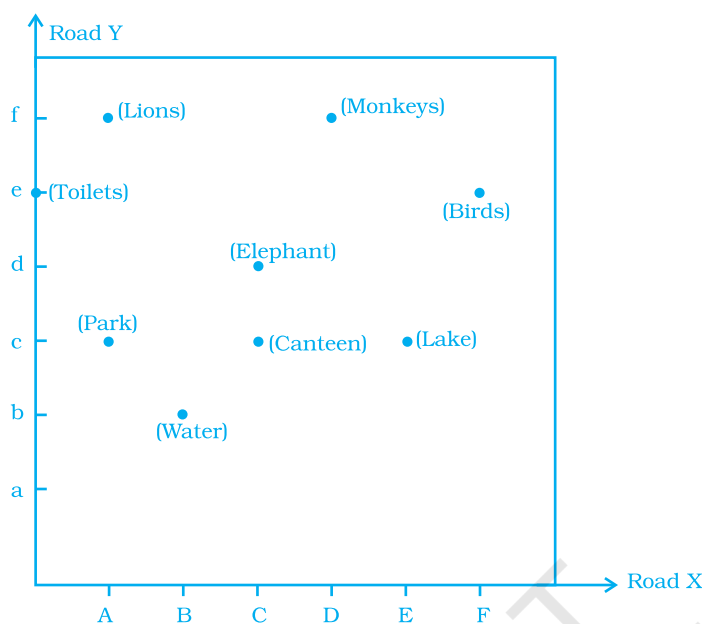
- 38.** Find the coordinates of all letters in the graph given below.



- 39.** Plot the given points on a graph sheet.

- (a) $(5, 4)$ (b) $(2, 0)$ (c) $(3, 1)$ (d) $(0, 4)$ (e) $(4, 5)$

40. Study the given map of a zoo and answer the following questions.



- Give the location of lions in the zoo.
 - (D, f) and (C, d) represent locations of which animals in the zoo?
 - Where are the toilets located?
 - Give the location of canteen.
41. Write the x -coordinate (abscissa) of each of the given points.
- $(7, 3)$
 - $(5, 7)$
 - $(0, 5)$
42. Write the y -coordinate (ordinate) of each of the given points.
- $(3, 5)$
 - $(4, 0)$
 - $(2, 7)$

Make a Plan :

- Do you need an estimate or an exact answer?**

When you are solving a word problem, ask yourself whether you need an exact answer or whether an estimate is sufficient. For example, if the amounts given in the problem are approximate, only an approximate answer can be given. If an estimate is sufficient, you may wish to use estimation techniques to save time in your calculations.

43. Plot the given points on a graph sheet and check if the points lie on a straight line. If not, name the shape they form when joined in the given order.

(a) (1, 2), (2, 4), (3, 6), (4, 8).

(b) (1, 1), (1, 2), (2, 1), (2, 2).

(c) (4, 2), (2, 4), (3, 3), (5, 4).

44. If y -coordinate is 3 times x -coordinate, form a table for it and draw a graph.

45. Make a line graph for the area of a square as per the given table.

Side (in cm)	1	2	3	4
Area (in cm^2)	1	4	9	16

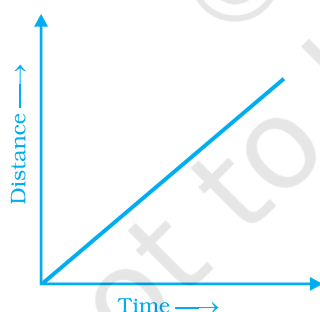
Is it a linear graph?

46. The cost of a note book is Rs 10. Draw a graph after making a table showing cost of 2, 3, 4, ... note books. Use it to find

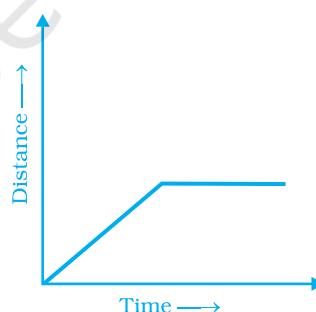
(a) the cost of 7 notebooks.

(b) The number of note books that can be purchased with Rs 50.

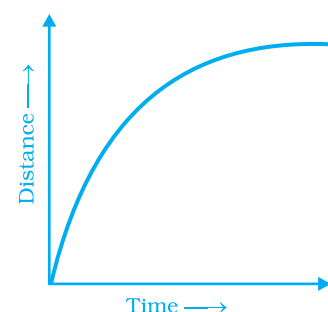
47. Explain the situations represented by the following distance-time graphs.



(a)



(b)



(c)

48. Complete the given tables and draw a graph for each.

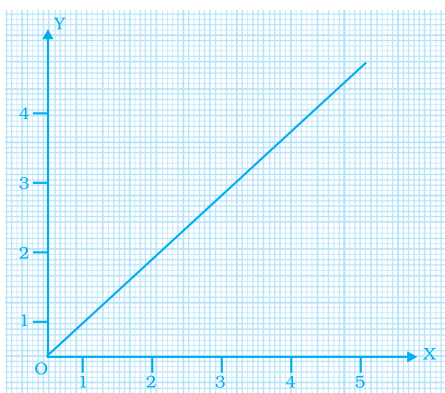
(a)

x	0	1	2	3
$y = 3x + 1$	1	4	-	-

(b)

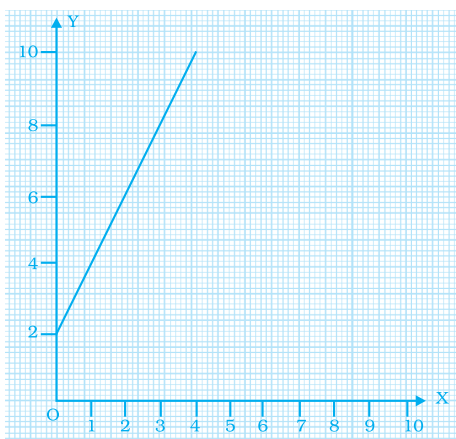
x	1	2	4	6
$y = x - 1$	0			

49. Study the given graphs (a) and (b) and complete the corresponding tables below.



(a)

x	0	1	2	3
y				



(b)

x	0	1	2	3	4
y					

50. Draw a graph for the radius and circumference of circle using a suitable scale.

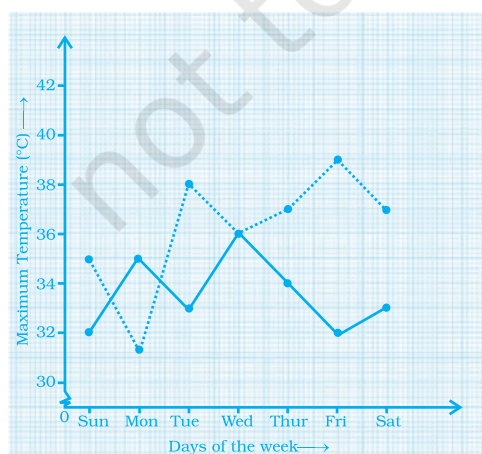
(Hint : Take radius = 7, 14, 21 units and so on)

From the graph,

(a) Find the circumference of the circle when radius is 42 units.

(b) At what radius will the circumference of the circle be 220 units?

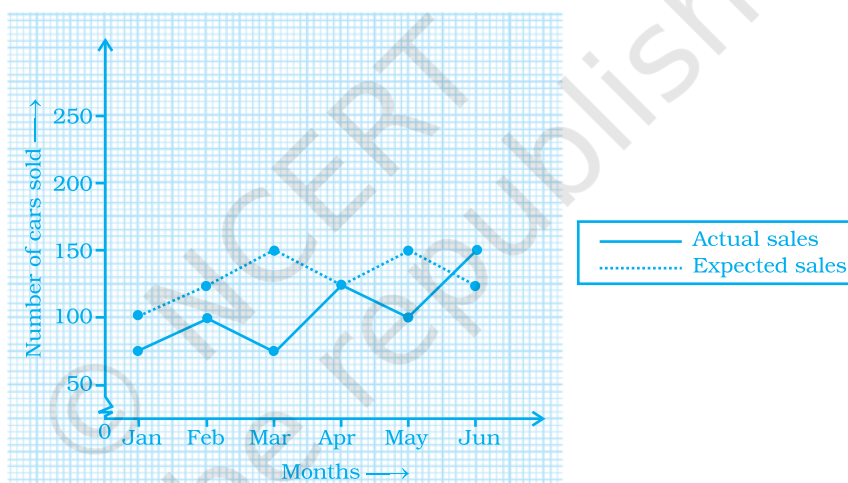
51. The graph shows the maximum temperatures recorded for two consecutive weeks of a town. Study the graph and answer the questions that follow.



..... First week
 — Second week

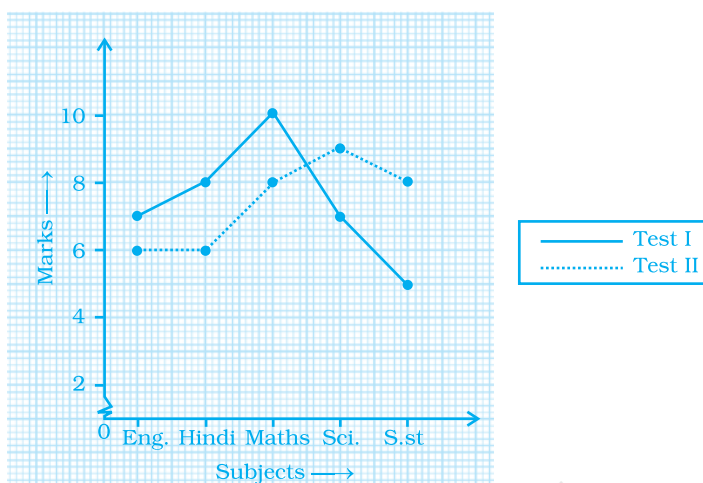
- What information is given by the two axes?
- In which week was the temperature higher on most of the days?
- On which day was the temperature same in both the weeks?
- On which day was the difference in temperatures the maximum for both the weeks?
- What were the temperatures for both the weeks on Thursday?
- On which day was the temperature 35°C for the first week?
- On which day was the temperature highest for the second week?

- 52.** The graph given below gives the actual and expected sales of cars of a company for 6 months. Study the graph and answer the questions that follow.

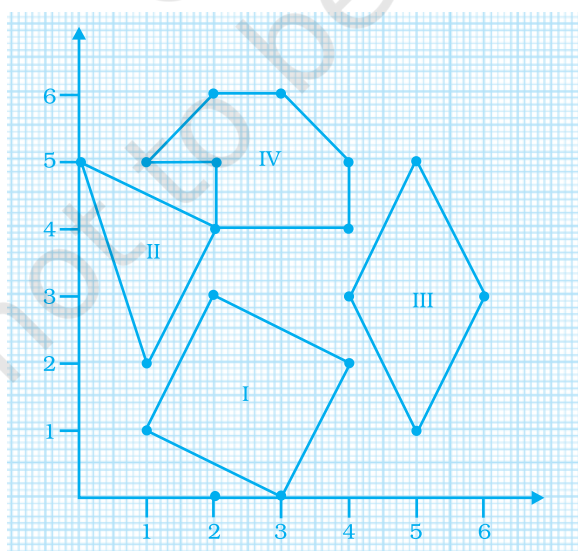


- In which month was the actual sales same as the expected sales?
- For which month(s) was (were) the difference in actual and expected sales the maximum?
- For which month(s) was (were) the difference in actual and expected sales the least?
- What was the total sales of cars in the months–Jan, Feb. and March?
- What is the average sales of cars in the last three months?
- Find the ratio of sales in the first three months to the last three months.

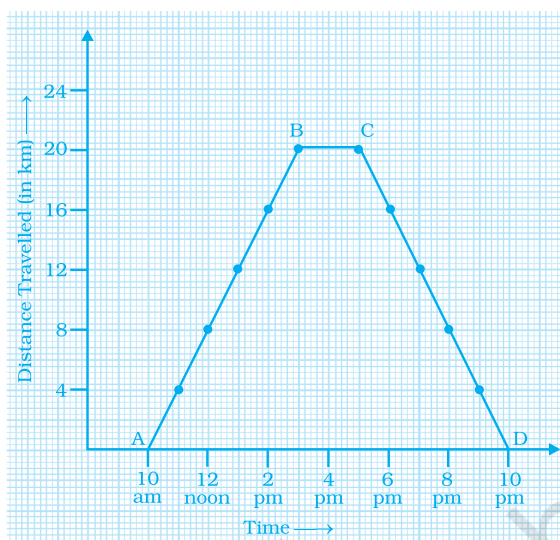
- 53.** The graph given below shows the marks obtained out of 10 by Sonia in two different tests. Study the graph and answer the questions that follow.



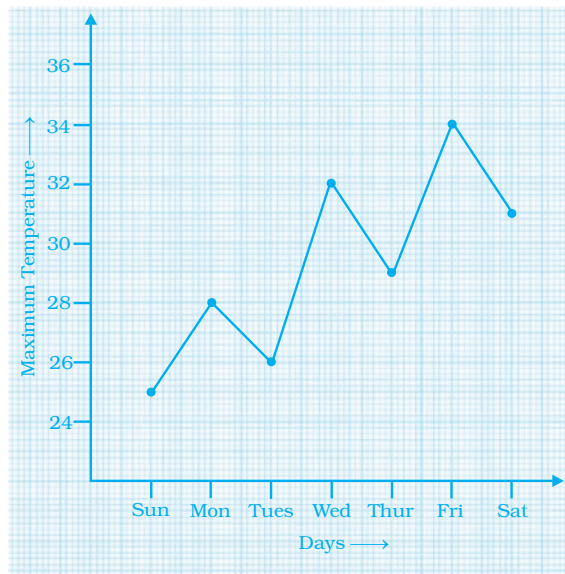
- What information is represented by the axes?
 - In which subject did she score the highest in Test I?
 - In which subject did she score the least in Test II?
 - In which subject did she score the same marks in both the Tests?
 - What are the marks scored by her in English in Test II?
 - In which test was the performance better?
 - In which subject and which test did she score full marks?
- 54.** Find the coordinates of the vertices of the given figures.



55. Study the graph given below of a person who started from his home and returned at the end of the day. Answer the questions that follow.

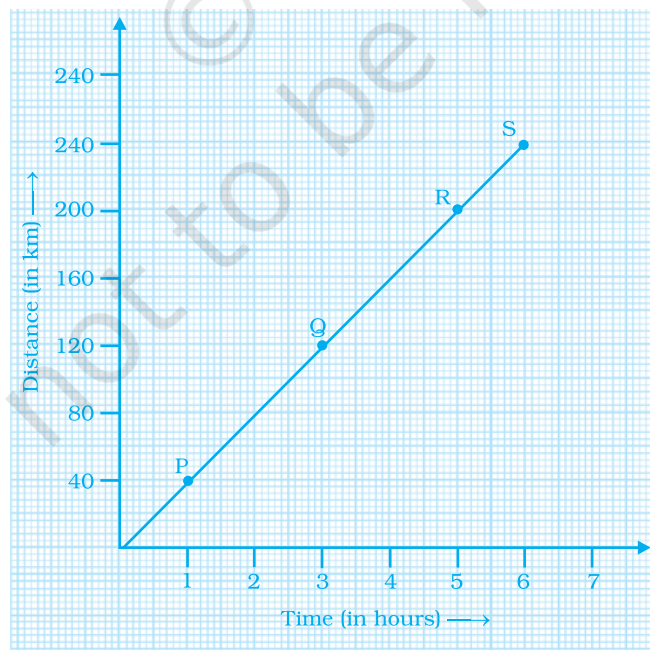


- At what time did the person start from his home?
 - How much distance did he travel in the first four hours of his journey?
 - What was he doing from 3 pm to 5 pm?
 - What was the total distance travelled by him throughout the day?
 - Calculate the distance covered by him in the first 8 hours of his journey.
 - At what time did he cover 16 km of his journey?
 - Calculate the average speed of the man from (a) A to B (b) B to C (c) At what time did he return home?
56. Plot a line graph for the variables p and q where p is two times q i.e., the equation is $p = 2q$. Then find.
- the value of p when $q = 3$
 - the value of q when $p = 8$
57. Study the graph and answer the questions that follow.
- What information does the graph give?
 - On which day was the temperature the least?
 - On which day was the temperature 31°C ?
 - Which was the hottest day?

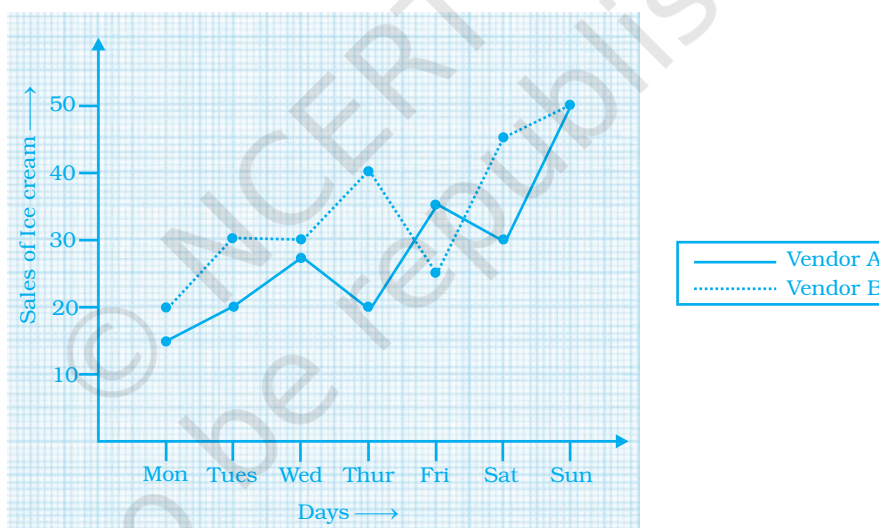


58. Study the distance-time graph given below for a car to travel to certain places and answer the questions that follow.

- How far does the car travel in 2 hours?
- How much time does the car take to reach R?
- How long does the car take to cover 80 km?
- How far is Q from the starting point?
- When does the car reach the place S after starting?



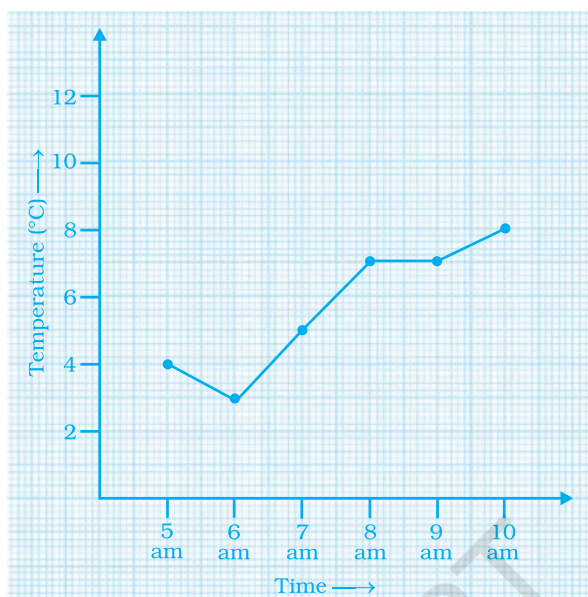
59. Locate the points A (1,2), B (4,2) and C (1,4) on a graph sheet taking suitable axes. Write the coordinates of the fourth point D to complete the rectangle ABCD.
60. Locate the points A(1,2), B (3,4) and C (5,2) on a graph sheet taking suitable axes. Write the coordinates of the fourth point D to complete the rhombus ABCD. Measure the diagonals of this rhombus and find whether they are equal or not.
61. Locate the points P (3,4), Q (1,0), R (0,4), S (4,1) on a graph sheet and write the coordinates of the point of intersection of line segments PQ and RS.
62. The graph given below compares the sales of ice creams of two vendors for a week.



Observe the graph and answer the following questions.

- (a) Which vendor has sold more icecreams on Friday?
- (b) For which day was the sales same for both the vendors?
- (c) On which day did the sale of vendor A increase the most as compared to the previous day?
- (d) On which day was the difference in sales the maximum?
- (e) On which two days was the sales same for vendor B?

- 63.** The table given below shows the temperatures recorded on a day at different times.



Observe the table and answer the following questions.

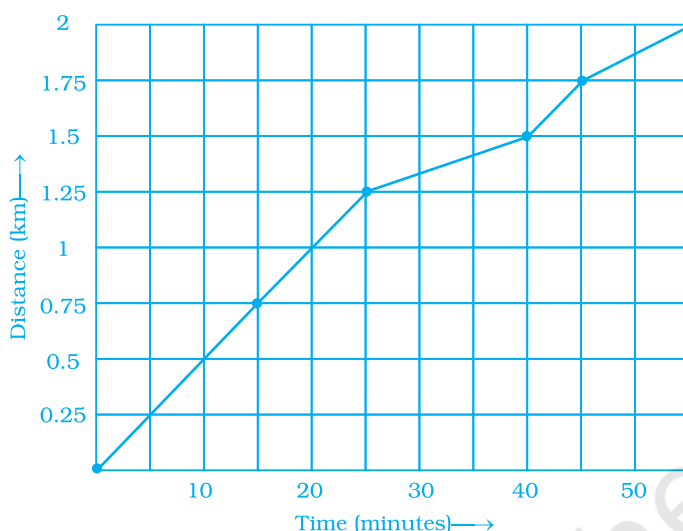
- What is the temperature at 8 am?
 - At what time is the temperature 3°C ?
 - During which hour did the temperature fall?
 - What is the change in temperature between 7 am and 10 am?
 - During which hour was there a constant temperature?
- 64.** The following table gives the growth chart of a child.

Height (in cm)	75	90	110	120	130
Age (in years)	2	4	6	8	10

Draw a line graph for the table and answer the questions that follow.

- What is the height at the age of 5 years?
- How much taller was the child at the age of 10 than at the age of 6?
- Between which two consecutive periods did the child grow more faster?

65. The following is the time-distance graph of Sneha's walking.



- (a) When does Sneha make the least progress? Explain your reasoning.
- (b) Find her average speed in km/hour.
66. Draw a parallelogram ABCD on a graph paper with the coordinates given in Table I. Use this table to complete Tables II and III to get the coordinates of E, F, G, H and J, K, L, M.

Point	(x, y)
A	(1, 1)
B	(4, 4)
C	(8, 4)
D	(5, 1)

Table I

Point	$(0.5x, 0.5y)$
E	(0.5, 0.5)
F	
G	
H	

Table II

Point	$(2x, 1.5y)$
J	(2, 1.5)
K	
L	
M	

Table III

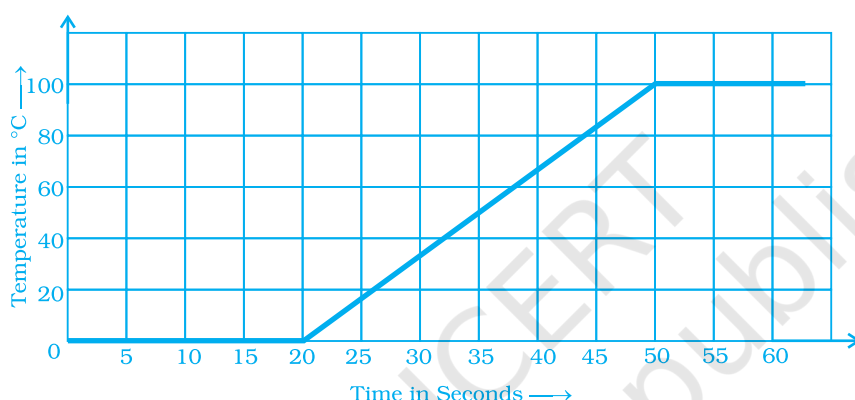
Draw parallelograms EFGH and JKLM on the same graph paper.

Plot the points (2, 4) and (4, 2) on a graph paper, then draw a line segment joining these two points.

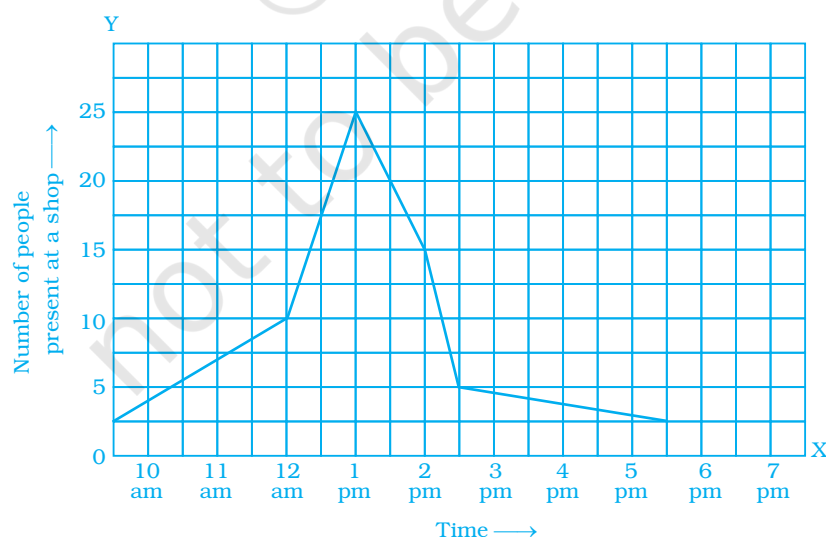
67. Extend the line segment on both sides to meet the coordinate axes. What are the coordinates of the points where this line meets the x -axis and the y -axis?

68. The following graph shows the change in temperature of a block of ice when heated. Use the graph to answer the following questions:

- For how many seconds did the ice block have no change in temperature?
- For how long was there a change in temperature?
- After how many seconds of heating did the temperature become constant at 0°C ?
- What was the temperature after 25 seconds?
- What will be the temperature after 1.5 minutes? Justify your answer.



69. The following graph shows the number of people present at a certain shop at different times. Observe the graph and answer the following questions.

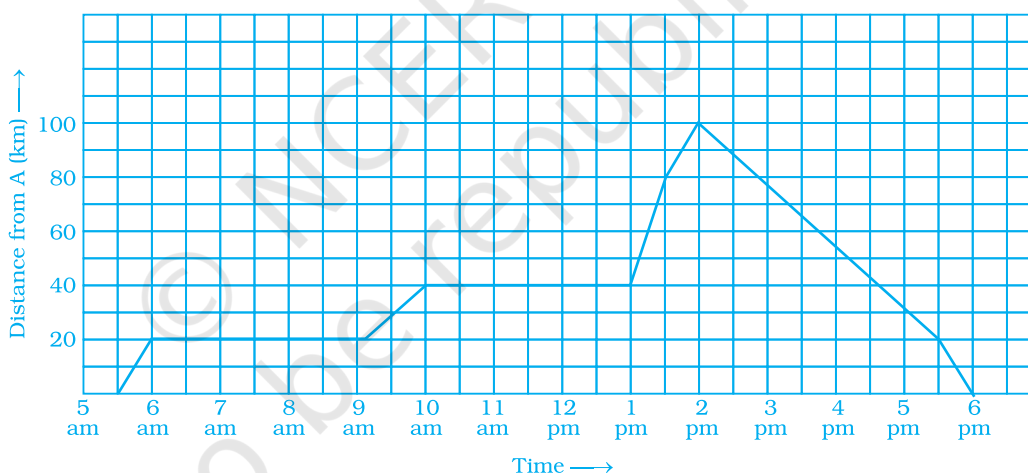


- What type of a graph is this?

- (b) What information does the graph give?
- (c) What is the busiest time of day at the shop?
- (d) How many people enter the shop when it opens?
- (e) About how many people are there in the shop at 1:30 pm?

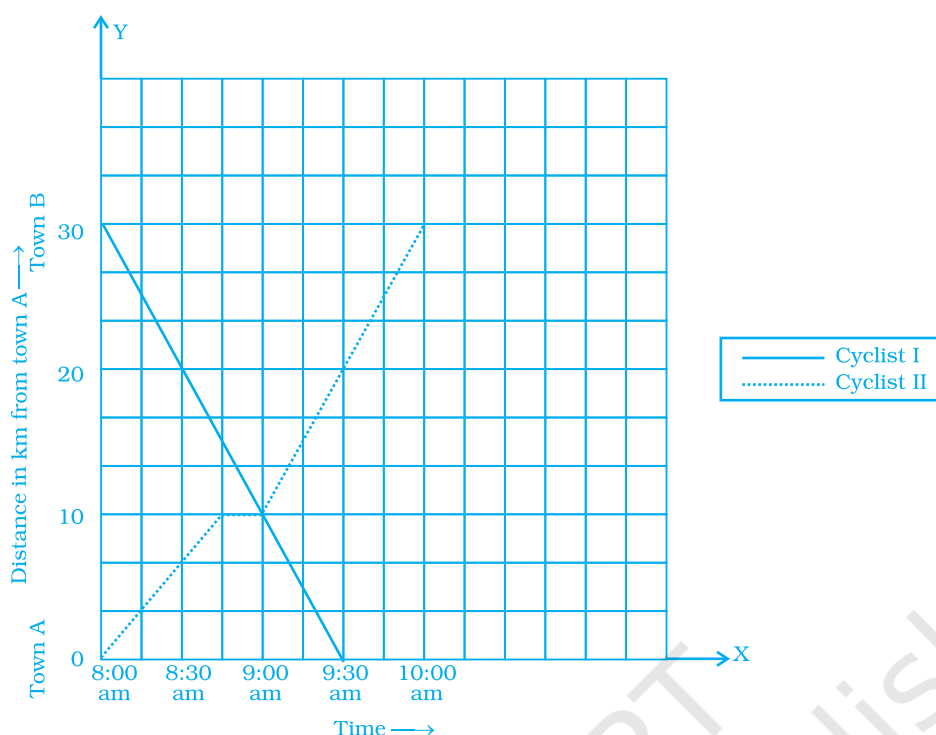
70. A man started his journey on his car from location A and came back. The given graph shows his position at different times during the whole journey.

- (a) At what time did he start and end his journey?
- (b) What was the total duration of journey?
- (c) Which journey, forward or return, was of longer duration?
- (d) For how many hours did he not move?
- (e) At what time did he have the fastest speed?



71. The following graph shows the journey made by two cyclists, one from town A to B and the other from town B to A.

- (a) At what time did cyclist II rest? How long did the cyclist rest?
- (b) Was cyclist II cycling faster or slower after the rest?
- (c) At what time did the two cyclists meet?
- (d) How far had cyclist II travelled when he met cyclist I?
- (e) When cyclist II reached town A, how far was cyclist I from town B?



- 72.** Ajita starts off from home at 07.00 hours with her father on a scooter that goes at a uniform speed of 30 km/h and drops her at her school after half an hour. She stays in the school till 13.30 hours and takes an auto rickshaw to return home. The rickshaw has a uniform speed of 10 km/h. Draw the graph for the above situation and also determine the distance of Ajita's school from her house.
- 73.** Draw the line graph using suitable scale to show the annual gross profit of a company for a period of five years.

Year	1st	2nd	3rd	4th	5th
Gross Profit (in Rs)	17,00,000	15,50,000	11,40,000	12,10,000	14,90,000

- 74.** The following chart gives the growth in height in terms of percentage of full height of boys and girls with their respective ages.

Age (in years)	8	9	10	11	12	13	14	15	16	17	18
Boys	72%	75%	78%	81%	84%	88%	92%	95%	98%	99%	100%
Girls	77%	81%	84%	88%	91%	95%	98%	99%	99.5%	100%	100%

Draw the line graph of above data on the same sheet and answer the following questions.

- In which year both the boys and the girls achieve their maximum height?
- Who grows faster at puberty (14 years to 16 years of age)?

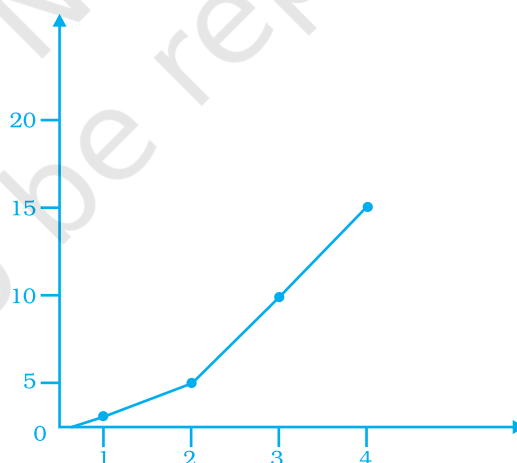
75. The table shows the data collected for Dhruv's walking on a road.

Time (in minutes)	0	5	10	15	20	25
Distance (in km)	0	0.5	1	1.25	1.5	1.75

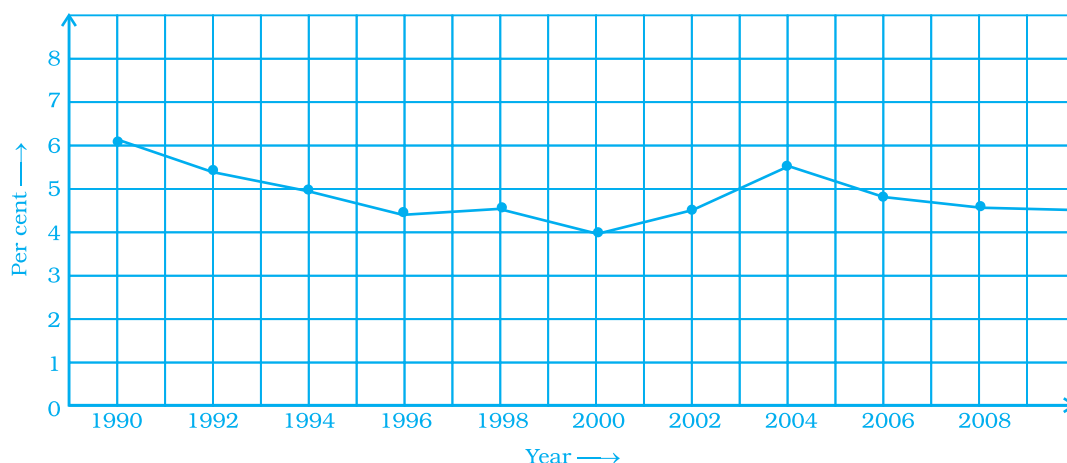
- Plot a line graph for the given data using a suitable scale.
- In what time periods did Dhruv make the most progress?

76. Observe the given graph carefully and complete the table given below.

x	1	2	3	4	5
y					



77. This graph shows the per cent of students who dropped out of school after completing High School. The point labelled A shows that, in 1996, about 4.7% of students dropped out.



- In which year was the dropout the rate highest? In which year was it the lowest?
- When did the per cent of students who dropped out of high school first fall below 5%?
- About what per cent of students dropped out of high school in 2007? About what per cent of students stayed in high school in 2008?

78. Observe the toothpick pattern given below:



Pattern 1



Pattern 2



Pattern 3



Pattern 4

- Imagine that this pattern continues. Complete the table to show the number of toothpicks in the first six terms.

Pattern	1	2	3	4	5	6
Toothpicks	4			13		

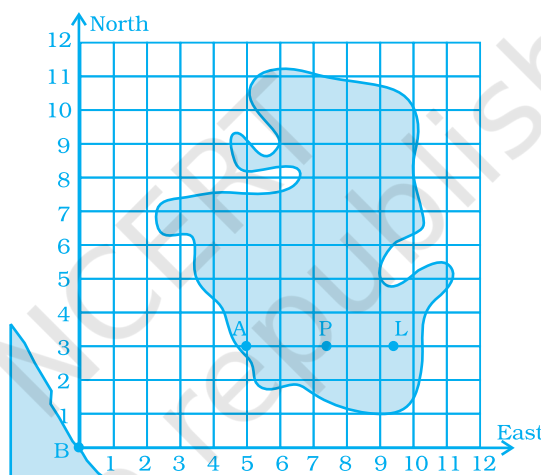
- Make a graph by taking the pattern numbers on the horizontal axis and the number of toothpicks on the vertical axis. Make the horizontal axis from 0 to 10 and the vertical axis from 0 to 30.
- Use your graph to predict the number of toothpicks in patterns 7 and 8. Check your answers by actually drawing them.
- Would it make sense to join the points on this graph? Explain.

79. Consider this input/output table.

Input	1	2	4	5	7
Output	2	5	11	14	20

- Graph the values from the table by taking Input along horizontal axis from 0 to 8 and Output along vertical axis from 0 to 24.
- Use your graph to predict the outputs for inputs of 3 and 8.

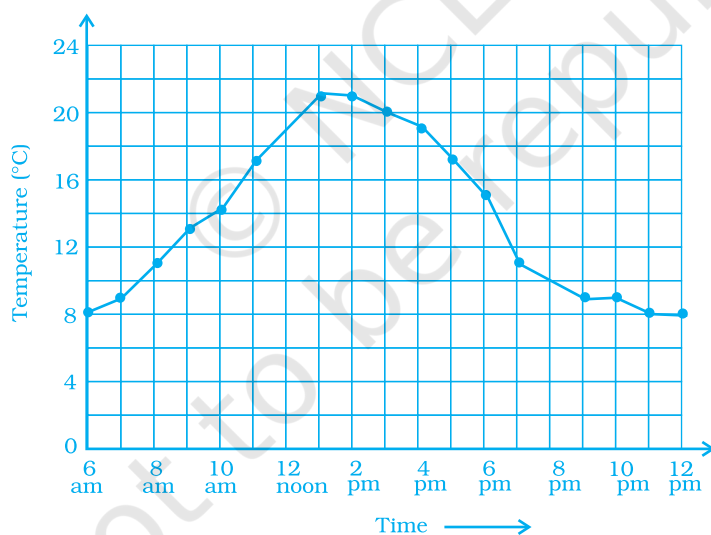
80. This graph shows a map of an island just off the coast of a continent. The point labelled B represents a major city on the coast. The distance between grid lines represents 1 km.



Point A represents a resort that is located 5 km East and 3 km North of Point B. The values 5 and 3 are the coordinates of Point A. The coordinates can be given as the ordered pair (5, 3), where 5 is the horizontal coordinate and 3 is the vertical coordinate.

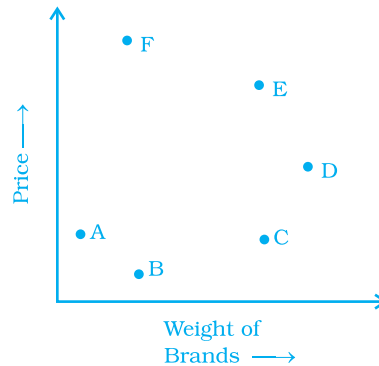
- On a copy of the map, mark the point that is 3 km East and 5 km North of Point B and label it S. Is Point S in the water or on the island? Is Point S in the same place as Point A?
- Mark the point that is 7 km east and 5 km north of Point B and label it C. Then mark the point that is 5 km east and 7 km north of Point B and label it D. Are Points C and D in the same place? Give the coordinates of Points C and D.

- (iii) Which point is in the water, (2, 7) or (7, 2)? Mark the point which is in water on your map and label it E.
 - (iv) Give the coordinates of two points on the island that are exactly 2 km from Point A.
 - (v) Give the coordinates of the point that is halfway between Points L and P.
 - (vi) List three points on the island with their x -coordinates greater than 8.
 - (vii) List three points on the island with a y -coordinate less than 4.
- 81.** As part of his science project, Prithvi was supposed to record the temperature every hour one Saturday from 6 am to midnight. At noon, he was taking lunch and forgot to record the temperature. At 8:00 pm, his favourite show came on and so forgot again. He recorded the data so collected on a graph sheet as shown below.

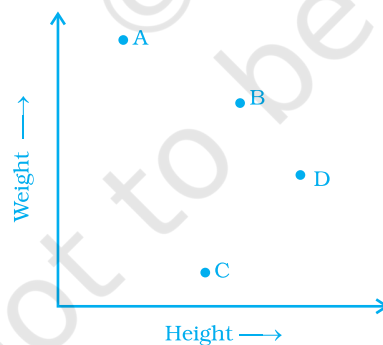


- (a) Why does it make sense to connect the points in this situation?
- (b) Describe the overall trend, or pattern, in the way the temperature changes over the time period shown on the graph.
- (c) Estimate the temperature at noon and 8 pm.

- 82.** The graph given below compares the price (in Rs) and weight of 6 bags (in kg) of sugar of different brands A, B, C, D, E, F.

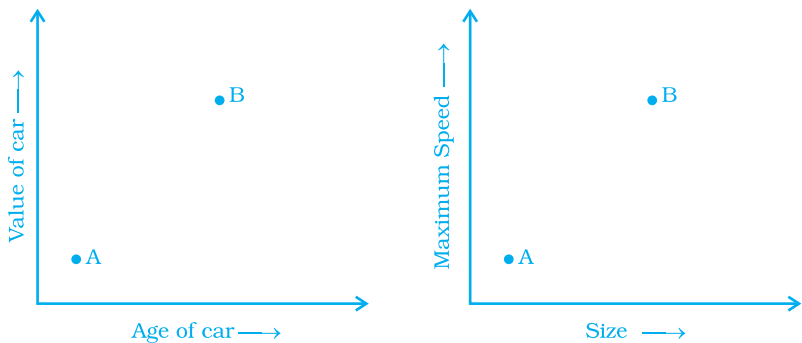


- Which brand(s) costs/cost more than Brand D?
 - Bag of which brand of sugar is the heaviest?
 - Which brands weigh the same?
 - Which brands are heavier than brand B?
 - Which bag is the lightest?
 - Which bags are of the same price?
- 83.** The points on the graph below represent the height and weight of the donkey, dog, crocodile, and ostrich shown in the drawing.



- What are the two variables represented in the graph?
- Which point represents each animals? Explain.

84. The two graphs below compare Car A and Car B. The left graph shows the relationship between age and value. The right graph shows the relationship between size and maximum speed.

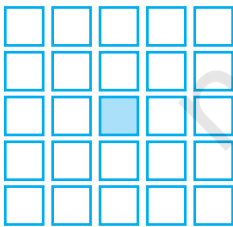


Use the graphs to determine whether each statement is true or false, and explain your answer.

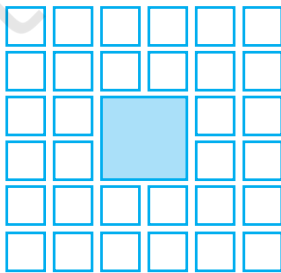
- (a) The older car is less valuable.
 - (b) The faster car is larger.
 - (c) The larger car is older.
 - (d) The faster car is older.
 - (e) The more valuable car is slower.
85. Sonal and Anmol made a sequence of tile designs from square white tiles surrounding one square purple tile. The purple tiles come in many sizes. Three of the designs are shown below.

(a) Copy and complete the table

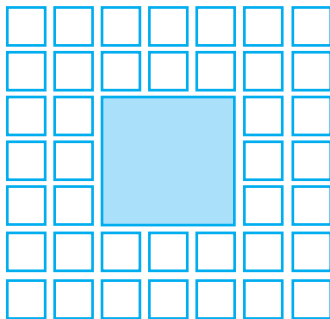
Side Length of Purple Tiles	1	2	3	4	5	10	100
Number of white Tiles in Border							



Side length 1



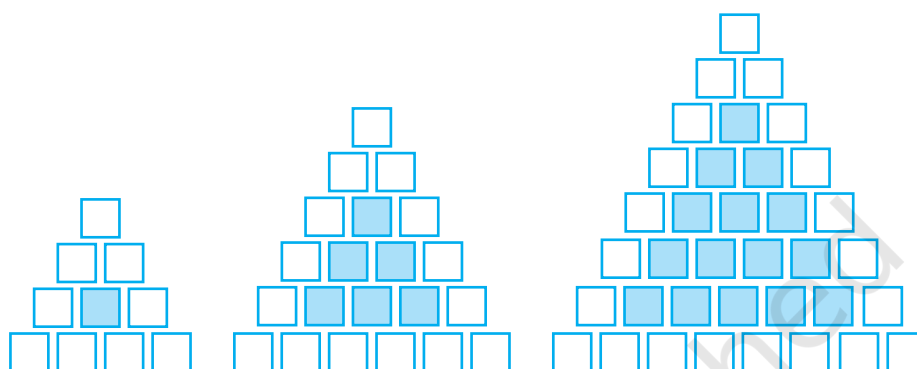
Side length 2



Side length 3

- (b) Draw a graph using the first five pairs of numbers in your table.
 (c) Do the points lie on a line?

86. Sonal and Anmol then made another sequence of the designs. Three of the designs are shown below.



- (a) Complete the table.

Rows, r	4	6	8
Number of white Tiles, w	9		
Number of Purple Tiles, p	1		

- (b) Draw a graph of rows and number of white tiles. Draw another graph of the number of rows and the number of purple tiles. Put the number of rows on the horizontal axis.
 (c) Which graph is linear?

(D) Activities

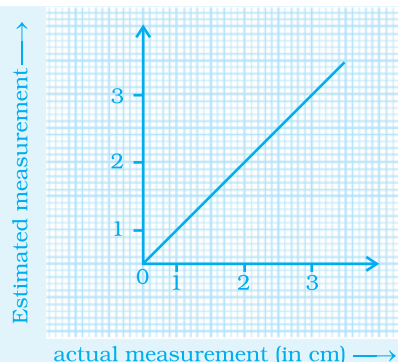
Create a table like the one shown.

Object	Estimate	Actual
	(in cm) (guessed measurement)	Measurement (in cm)
* Length of a pen		
* Length of an eraser		
* Length of your palm		

* Length of your geometry box

* Length of your maths notebook

If an estimate is the same as the actual measurement then the point (actual measurement, estimate) lies on the line, straight line p . For example, if an object measures 5 cm and you estimate it to 5 cm, then the graph of its point lies on line p in the figure below.



Using your completed table,

- (i) Plot the data from the table where the coordinates of the points are (measurement, estimate).
- (ii) Identify the objects overestimated.
- (iii) Identify the objects underestimated.
- (iv) By looking at the graph, how can overestimates and underestimates be determined? How accurate is your estimation?

Activity 2

Clues

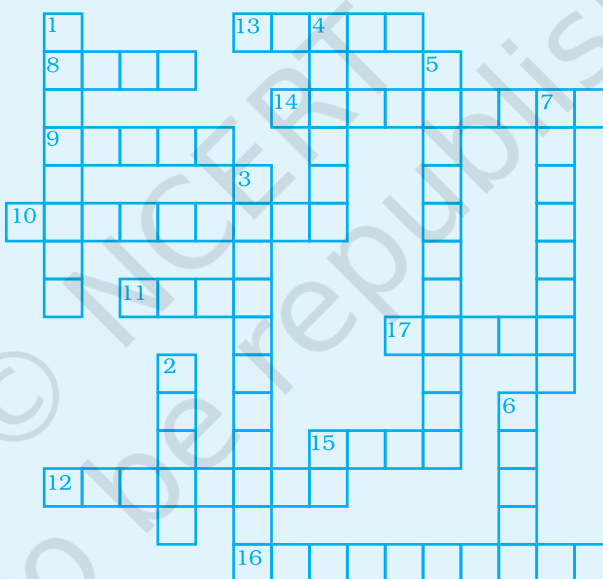
Down

- (1) A graph used to show comparison among categories. (2 words)
- (2) The point (0, 4) lies on the _____.
- (3) A line graph which is a whole unbroken line.
- (4) The point where the two axes meet.
- (5) _____ of a point are required to locate the point on a graph.
- (6) The x -axis and y -axis are at _____ angles to each other.
- (7) x -coordinate of a point.

Across

- (8) The plural of Axis.
- (9) The sheet of paper on which coordinates of any given point are plotted.

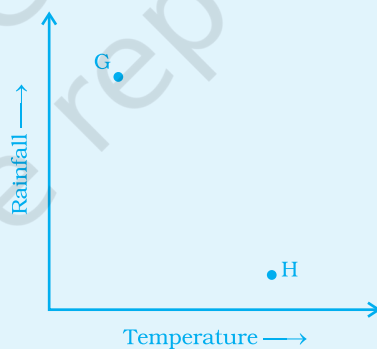
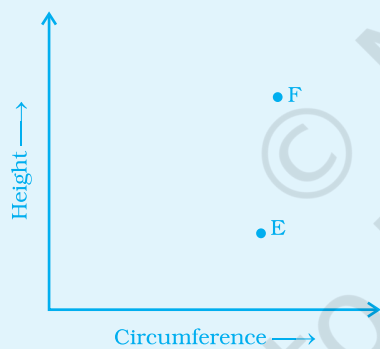
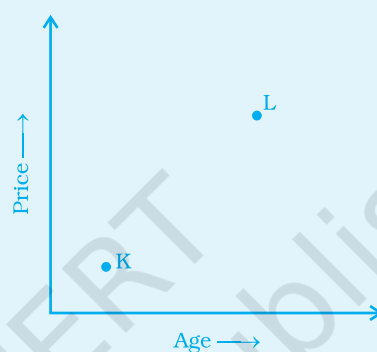
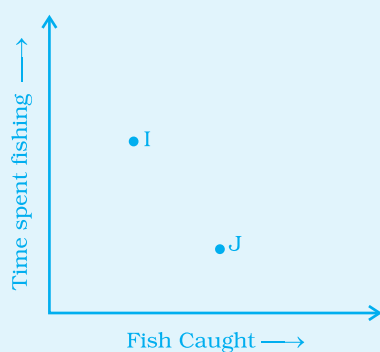
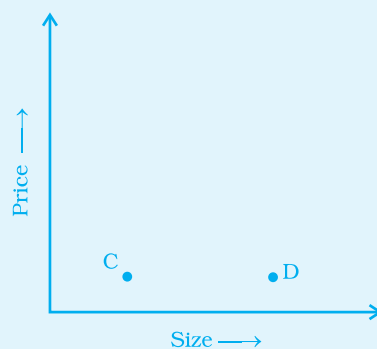
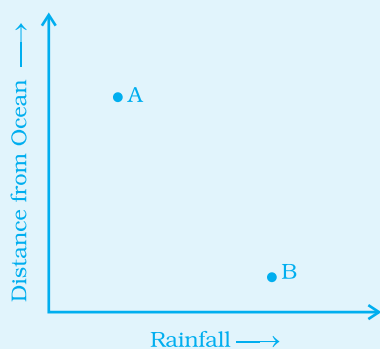
- (10) The system of fixing points on a graph with the help of coordinates.
- (11) A _____ graph displays the data that changes continuously over time.
- (12) y -coordinate of a point.
- (13) A pie chart is used to compare parts of a _____.
- (14) A bar graph that shows data in intervals.
- (15) In a histogram there are no _____ between the bars.
- (16) The x -axis is a _____ line on a plane.
- (17) y -coordinate of a point represents the distance of the point from the _____.



Activity 3

Complete Parts (a) and (b) for each following graphs.

- (a) Tell what two variables does the graph show.
- (b) Describe what the graph tells you about the things represented by the points. Then try to come up with an idea about what the points could represent.



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UNIT 13

PLAYING WITH NUMBERS

(A) Main Concepts and Results

- Numbers can be written in general form. For example, a two digit number ab is written as $ab = 10a + b$; a three digit number abc is written as $abc = 100a + 10b + c$.
- The general form of numbers are helpful in solving various problems related to numbers.
- Rationale for the divisibility of numbers by 11, 10, 5, 2, 9 or 3 can be explained by writing the numbers in general form.
- Many number puzzles involving different letters for different digits are solved using rules of number operations.

(B) Solved Examples

In examples 1 to 4, out of four options only one is correct. Write the correct answer.

Example 1 : Generalised form of a three-digit number xyz is

- (a) $x + y + z$ (b) $100x + 10y + z$
(c) $100z + 10y + x$ (d) $100y + 10x + z$

Solution : The correct answer is (b).

Example 2 : The usual form of $100a + b + 10c$ is

- (a) abc (b) cab (c) bac (d) acb

Solution : The correct answer is (d).

Example 3 : If $5 \times A = CA$ then the values of A and C are

(a) $A = 5, C = 1$

(b) $A = 4, C = 2$

(c) $A = 5, C = 2$

(d) $A = 2, C = 5$

Solution : The correct answer is (c).

Example 4 : If $5A + 25$ is equal to $B2$, then the value of $A + B$ is

(a) 15

(b) 10

(c) 8

(d) 7

Solution : The correct answer is (a).

In examples 5 to 7, fill in the blanks to make the statements true.

Example 5 : The number $ab - ba$ where a and b are digits and $a > b$ is divisible by _____.

Solution : 9.

Example 6 : When written in usual form $100a + 10c + 9$ is equal to _____.

Solution : $ac9$

Example 7 : If $AB \times B = 9B$, then $A = \underline{\hspace{2cm}}$, $B = \underline{\hspace{2cm}}$.

Solution : 9, 1

In examples 8 to 10, state whether the statements are true (T) or false (F).

Example 8 : If abc, cab, bca are three digit numbers formed by the digits a, b , and c then the sum of these numbers is always divisible by 37.

Solution : True.

Example 9 : Let ab be a two-digit number, then $ab + ba$ is divisible by 9.

Solution : False.

Example 10 : If a number is divisible by 2 and 4, then it will be divisible by 8.

Solution : False.

Example 11 : A three-digit number $42x$ is divisible by 9. Find the value of x .

Solution : Since $42x$ is divisible by 9, the sum of its digits, i.e. $4 + 2 + x$ must be divisible by 9.

i.e. $6 + x$ is divisible by 9

i.e. $6 + x = 9$ or 18 , ____.

Since x is a digit, therefore $6 + x = 9$ or, $x = 3$.

Example 12 : Find the value of A and B if

$$\begin{array}{r} 41A \\ + B4 \\ \hline 512 \end{array}$$

Solution : From ones column $A + 4$ gives a number whose ones digit is 2. So, $A = 8$. The value of B can be obtained by solving $2 + B$ is a number whose ones digit is 1. So, $B = 9$.

$$\begin{array}{r} 418 \\ + 94 \\ \hline 512 \end{array}$$

Example 13 : Suppose that the division $x \div 5$ leaves a remainder 4 and the division $x \div 2$ leaves a remainder 1. Find the ones digit of x .

Solution : Since $x \div 5$ leaves a remainder 4, so ones digit of x can be 4 or 9. Also, since $x \div 2$ leaves a remainder 1, so ones digit must be 9 only.

Application on Problem Solving Strategy



Example 14 :

If $756x$ is divisible by 11, where x is a digit find the value of x .



Understand and Explore the problem

- What is given in the question?
A four digit number $756x$ is divisible by 11.
- Which property is required to solve the problem?
Divisibility of a number by 11.



Plan a Strategy

- Find the sum of the digits of given number $756x$ at odd places.
- Find the sum of the digits of $756x$ at even places.
- Find the difference of step 1 and step 2.



Solve

- Given $y = 2x$
- Sum of digits at odd places = $x + 5$
- Sum of digits at even places = $6 + 7 = 13$
- Difference = $(x + 5) - 13$
 $= x - 8$

Now $(x - 8)$ should be equal to 0 or a multiple of 11 (i.e. 11, 22, 33, ..., etc.)

$$x - 8 = 0$$

$$x = 8 \text{ or } x - 8 = 11$$

$$x = 11 + 8 = 19$$

- Since x is a digit so it can take values from 0 – 9
Hence $x = 8$

Required number is 7568.



Revise

- 7568
Sum of digits at odd places = $5 + 8 = 13$
Sum of digits at even places = $6 + 7 = 13$
Difference = $13 - 13$
 $= 0$
So Value of x is correct.

Think and Discuss



1. What would be the value of y , if $277y$ is divisible by 11?

(C) Exercise

In each of the questions 1 to 17, out of the four options, only one is correct. Write the correct answer.

1. Generalised form of a four-digit number $abdc$ is
 - (a) $1000 a + 100 b + 10 c + d$
 - (b) $1000 a + 100 c + 10 b + d$
 - (c) $1000 a + 100 b + 10 d + c$
 - (d) $a \times b \times c \times d$

2. Generalised form of a two-digit number xy is
(a) $x + y$ (b) $10x + y$ (c) $10x - y$ (d) $10y + x$
3. The usual form of $1000a + 10b + c$ is
(a) abc (b) $abco$ (c) $aobc$ (d) $aboc$
4. Let abc be a three-digit number. Then $abc - cba$ is not divisible by
(a) 9 (b) 11 (c) 18 (d) 33
5. The sum of all the numbers formed by the digits x, y and z of the number xyz is divisible by
(a) 11 (b) 33 (c) 37 (d) 74
6. A four-digit number $aabb$ is divisible by 55. Then possible value(s) of b is/are
(a) 0 and 2 (b) 2 and 5 (c) 0 and 5 (d) 7
7. Let abc be a three digit number. Then $abc + bca + cab$ is not divisible by
(a) $a + b + c$ (b) 3 (c) 37 (d) 9
8. A four-digit number $4ab5$ is divisible by 55. Then the value of $b - a$ is
(a) 0 (b) 1 (c) 4 (d) 5
9. If abc is a three digit number, then the number $abc - a - b - c$ is divisible by
(a) 9 (b) 90 (c) 10 (d) 11
10. A six-digit number is formed by repeating a three-digit number. For example 256256, 678678, etc. Any number of this form is divisible by
(a) 7 only (b) 11 only (c) 13 only (d) 1001
11. If the sum of digits of a number is divisible by three, then the number is always divisible by
(a) 2 (b) 3 (c) 6 (d) 9
12. If $x + y + z = 6$ and z is an odd digit, then the three-digit number xyz is
(a) an odd multiple of 3 (b) odd multiple of 6
(c) even multiple of 3 (d) even multiple of 9

- 13.** If $5A + B3 = 65$, then the value of A and B is
 (a) $A = 2, B = 3$ (b) $A = 3, B = 2$
 (c) $A = 2, B = 1$ (d) $A = 1, B = 2$
- 14.** If $A3 + 8B = 150$, then the value of $A + B$ is
 (a) 13 (b) 12 (c) 17 (d) 15
- 15.** If $5A \times A = 399$, then the value of A is
 (a) 3 (b) 6 (c) 7 (d) 9
- 16.** If $6A \times B = A8B$, then the value of $A - B$ is
 (a) -2 (b) 2 (c) -3 (d) 3
- 17.** Which of the following numbers is divisible by 99
 (a) 913462 (b) 114345 (c) 135792 (d) 3572406

In questions 18 to 33, fill in the blanks to make the statements true.

- 18.** 3134673 is divisible by 3 and _____.
- 19.** $20x3$ is a multiple of 3 if the digit x is _____ or _____ or _____.
- 20.** $3x5$ is divisible by 9 if the digit x is _____.
- 21.** The sum of a two-digit number and the number obtained by reversing the digits is always divisible by _____.
- 22.** The difference of a two-digit number and the number obtained by reversing its digits is always divisible by _____.
- 23.** The difference of three-digit number and the number obtained by putting the digits in reverse order is always divisible by 9 and _____.

24. If
$$\begin{array}{r} 2 \quad B \\ + A \quad B \\ \hline 8 \quad A \end{array}$$
 then $A = \underline{\hspace{1cm}}$ and $B = \underline{\hspace{1cm}}$.

25. If
$$\begin{array}{r} A \quad B \\ \times B \\ \hline 9 \quad 6 \end{array}$$
 then $A = \underline{\hspace{1cm}}$ and $B = \underline{\hspace{1cm}}$.

26. If
$$\begin{array}{r} B \quad 1 \\ \times B \\ \hline 4 \quad 9B \end{array}$$
 then $B = \underline{\hspace{1cm}}$.

27. 1×35 is divisible by 9 if $x = \underline{\hspace{2cm}}$.
28. A four-digit number $abcd$ is divisible by 11, if $d + b = \underline{\hspace{2cm}}$ or $\underline{\hspace{2cm}}$.
29. A number is divisible by 11 if the differences between the sum of digits at its odd places and that of digits at the even places is either 0 or divisible by $\underline{\hspace{2cm}}$.
30. If a 3-digit number abc is divisible by 11, then $\underline{\hspace{2cm}}$ is either 0 or multiple of 11.
31. If $A \times 3 = 1A$, then $A = \underline{\hspace{2cm}}$.
32. If $B \times B = AB$, then either $A = 2, B = 5$ or $A = \underline{\hspace{2cm}}, B = \underline{\hspace{2cm}}$.
33. If the digit 1 is placed after a 2-digit number whose tens is t and ones digit is u , the new number is $\underline{\hspace{2cm}}$.

State whether the statements given in questions 34 to 44 are true (T) or false (F):

34. A two-digit number ab is always divisible by 2 if b is an even number.
35. A three-digit number abc is divisible by 5 if c is an even number.
36. A four-digit number $abcd$ is divisible by 4 if ab is divisible by 4.
37. A three-digit number abc is divisible by 6 if c is an even number and $a + b + c$ is a multiple of 3.
38. Number of the form $3N + 2$ will leave remainder 2 when divided by 3.
39. Number $7N + 1$ will leave remainder 1 when divided by 7.
40. If a number a is divisible by b , then it must be divisible by each factor of b .
41. If $AB \times 4 = 192$, then $A + B = 7$.
42. If $AB + 7C = 102$, where $B \neq 0, C \neq 0$, then $A + B + C = 14$.
43. If $213x27$ is divisible by 9, then the value of x is 0.
44. If $N \div 5$ leaves remainder 3 and $N \div 2$ leaves remainder 0, then $N \div 10$ leaves remainder 4.

Solve the following :

45. Find the least value that must be given to number a so that the number $91876a2$ is divisible by 8.

46. If $\begin{array}{r} 1 \ P \\ \times \ P \\ \hline Q \ 6 \end{array}$ where $Q - P = 3$, then find the values of P and Q.
47. If $1AB + CCA = 697$ and there is no carry-over in addition, find the value of $A + B + C$.
48. A five-digit number AABAA is divisible by 33. Write all the numbers of this form.
49. Find the value of the letters in each of the following questions.

$$\begin{array}{r} A \ A \\ +A \ A \\ \hline XA \ Z \end{array}$$

50.
$$\begin{array}{r} 8 \ 5 \\ +4 \ A \\ \hline B \ C \ 3 \end{array}$$

51.
$$\begin{array}{r} B \ 6 \\ +8 \ A \\ \hline C \ A \ 2 \end{array}$$

52.
$$\begin{array}{r} 1 \ B \ A \\ + \ A \ B \ A \\ \hline 8 \ A \ 2 \end{array}$$

53.
$$\begin{array}{r} C \ B \ A \\ +C \ B \ A \\ \hline 1 \ A \ 3 \ 0 \end{array}$$

54.
$$\begin{array}{r} B \ A \ A \\ +B \ A \ A \\ \hline 3 \ A \ 8 \end{array}$$

55.
$$\begin{array}{r} A \ 0 \ 1 \ B \\ +1 \ 0 \ A \ B \\ \hline B \ 1 \ 0 \ 8 \end{array}$$

56.
$$\begin{array}{r} A \ B \\ \times \ 6 \\ \hline C \ 6 \ 8 \end{array}$$

57.
$$\begin{array}{r} A \ B \\ \times \ A \ B \\ \hline 6 \ A \ B \end{array}$$

58.
$$\begin{array}{r} A \ A \\ \times \ A \\ \hline C \ A \ B \end{array}$$

- and $B - A = 1$
61. If $2A7 \div A = 33$, then find the value of A.

59.
$$\begin{array}{r} A \ B \\ -B \ 7 \\ \hline 4 \ 5 \end{array}$$

60.
$$\begin{array}{r} 8 \ A \ B \ C \\ -A \ B \ C \ 5 \\ \hline D \ 4 \ 8 \ 8 \end{array}$$

62. 212×5 is a multiple of 3 and 11. Find the value of x.
63. Find the value of k where $31k2$ is divisible by 6.
64. $1y3y6$ is divisible by 11. Find the value of y.
65. $756 \times$ is a multiple of 11, find the value of x.
66. A three-digit number $2a3$ is added to the number 326 to give a three-digit number $5b9$ which is divisible by 9. Find the value of $b - a$.

67. Let $E = 3$, $B = 7$ and $A = 4$. Find the other digits in the sum

$$\begin{array}{r} B A S E \\ + B A L L \\ \hline G A M E S \end{array}$$

68. Let $D = 3$, $L = 7$ and $A = 8$. Find the other digits in the sum

$$\begin{array}{r} M A D \\ + A S \\ + A \\ \hline B U L L \end{array}$$

69. If from a two-digit number, we subtract the number formed by reversing its digits then the result so obtained is a perfect cube. How many such numbers are possible? Write all of them.
70. Work out the following multiplication.

$$\begin{array}{r} 12345679 \\ \times 9 \\ \hline \\ \hline \end{array}$$

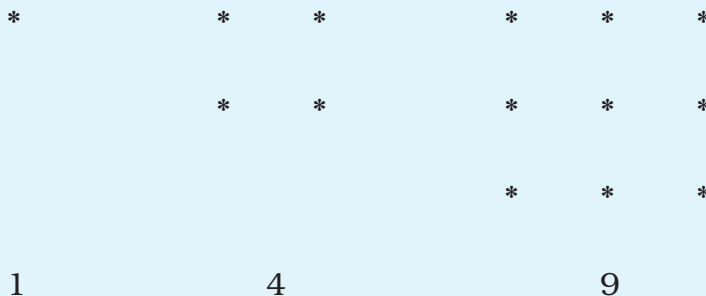
Use the result to answer the following questions.

- (a) What will be 12345679×45 ?
- (b) What will be 12345679×63 ?
- (c) By what number should 12345679 be multiplied to get 888888888?
- (d) By what number should 12345679 be multiplied to get 999999999?
71. Find the value of the letters in each of the following:
- (i)
$$\begin{array}{r} P Q \\ \times 6 \\ \hline Q Q Q \end{array}$$
- (ii)
$$\begin{array}{r} 2 L M \\ + L M 1 \\ \hline M 1 8 \end{array}$$
72. If 148101B095 is divisible by 33, find the value of B.
73. If 123123A4 is divisible by 11, find the value of A.
74. If $56x32y$ is divisible by 18, find the least value of y .

(D) Application, Games and Puzzles

1. Polygonal Numbers

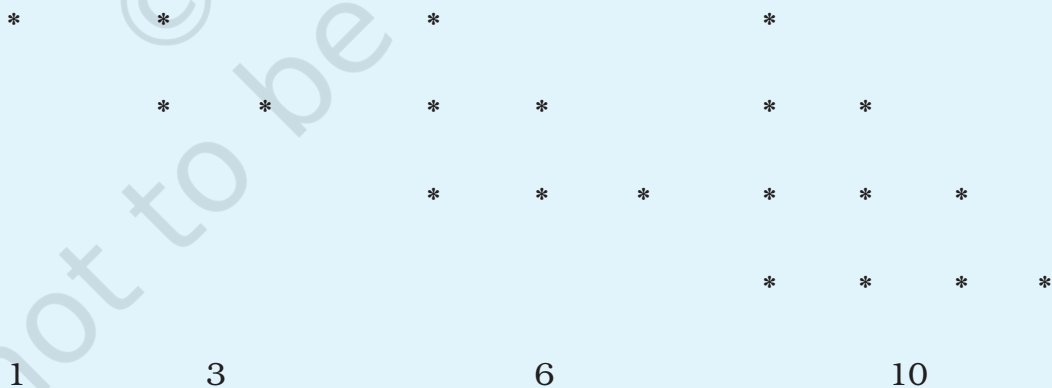
Study the patterns given below and extend it. We already know about square numbers.



Draw two more.

Here for the first square number, use 1^2 ; for the second square number, use 2^2 . To find the third square number use 3^2 and so on. Write the n th square number.

Now let's move to triangular numbers.

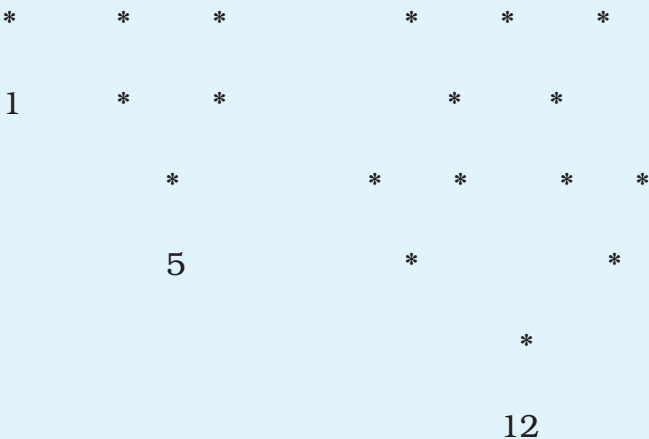


Find the next triangular number.

To find the n th triangular number we use the formula $\frac{n \times (n+1)}{2}$

Are you familiar with pentagonal numbers?

First three are given to you. Write the next one

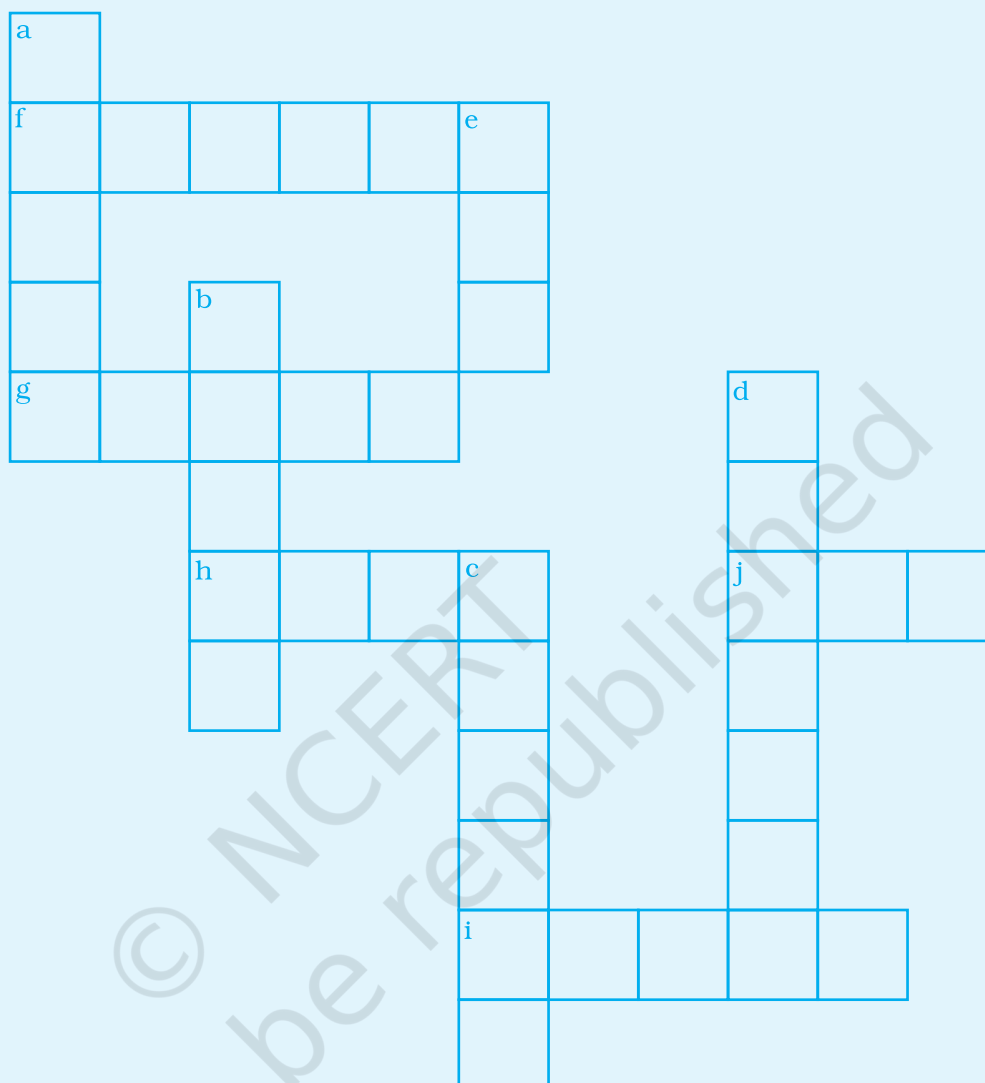


Draw the dot patterns for the next pentagonal number. Count the number of dots inside the entire shape and write the number under the shape.

2. Put tick mark in the appropriate boxes if the given numbers are divisible by any of 2, 3, 4, 5, 6, 8, 10, 11 numbers.

S.No.	Number	Divisible by									
		2	3	4	5	6	7	8	9	10	11
1.	40185										
2.	92286										
3.	56390										
4.	419562										
5.	10593248										

3. Cross Number Puzzle



Fill in the blank spaces in the cross number puzzle using following clues.

Down

- (a) 59 _____ $63 \div 33$
- (b) 81 _____ $42 \div 6$
- (c) 7 _____ $6988 \div 11$
- (d) 37604 _____ $5 \div 15$

(e) $56 \underline{\hspace{1cm}} \div 10$

Across

(f) $90 \underline{\hspace{1cm}} 815 \div 15$

(g) $3514 \underline{\hspace{1cm}} \div 12$

(h) $4 \underline{\hspace{1cm}} 07 \div 7$

(i) $8 \underline{\hspace{1cm}} 558 \div 6$

(j) $6 \underline{\hspace{1cm}} 5 \div 55$

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ANSWERS

Unit 1

1. (d) 2. (b) 3. (a) 4. (d) 5. (a) 6. (a)
 7. (a) 8. (c) 9. (b) 10. (b) 11. (a) 12. (c)
 13. (b) 14. (d) 15. (a) 16. (a) 17. (b) 18. (d)
 19. (d) 20. (a) 21. (a) 22. (a) 23. (d) 24. (a)
 25. (b) 26. $\frac{45}{63}$ 27. $\frac{35}{45}$ 28. $\frac{35}{40}$
 29. positive rational number 30. negative rational number
 31. no 32. 1, -1 33. x^2 34. $-\frac{45}{8}$ or $-5\frac{5}{8}$
 35. $(657)^{-1}$ 36. -1 37. $\frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$
 38. more 39. infinitely many 40. opposite
 41. positive 42. order 43. $-\frac{7}{5}$ 44. $\frac{3}{4}$
 45. $\frac{1011}{100}$ 46. $\frac{1}{5} \times \frac{3}{8}$ 47. -3 -4 48. False
 49. False 50. True 51. True 52. False 53. True 54. True
 55. True 56. False 57. False 58. True 59. False 60. False
 61. False 62. False 63. True 64. True 65. False 66. False
 67. True 68. False 69. False 70. False 71. False 72. False
 73. False 74. False 75. True 76. False 77. False 78. False

79. False 80. True 81. False 82. False 83. True 84. False
 85. False 86. False 87. False 88. True 89. True 90. True
 91. True 92. True 93. True 94. False 95. True 96. True
 97. False 98. True 99. True
100. $\frac{8}{4}, \frac{9}{3}, \frac{6}{3}, \frac{4}{2}, \frac{1}{1}, \frac{0}{1}, \frac{-1}{1}, \frac{-2}{1}, \frac{-4}{2}, \frac{-6}{2}$ 101. $\frac{64}{16}, \frac{36}{-12}, \frac{5}{-4}, \frac{140}{28}$
102. a) $\frac{-8}{9}$ b) $\frac{-256}{35}$ 106. (a) $\frac{25}{8}$ (b) $\frac{-4}{75}$ (c) $\frac{17}{70}$ (distributive law)
107. Associative property
111. (a) $6\frac{7}{8}$ (b) $-3\frac{1}{3}$ (c) $\frac{-11}{8}$ or $-1\frac{3}{8}$ (d) $\frac{-88}{3}$ or $-29\frac{1}{3}$
112. (a) $\frac{142}{15}$ or $9\frac{7}{15}$ (b) $\frac{2}{7}$ (c) $\frac{32}{63}$ (d) $\frac{41}{48}$
113. $\frac{-7}{3}$ as it is smaller than -1 whereas rest of the numbers are greater than -1
114. Rs 18 115. 85 km/h 116. $\frac{3}{2}$ m or 1.5m
117. Rs. 77,000 118. 16 pieces 119. 28 120. 1920
121. Rs 864, Rs 720, Rs 432
122. Rs 32,000, Rs 12,000, Rs 16,000
123. Associative and commutative property.
124. (i) Commutative property.
 (ii) Distributive property of multiplication over addition.
 (iii) Associative property.
 (iv) Additive identity of rational number.
 (v) Multiplicative identity of rational number.
125. (i) $\frac{-8}{9}$ (ii) $\frac{3}{10}$ 126. $\frac{13}{16} > \frac{5}{8} > \frac{1}{4}$
127. $\frac{-2}{3}$ 128. $\frac{20}{21}$ 129. -39 130. $\frac{7}{5}$
131. No. 132. $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$ 133. $\frac{1}{5}$ and $\frac{-1}{5}$

134. 12 135. $\frac{11}{24}$ m 136. $\frac{8}{7} > \frac{2}{5} > 0 > \frac{-9}{8} > \frac{-3}{2}$
137. (i) 0 (ii) $\frac{1}{2}$ 138. 3.2°F 139. $\frac{-48}{7}$ or $-6\frac{6}{7}$
140. -1 141. a) $\frac{19}{10}$ m b) $\frac{209}{100}$ m
142. 7; $\frac{75}{32}$ sqm or $2\frac{11}{32}$ sqcm 143. $\frac{3}{8}$ cup
144. a) $\frac{3}{160}$ km b) $\frac{13}{200}$ km c) Nancy
145. a) $58\frac{1}{2}$ km b) $117\frac{1}{3}$ km
146. (a) Less than (b) Paper Glass
(c) More $\frac{1}{2}$ (d) Paper > Glass > Scrap > Aluminium cans
147. $97\frac{7}{25}$ cm, $98\frac{4}{9}$ cm, $98\frac{1}{25}$ cm, $97\frac{47}{50}$ cm
 $97\frac{7}{25}$ cm < $97\frac{47}{50}$ cm < $98\frac{1}{25}$ cm < $98\frac{4}{9}$ cm 148. $\frac{2}{5}$ m
149. May : $2\frac{1731}{2500}$, June : $\frac{381}{625}$, July : $-6\frac{568}{625}$, August : $-8\frac{159}{250}$
150. AP : $\frac{616}{10} = \frac{308}{5}$, Assam : $\frac{571}{10}$, Bihar : $\frac{607}{10}$,
Gujarat : $\frac{619}{10}$, Haryana : $\frac{641}{10}$, HP : $\frac{651}{10}$,
Karnataka : $\frac{624}{10} = \frac{312}{5}$, Kerala : $\frac{706}{10} = \frac{353}{5}$, MP : $\frac{565}{10} = \frac{113}{2}$,
Maharashtra : $\frac{645}{10} = \frac{129}{2}$, Orissa : $\frac{576}{10} = \frac{288}{5}$, Punjab : $\frac{669}{10}$

Rajasthan : $\frac{598}{10} = \frac{299}{5}$, Tamil Nadu : $\frac{637}{10}$, U.P. : $\frac{589}{10}$,

West Bengal : $\frac{628}{10} = \frac{314}{5}$

Kerala; Punjab; HP; Maharashtra; Haryana; Tamil Nadu; West Bengal; Karnataka; Gujarat; Andhra Pradesh; Bihar; Rajasthan; UP; Orissa; Assam; MP.

152. 39 cm 153. Manavi : Rs 315, Kuber : Rs 84

(D) Games and Puzzles

1.

$\frac{32}{38}$	$\frac{18}{38}$	$\frac{4}{38}$	$\frac{-14}{-38}$
$\frac{-18}{-57}$	$\frac{-21}{-133}$	$\frac{24}{38}$	$\frac{104}{152}$
$\frac{22}{38}$	$\frac{70}{95}$	$\frac{25}{95}$	$\frac{-20}{-95}$
$\frac{1}{19}$	$\frac{-16}{-38}$	$\frac{45}{57}$	$\frac{60}{114}$

2.

Down 1: Rational

Down 3: Commutative

Down 5: Indefinitely

Down 7: 1

Across 1: infinite

Across 3: Multiplication

Across 5: Not defined

Down 2: Additive

Down 4: Reciprocal (or Inverse)

Down 6: Division

Down 8: Number

Across 2: Associative

Across 4: Natural

Across 6: Inverse

3. Riddle

S.No. Lowest Term

Word

(1) $\frac{-2}{5}$

S(P)IN

(2) $\frac{2}{3}$

T(Y)PE

(3) $\frac{3}{4}$

WI(T)H

S.No. Lowest Term

Word

(4) $\frac{-1}{3}$

(H)OST

(5) $\frac{3}{10}$

SH(A)RP

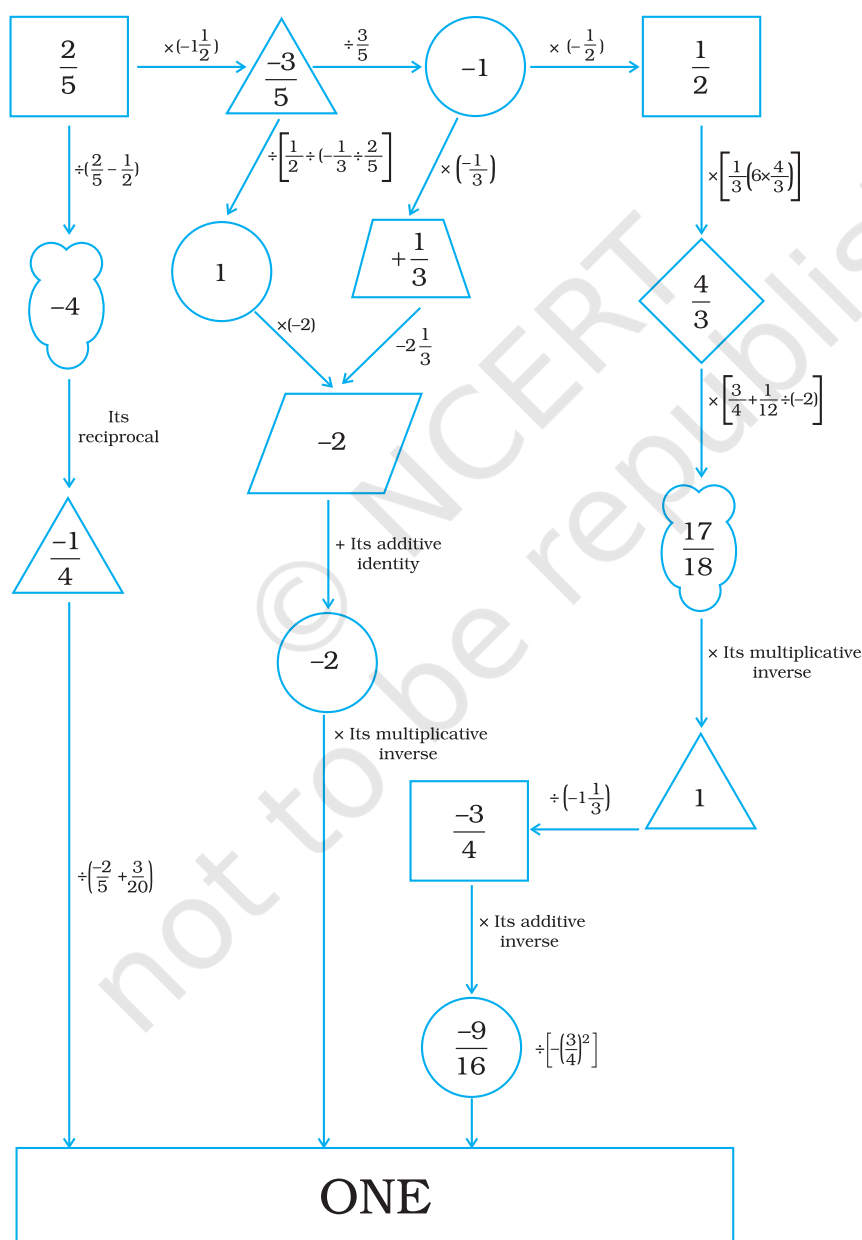
(6) $\frac{1}{5}$

(G)AIN

3. Riddle

S.No.	Lowest Term	Word	S.No.	Lowest Term	Word				
(7)	$\frac{4}{5}$	PRO O F	(9)	$\frac{1}{4}$	A WAY				
(8)	$\frac{-1}{2}$	R AIN	(10)	$\frac{-1}{3}$	S WEET				
<u>P</u>	<u>Y</u>	<u>T</u>	<u>H</u>	<u>A</u>	<u>G</u>	<u>O</u>	<u>R</u>	<u>A</u>	<u>S</u>
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

4.



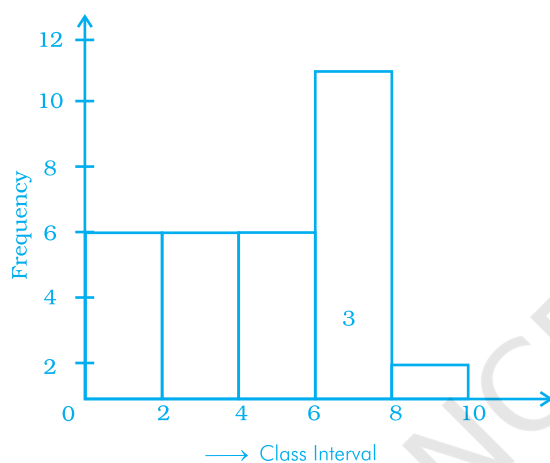
Unit 2

1. (d) 2. (a) 3. (b) 4. (c) 5. (d) 6. (b)
7. (b) 8. (c) 9. (c) 10. (d) 11. (b) 12. (c)
13. (d) 14. (d) 15. (b) 16. (c) 17. (b) 18. (a)
19. (a) 20. (d) 21. (d) 22. (b) 23. (b) 24. (d)
25. (d) 26. (c) 27. (d) 28. (d) 29. (c) 30. (d)
31. (c) 32. (b) 33. (d) 34. (d) 35. (c) 36. Raw
37. 20 38. Upper class limit 39. 19 40. Parts
41. Head, tail 42. 1, 2, 3, 4, 5, 6 43. event
44. Random 45. Size/Width 46. 35-40
47. 40 48. 8 49. 22 50. 14 51. Frequency
52. Class Intervals 53. 2 54. 5 55. Bars 56. likely
57. X, Y 58. 20-30 59. True 60. False 61. True 62. True
63. True 64. True 65. True 66. False 67. True 68. True
69. False 70. False 71. True 72. False 73. True 74. False
75. False 76. True 77. False 78. False 79. True 80. False
81. False 82. a) 20 b) 60 c) 4 d) 20-30 e) 30-40 f) 10
84. a) 329 b) 168 c) 301 d) 2 hours or more
85. a) Bus b) $\frac{1}{4}$ c) 72 d) 6 e) car and Walk
86. a) $\frac{1}{2}$ b) $\frac{1}{6}$ c) $\frac{2}{6}$ or $\frac{1}{3}$ d) 0 e) $\frac{5}{6}$ f) $\frac{4}{6}$ or $\frac{2}{3}$
87. a) Certain to happen (b) May or may not happen
c) Certain to happen (d) Impossible to happen
e) Impossible to happen (f) May or may not happen
88. Mathematics 180, English 135, Social Science 30
Science 105, Hindi 90 89. 28
90. (a) 42 (b) 150-155 (c) 5 (d) 28

91.

Class interval	Tally marks	Frequency
0 – 2		0
2 – 4		6
4 – 6		6
6 – 8		11
8 – 10		2
	Total	25

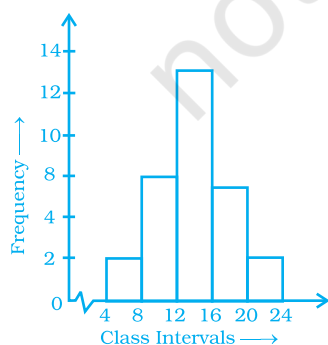
92.



93.

Class interval	Tally marks	Frequency
4 – 8		2
8 – 12		8
12 – 16		13
16 – 20		5
20 – 24		2
	Total	30

94.



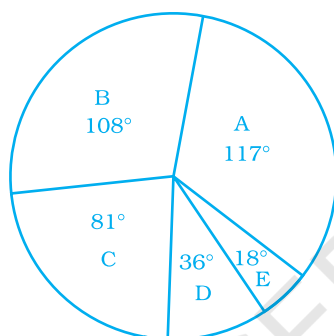
95.

Class interval	Tally marks	Frequency
25 – 30		2
30 – 35		8
35 – 40		10
40 – 45		7
45 – 50		3
	Total	30

a) 25 - 30

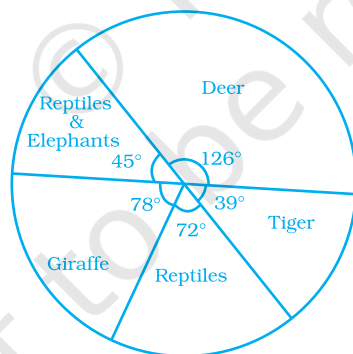
b) 35 - 40

96.



97. (i) 1 Crore
(ii) 2.5 times
(iii) $\frac{3}{10}$

98.



99. (a) $\frac{1}{8}$
(b) $\frac{4}{16}, \frac{4}{16}$

100.

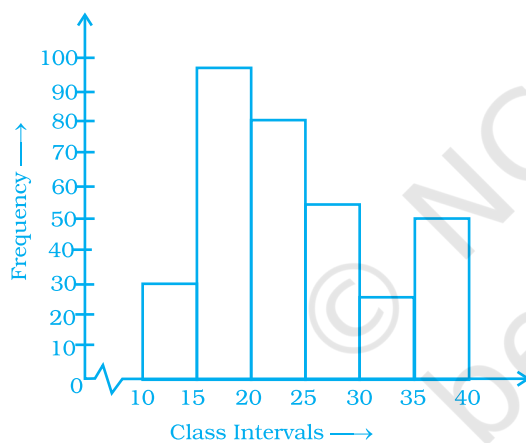
Class interval	Tally marks	Frequency
30 – 35		3
35 – 40		3
40 – 45		3
45 – 50		3
50 – 55	≡	5
55 – 60		4
60 – 65	≡	5
65 – 70		2
70 – 75	≡	7
	Total	35

a) 9

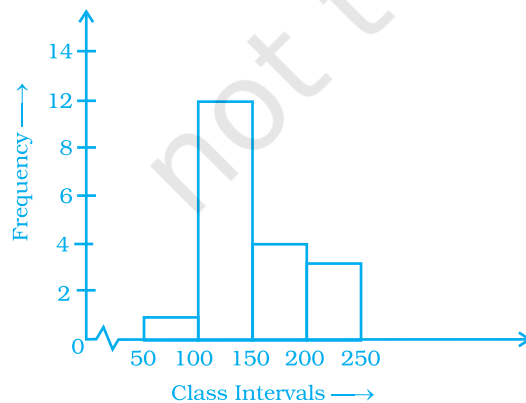
b) 70 – 75

101. 12, 14, 06, 2, 1

102.

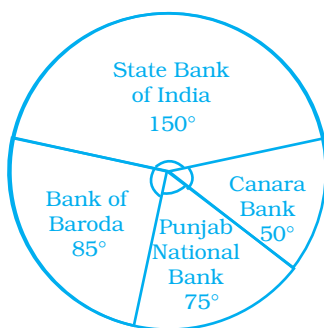


103.

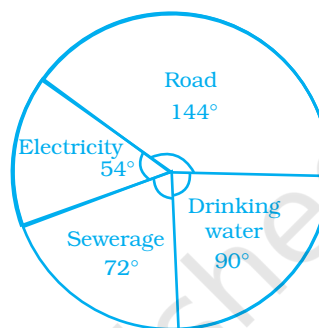


104. a) 10-15, 15-20, 20-25, 25-30, 30-35, 35-40
 b) 5
 c) 10-15
 d) 15-20
105. a) 5
 b) maximum experience 2, minimum experiences 5
 c) 9

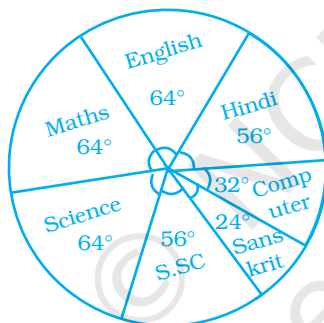
106.



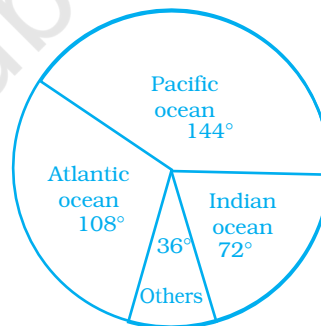
107.



108.



109.



110. (i) Cold drinks
 (ii) 300

111. a) $\frac{1}{4}$ b) $\frac{3}{8}$ c) $\frac{7}{8}$
112. a) $\frac{1}{2}$ b) $\frac{3}{10}$ c) $\frac{1}{10}$ d) 0

113. a) 32%  b) 28%  c) 22%  d) 18% 

114. a) 32%  Red b) 38%  Yellow c) 30%  Pink

115. Housing	-Rs 15,000
Food	-Rs 10,000
Car loan	-Rs 12,500
Utilities	-Rs 5,000
Phone	-Rs 2,500
Clothing	-Rs 2,500
Entertainment	-Rs 2,500

- 116.** a) Newspaper
 b) Radio
 c) 39%
 d) 63%
 e) Internet, Webmedia

(D) Application, Games and Puzzles

(I)	K	Q	J	10	9	8	7	6	5	4	3	2	A	Total
Spade	1	1	1	1	1	1	1	1	1	1	1	1	1	13
Heart	1	1	1	1	1	1	1	1	1	1	1	1	1	13
Diamond	1	1	1	1	1	1	1	1	1	1	1	1	1	13
Club	1	1	1	1	1	1	1	1	1	1	1	1	1	13
														52

1) 2 2) 52 3) 13 4) 4 Spade, Heart, Diamond, Club

5) 26 6) 26 7) 3 of each type 8) 12

9) (i) $\frac{6}{52}$ or $\frac{3}{26}$ (ii) $\frac{2}{52}$ or $\frac{1}{26}$ (iii) $\frac{1}{52}$

(iv) $\frac{12}{52}$ or $\frac{6}{26}$ or $\frac{3}{13}$ (v) $\frac{2}{52}$ or $\frac{1}{26}$ (vi) $\frac{2}{52}$ or $\frac{1}{26}$

(vii) 1 (viii) $\frac{3}{52}$ (ix) $\frac{1}{52}$

(x) $\frac{4}{52}$ or $\frac{1}{13}$ (xi) $\frac{13}{52}$ or $\frac{1}{4}$ (xii) $\frac{1}{2}$

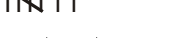
(II) (a)

Outcomes	Sum	Outcomes	Sum	Outcomes	Sum
(1, 5)	6	(4, 1)	5	(6, 3)	9
(1, 6)	7	(4, 2)	6	(6, 4)	10
(2, 1)	3	(4, 3)	7	(6, 5)	11
(2, 2)	4	(4, 4)	8	(6, 6)	12
(2, 3)	5	(4, 5)	9		
(2, 4)	6	(4, 6)	10		
(2, 5)	7	(5, 1)	6		
(2, 6)	8	(5, 2)	7		
(3, 1)	4	(5, 3)	8		
(3, 2)	5	(5, 4)	9		
(3, 3)	6	(5, 5)	10		
(3, 4)	7	(5, 6)	11		
(3, 5)	8	(6, 1)	7		
(3, 6)	9	(6, 2)	8		

(b)

Sum of Dots	Tally Marks	No. of Outcomes	Probability
1	0	0	0
2	I	1	$\frac{1}{36}$
3	II	2	$\frac{1}{18}$
4	III	3	$\frac{1}{12}$
5	IIII	4	$\frac{1}{9}$
6	≡	5	$\frac{5}{36}$
7	≡ I	6	$\frac{1}{6}$
8	≡	5	$\frac{5}{36}$
9	IIII	4	$\frac{1}{9}$

10		2	$\frac{1}{12}$
11		2	$\frac{1}{18}$
12		1	$\frac{1}{36}$

- (III) 1. A 
 B 
 C 
 D 
 E 
 F 
 G 
 H 
 I 
 J 
 K 
 L 
 M 
 N 
 O 
 P 
 Q 
 R 
 S 
 T 
 U 
 V 
 W 
 X 
 Y 
 Z 

2. 2 letters	≡ ≡ ≡ ≡	20
3 letters	≡ ≡ ≡	18
4 letters	≡ ≡ ≡	18
5 letters	≡	08
6 letters	≡	09
more than 6 letters	≡ ≡ ≡ ≡ ≡ ≡	33
		106

Crossword Answers

Across

1. Pie Chart
5. Five
7. Range
8. Event
9. Whole
10. One
12. Equal

Down

2. Histogram
3. Raw
4. Class Size
6. Frequency
11. Zero

Unit 3

1. (c)	2. (a)	3. (c)	4. (d)	5. (b)	6. (c)
7. (b)	8. (b)	9. (c)	10. (b)	11. (b)	12. (b)
13. (b)	14. (a)	15. (b)	16. (d)	17. (b)	18. (b)
19. (a)	20. (d)	21. (b)	22. (a)	23. (c)	24. (d)

- 25.** 8 **26.** 8 **27.** 6 **28.** 4 **29.** $2n$ **30.** 3
31. 30.25 **32.** 5.3 **33.** 6 **34.** 10000 **35.** 1000000
36. 2 **37.** 0.49 **38.** 36 **39.** 9 **40.** 8, 15 **41.** 1.4
42. 1.728 **43.** odd **44.** $\sqrt[3]{x}$ or $x^{1/3}$ **45.** 5 **46.** 2
47. 2 **48.** 3 **49.** True **50.** False **51.** True **52.** True
53. False **54.** True **55.** False **56.** True **57.** False **58.** False
59. False **60.** False **61.** True **62.** True **63.** False **64.** False
65. False **66.** False **67.** False **68.** False **69.** True **70.** False
71. False **72.** True **73.** True **74.** False **75.** True **76.** False
77. False **78.** False **79.** True **80.** True **81.** False **82.** False
83. False **84.** False **85.** False **86.** False **87.** 1, 4, 9, 16, 25
88. 27, 216, 729 **90.** $1+3+5+7+9+11+13+15+17$
91. a) $484 = 2 \times 2 \times 11 \times 11$; perfect square
 b) $11250 = 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$; not a perfect square
 c) $841 = 29 \times 29$; a perfect square
 d) $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$; a perfect square.
92. a) $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$; not a perfect cube
 b) $343 = 7 \times 7 \times 7$; a perfect cube
 c) $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$; a perfect cube
 d) $1331 = 11 \times 11 \times 11$; a perfect cube
93. a) $101^2 = 10201$ b) $72^2 = 5184$ **94.** Yes, because $6^2 + 8^2 = 10^2$
95. (3, 4, 5) **96.** a) 105 b) 69 **97.** a) 8 b) 13
98. No, 11 **99.** No, 75 **100.** 3, 4, 5 and 5, 12, 13
101. 6; 6 **102.** 60; 60 **103.** a) 37 b) 75
104. a) 5.2 b) 1.2 **105.** 16; 37
106. 41, 79 **107.** 1024 **108.** 961 **109.** 3600
110. $\sqrt{50}$ or $5\sqrt{2}$ **111.** 7.2 **112.** 9.2 **113.** 22500 m^2

- 114.** 16 **115.** 3,375 **116.** 82m **117.** 576 m²
118. 8 cm **119.** 5, 10 and 15 **120.** 42.25 m²
121. 4 **122.** 6 **123.** 32 **124.** 52
125. 104 **126.** 93 **127.** 37 m **128.** 3.3 m
129. 900 **130.** 8, 12, 20 **131.** 3600 **132.** $10\frac{1}{2}$ m
133. 18 **134.** 0.3, 0.45, 0.6 **135.** 3.6
136. 50,653 **137.** 85, 184
138. 8836 **139.** 6, 19, 30 **140.** 104 **141.** 196, 961
142. 12, 21, 102, 201

Cross Number Puzzle

¹ 7	² 3	6	1	⁶ 3
³ 2	⁵ 5	0	0	0
⁴ 9	⁷ 6	¹⁰ 7	1	2
6	2	5	⁹ 3	5
¹¹ 1	5	1	⁸ 6	4

Unit 4

- 1.** (c) **2.** (c) **3.** (c) **4.** (a) **5.** (b) **6.** (c)
7. (a) **8.** (c) **9.** (d) **10.** (a) **11.** (b) **12.** (c)
13. (a) **14.** (d) **15.** (a) **16.** highest **17.** 1
18. $\frac{6}{5}$ **19.** solution **20.** 3 **21.** 3, 4 and 5
22. Rs 16.50 **23.** sign **24.** 10 **25.** - 60 **26.** -24
27. 5 **28.** 7 **29.** 6 years **30.** $4x + 15 = 39$
31. $x + 9$ **32.** 100 **33.** False **34.** False **35.** False **36.** True
37. False **38.** True **39.** False **40.** False **41.** False **42.** False
43. False **44.** True **45.** False **46.** False **47.** False **48.** False

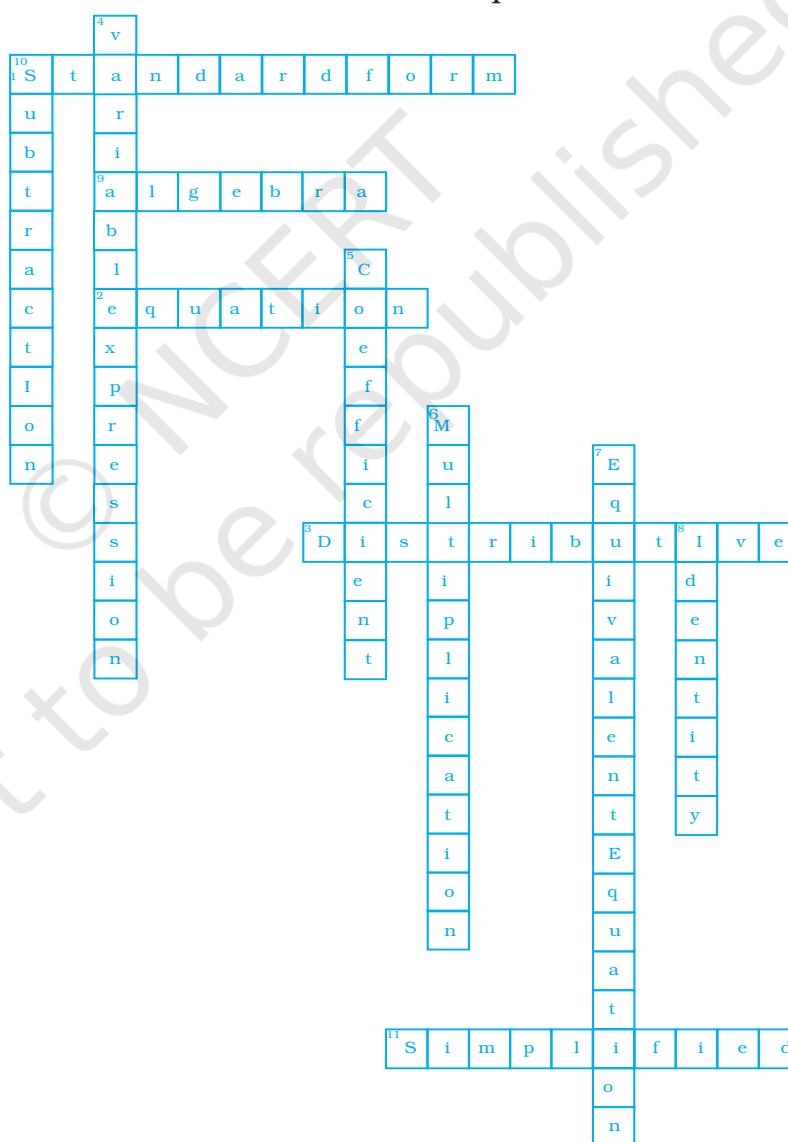
- 49.** $x = 8$ **50.** $x = -2$ **51.** $x = 7$ **52.** $x = \frac{8}{3}$
53. $x = 0$ **54.** $x = \frac{31}{6}$ **55.** $y = \frac{17}{22}$ **56.** $x = -5$
57. $x = 2$ **58.** $x = 4$ **59.** $x = -6$ **60.** $t = 0$
61. $x = 7$ **62.** $x = 2$ **63.** $x = \frac{-12}{5}$ **64.** $x = 11$
65. $x = \frac{-8}{9}$ **66.** $x = 5$ **67.** $x = \frac{43}{35}$ **68.** $t = 17$
69. $y = \frac{1}{2}$ **70.** $x = 37$ **71.** $y = \frac{-37}{57}$ **72.** $x = \frac{1}{18}$
73. $x = \frac{-3}{17}$ **74.** $t = \frac{1}{3}$ **75.** $m = \frac{7}{5}$ **76.** $P = \frac{-5}{22}$
77. $x = -96$ **78.** $x = 18.3$ **79.** 24 flowers **80.** Rs 4500
81. 50l, 100l **82.** 800 **83.** 24 **84.** 18
85. 23 **86.** 52 **87.** 1200 **88.** 12, 42
89. 56 **90.** 9m, 23m, 23m **91.** 12 years
92. Rs 3,00,000 **93.** $\frac{7}{4}$ **94.** 65, 66, 67, 68
95. $14\frac{1}{3}$ kg **96.** $l = 80$ cm, $b = 40$ cm
97. A = 20 years, B = 15 years **98.** $\frac{1}{5}$ **99.** 36
100. 20 days **101.** 9 km/hr
102. 500 Rs notes: 150, 1000 Rs note: 25 **103.** 15
104. $\frac{9}{5}$ **105.** 10 days **106.** 100 **107.** 11 km/hr
108. 22 km/hr, 30 km/hr **109.** 7 hr **110.** $x = 10$ cm
111. $x = 3$ cm **112.** Rs 80, Rs 120 **113.** 40

Application, Games and Puzzles

- 1.** (a) $x = 3$ (b) $Y = 2$ (c) $Z = 2$ (d) $P = 1$ (e) $Q = 6$ (f) $R = 2$
2.

$$\begin{array}{l}
 \diamond + \star = 8 \\
 \diamond + \diamond + \star = 10 \\
 \diamond + \star + \star + \star + \star = 26
 \end{array}$$

3. (c) (i) $x = 6\frac{1}{2}$ (ii) $x = 1$ (iii) $x = 1$ (iv) $\frac{2}{7}$ (v) $x = 60$
 (vi) $x = -5$ (vii) $x = \frac{-7}{5}$ (viii) $x = \frac{24}{5}$ (ix) $x = 5$ (x) $x = 42$
4. (1) $6\frac{1}{2}$ (2) 1 (3) -1 (4) $\frac{2}{7}$ (5) 60
 (6) -5 (7) $-\frac{7}{5}$ (8) $4\frac{4}{5}$ (9) 5 (10) 42
5. 1. Subtraction 2. Equation 3. Distributive
 4. Variable expression 5. Coefficient 6. Multiplication
 7. Equivalent equation 8. Identity 9. Algebra
 10. Standard form 11. Simplified



Unit 5

1. (b) 2. (a) 3. (a) 4. (b) 5. (d) 6. (c)
 7. (c) 8. (a) 9. (a) 10. (a) 11. (a) 12. (c)
 13. (b) 14. (c) 15. (d) 16. (a) 17. (a) 18. (a)
 19. (a) 20. (a) 21. (c) 22. (b) 23. (b) 24. (a)
 25. (b) 26. (c) 27. (d) 28. (b) 29. (a) 30. (b)
 31. (a) 32. (d) 33. (a) 34. (b) 35. (a) 36. (c)
 37. (a) 38. (a) 39. (c) 40. (a) 41. (a) 42. (a)
 43. (d) 44. (a) 45. (a) 46. (b) 47. (b) 48. (a)
 49. (c) 50. (c) 51. (b) 52. (c)
 53. HO and EP, PO and EH
 54. RO and OP, OP and PE, PE and ER, ER and RO
 55. $\angle W$ and $\angle Y$, $\angle X$ and $\angle Z$ 56. DF and EG
 57. Angles 58. 72° 59. 720° 60. 20°
 61. 10° 62. Concave Polygon 63. Kite
 64. 108° 65. An equilateral triangle 66. 9
 67. Line segments 68. Angles 69. $2n-4$
 70. 360° 71. Square 72. Trapezium
 73. Rhombus, Square 74. Right 75. 5
 76. 2 included 77. All 78. 1 79. Opposite
 80. 5 81. Parallelogram 82. 28cm 83. 9
 84. Equal 85. Decagon 86. Square 87. 6cm
 88. Supplementary 89. Kite 90. 80°
 91. Quadrilateral 92. False 93. True 94. False
 95. True 96. False 97. False 98. True
 99. True 100. False 101. True 102. True
 103. True 104. False 105. False 106. False
 107. False 108. False 109. False 110. False

- 111.** True **112.** False **113.** False **114.** False
115. False **116.** False **117.** True **118.** True
119. True **120.** True **121.** True **122.** False
123. False **124.** True **125.** False **126.** True
127. True **128.** True **129.** True **130.** True
131. True **132.** 8.5cm **133.** 45° , 135° , 45° , 135°
134. Trapezium, Others are parallelogram **135.** 2 : 3
136. 36° **137.** No, in a rectangle diagonals are equal.
138. 70° , 110° , 70° , 110°
139. No, diagonals of a parallelogram bisect each other i.e. in the ratio 1:1.
140. 12 **141.** Parallelogram **142.** Rhombus
143. 23 cm, 30 cm, 30 cm **144.** 30° , 60° , 120°
145. 55° , 70° , 70° **146.** 100° , 80° , 100°
147. 120° , 60° , 15 cm, 11 cm, 12 cm, 52 cm **148.** 20° , 20°
149. 45° , 75° , 35° **150.** 70° **151.** 15° each
152. (i)Yes, opposite sides of a rectangle are equal.
(ii)Yes, MY and RX are perpendicular to OE.
(iii)Yes, these are alternate interior angles.
(iv)Yes, $\triangle MYO \cong \triangle RXE$
153. 50° , 50° , 50° **154.** 120° **155.** 90°
156. 135° , 45° **157.** 100° **158.** 2.5 **159.** 90°
160. $x = 2$ **161.** $x = 10^\circ$, $y = 20^\circ$
162. $x = 80^\circ$, $y = 110^\circ$ **163.** $x = 80^\circ$
164. 105° each, Parallelogram **165.** 200° , concave
166. 90° **167.** 135°
168. Ext. angle of regular pentagon = $\frac{360^\circ}{5} = 72^\circ$
Ext. angle of regular decagon = $\frac{360^\circ}{10} = 36^\circ$
 $72^\circ = 2 \times 36^\circ$
169. 74° **170.** 80°

171. Yes, $\frac{1}{2}\angle E + \frac{1}{2}\angle P = 180^\circ - \angle PSE \Rightarrow \angle E + \angle P = 360^\circ - 2\angle PSE$

and $\angle E + \angle P + \angle O + \angle H = 360^\circ$
 $\Rightarrow 360^\circ - 2\angle PSE + \angle O + \angle H = 360^\circ$

172. $x = 90^\circ$, $y = 60^\circ$, $z = 30^\circ$

173. False

Trap ABCD

in which $AD \parallel BC$

174. $\angle A = 120^\circ$, $\angle B = 105^\circ$, $\angle C = 75^\circ$, $\angle D = 60^\circ$

175. $l \parallel m$

$\angle DXY = \angle XYA$ (alt int. $\angle$ s)

$\frac{\angle DXY}{2} = \frac{\angle XYA}{2}$ ($\div 2$)

$\angle 1 = \angle 2$ (XP and YQ are bisectors)

$\therefore XP \parallel QY$ (1)

Similarly $XQ \parallel PY$ (2)

From (1) and (2)

PXQY is a parallelogram

$\angle DXY + \angle XYB = 180^\circ$

$\frac{\angle DXY}{2} + \frac{\angle XYB}{2} = \frac{180^\circ}{2}$ ($\div$ by 2)

$\angle 1 + \angle 3 = 90^\circ$ (4)

In $\triangle XYP$

$\angle 1 + \angle 3 + \angle P = 180^\circ$

$90^\circ + \angle P = 180^\circ$ (from 4)

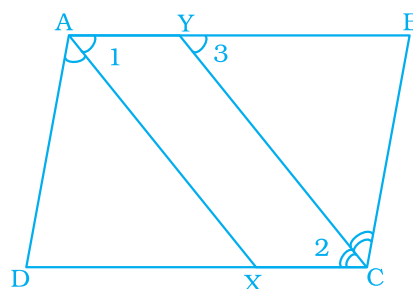
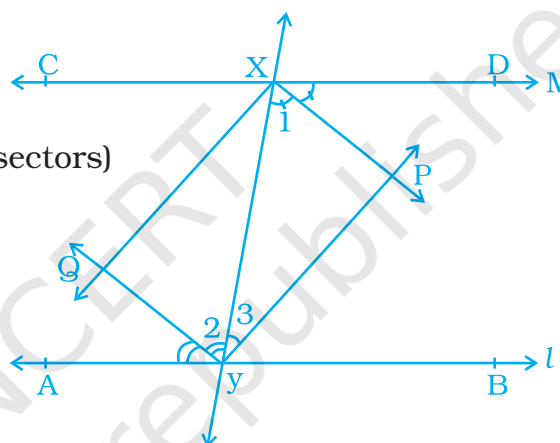
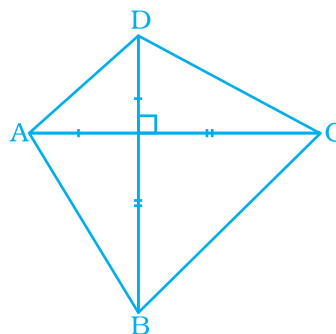
$\angle P = 90^\circ$

From (3) and (5), PXQY is a rectangle

176. $\frac{\angle A}{2} = \frac{\angle C}{2}$ (opp. $\angle$ s of a $\parallel^{\text{gm}}$)
 $\angle 1 = \angle 2$ ($\div 2$)

But $\angle 2 = \angle 3$ (all $\angle$ s)

$\therefore \angle 1 = \angle 3$



But they are a pair of corresponding $\angle$ s

$$\therefore AX \parallel YC \quad (1)$$

$$AY \parallel XC \quad (2) \quad (AB \parallel DC)$$

From (1) and (2)

$\square$ AXCY is a Parallelogram

177. Given: (i) ABCD is a $\parallel^{\text{gm}}$

$$(ii) \angle 1 = \angle 2$$

To Prove: (i) $\angle 3 = \angle 4$

(ii) ABCD is rhombus

Proof: (i) $\angle 1 = \angle 4$

$$\angle 2 = \angle 3 \quad (\text{alt } \angle \text{s})$$

$$\text{But } \angle 1 = \angle 2$$

$$\angle 3 = \angle 4$$

$$(ii) \angle 1 = \angle 2 \quad (\text{given alt.})$$

$$\angle 2 = \angle 3$$

$$\angle 1 = \angle 3$$

Hence $CD = DA$

$\therefore$ ABCD is a rhombus

178. $135^\circ, 45^\circ, 135^\circ, 45^\circ$

179. $60^\circ, 120^\circ, 60^\circ, 120^\circ$

180. 45°

181. Given: ABCD is a $\parallel^{\text{gm}}$, bisector of $\angle A$, bisects BC in F i.e. $\angle 1 = \angle 2$, $CF = FB$

Const: Draw $FE \parallel BA$

Proof: ABFE is a $\parallel^{\text{gm}}$ by const. ($FE \parallel BA$)

$$\angle 1 = \angle 6 \quad (\text{alt. } \angle)$$

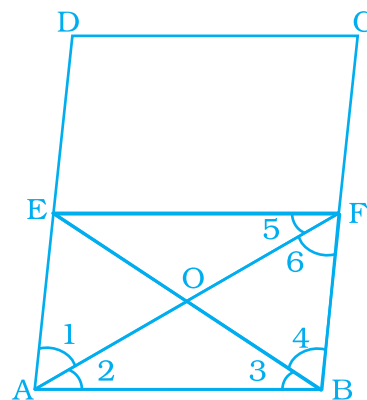
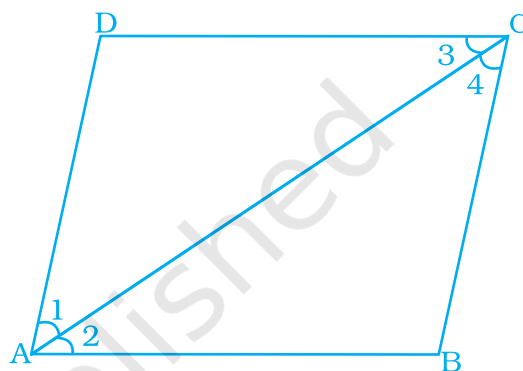
$$\text{But } \angle 1 = \angle 2 \quad (\text{given})$$

$$\therefore \angle 2 = \angle 6$$

$$AB = FB \quad (1) \quad (\text{sides opp to equal } \angle \text{s})$$

$\therefore$ ABFE is a rhombus

In $\triangle ABO$ and $\triangle BOF$



$AB = BF$ from (1)
 $BO = BO$ Common
 $AO = FO$ Diagonals bisect each other

$$\triangle ABO \cong \triangle BOF$$

$$\angle 3 = \angle 4$$

$$BF = \frac{1}{2} BC \quad (\text{given})$$

$$BF = \frac{1}{2} AD \quad (BC = AD)$$

$$AE = \frac{1}{2} AC \quad (BF = AE)$$

$\therefore E$ is mid point of AD

182. 90°

183. 3, 3, 3. So, maximum number of acute angles is always 3.

184. (a) 116°

185. 30cm

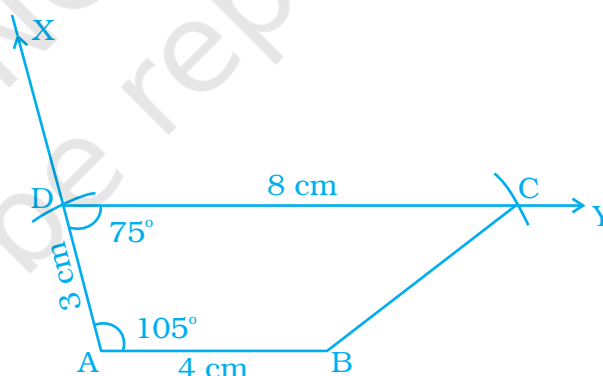
186. $\angle A + \angle D = 180^\circ$

$$105^\circ + \angle D = 180^\circ$$

$$\angle D = 75^\circ$$

Steps of construction

1. Draw $AB = 4$ cm
2. Draw $\overrightarrow{AX}$ such that $\angle BAX = 105^\circ$
3. Mark a point D on AX such that $AD = 3$ cm
4. Draw $\overrightarrow{DY}$ such that $\angle ADY = 75^\circ$
5. Mark a point C such that $CD = 8$ cm
6. Join BC . $ABCD$ is the required trapezium.

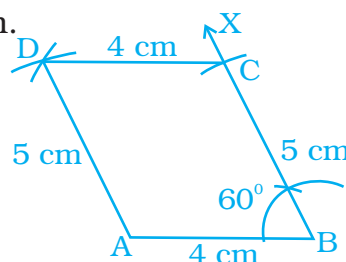


187. Opp sides of a $\parallel$ gm are equal.

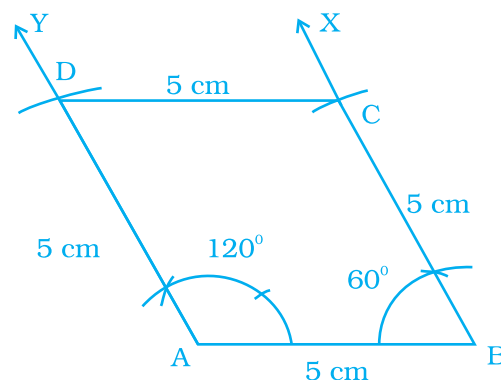
$$AB = DC = 4\text{cm}$$

$$BC = AD = 5\text{cm}$$

Steps of construction



1. Draw $AB = 4$ cm
2. Draw ray BX such that $\angle ABX = 60^\circ$
3. Mark a point C such that $BC = 5$ cm
4. With C and A as centre, draw arcs intersecting at a point D respectively $ABCD$ is the required parallelogram.

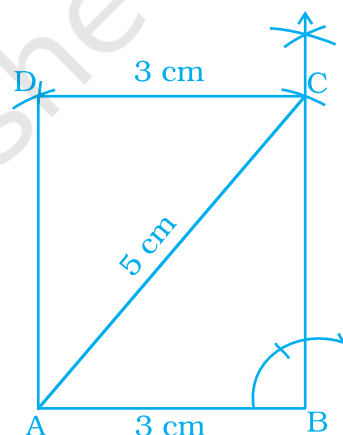


- 188.** $\angle B = 60^\circ$ (suppose)
 $\angle A + \angle B = 180^\circ$ (sum of co-interior angles)
 $\angle A + 60^\circ = 180^\circ$
 $\angle A = 120^\circ$
 $AB = BC = CD = DA = 5$ cm

Steps of construction

1. Draw $AB = 5$ cm
2. Draw ray AY such that $\angle BAY = 120^\circ$
3. Mark a point D such that $AD = 5$ cm
4. Draw ray BX such that $\angle ABX = 60^\circ$
5. Mark a point C such that $BC = 5$ cm
6. Join C and D

$\therefore ABCD$ is the required rhombus



- 189.** Diagonals of a rectangle are equal.

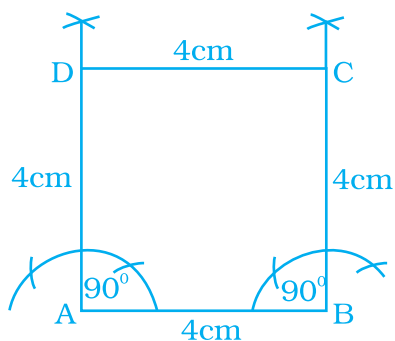
$$AC = BD = 5 \text{ cm}$$

Steps of construction

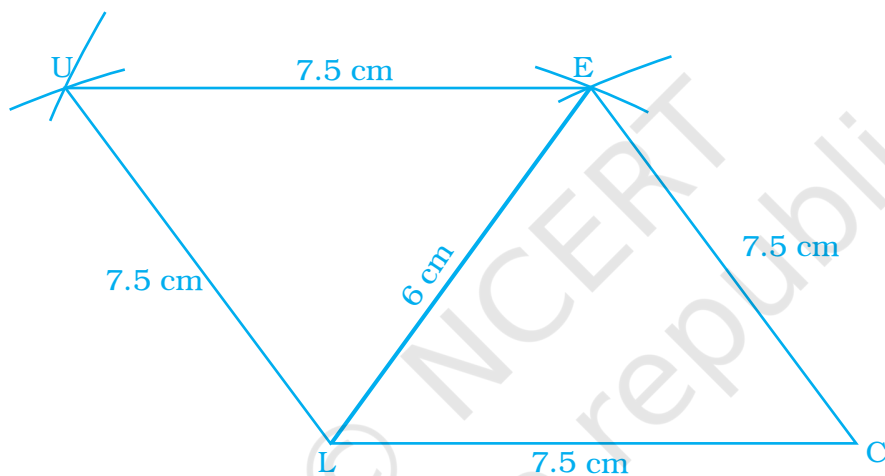
1. Draw $AB = 3$ cm
2. Draw a ray BX such that $\angle ABX = 90^\circ$
3. Draw an arc such that $AC = 5$ cm
4. With B as centre, draw an arc of radius 5 cm. With C as centre draw another arc of radius 3 cm which intersect first arc at a point, suppose D .
5. Join CD and AD

ABCD is the required rectangle.

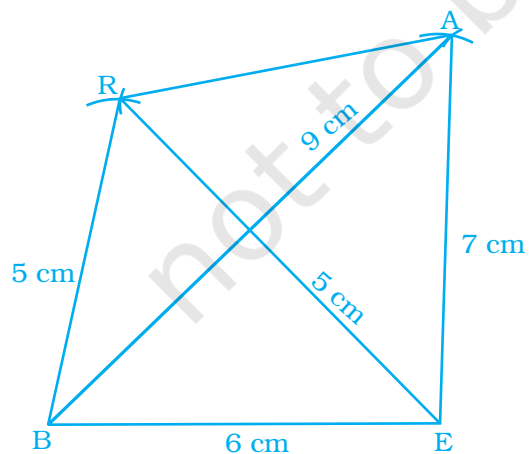
190.



191.



192.



RA = 5 cm

194. Cyclic quadrilateral

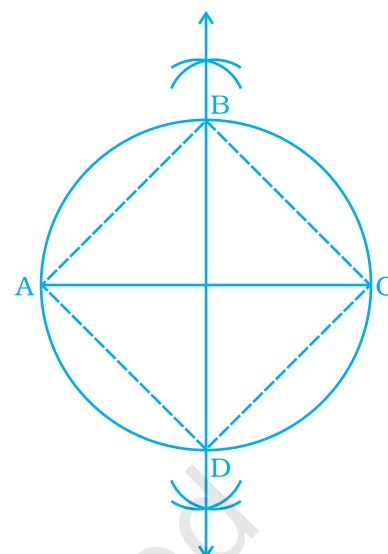
$$\angle B = \angle D = 90^\circ \quad (\text{Angle in a semicircle})$$

$$\angle A = \angle C = 90^\circ$$

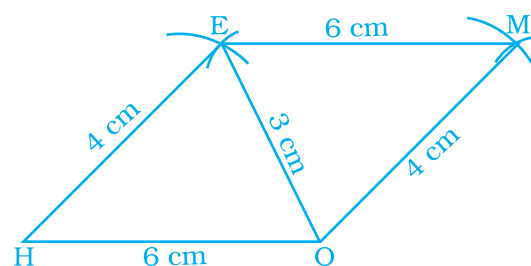
$$\angle B + \angle D = 180^\circ$$

$$\angle A + \angle C = 180^\circ$$

opposite $\angle$ s are supplementary.



195.



196. No,

In a Δ , sum of two sides always is greater than the third side.

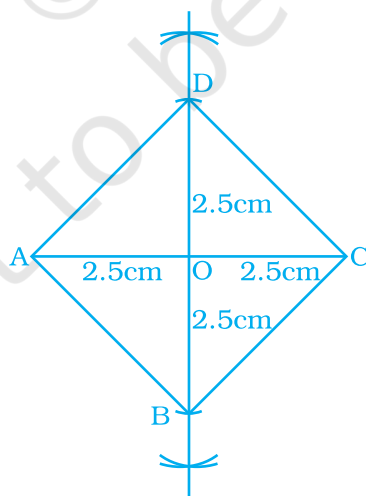
$$AB + BC > AC$$

197. No,

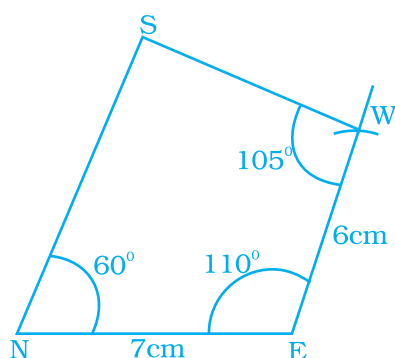
$$\angle O + \angle R + \angle A = 120^\circ + 105^\circ + 135^\circ = 360^\circ$$

198.

Diagonals bisect at right angle

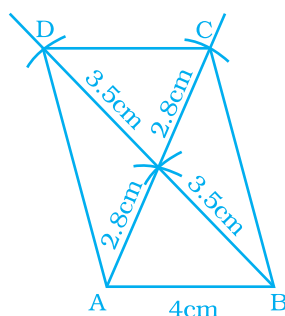


199.

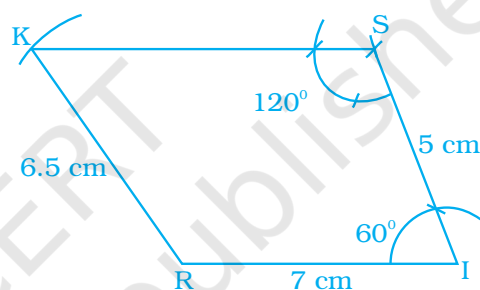


$$\begin{aligned}\text{Fourth angle} &= 360^\circ - (60^\circ + 110^\circ + 85^\circ) \\ &= 360^\circ - 255^\circ = 105^\circ\end{aligned}$$

200.



Other side = 5 cm



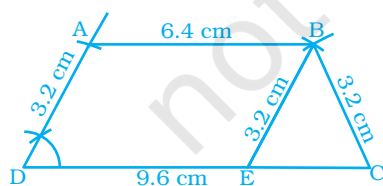
201. 72°

202. $\angle I + \angle S = 180^\circ$

$$60^\circ + \angle S = 180^\circ$$

$$\angle S = 120^\circ$$

203.



BEC is an equilateral triangle

$$\angle A = 120^\circ, \angle B = 60^\circ$$

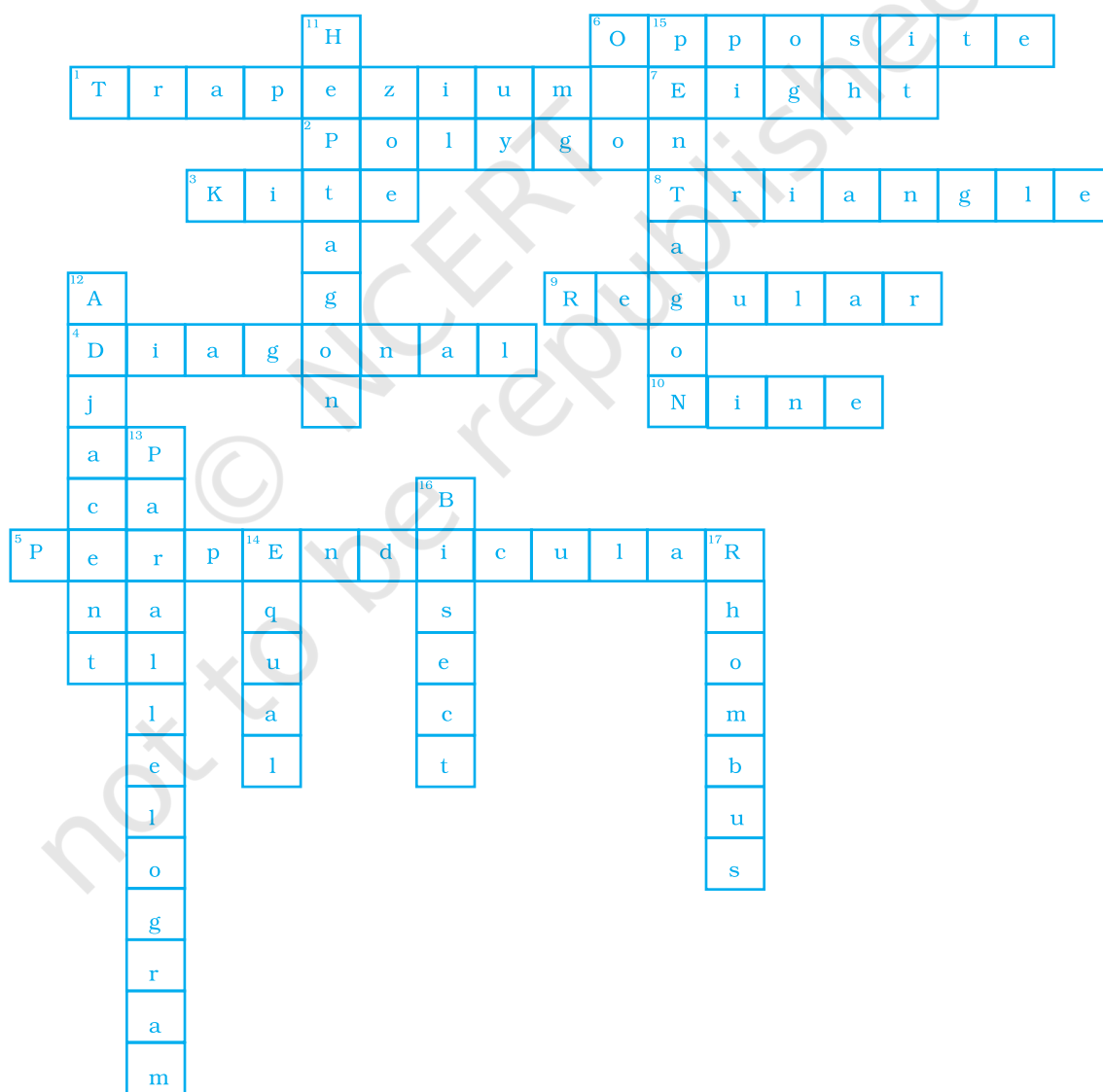
Application, Games and Puzzles

Across

1. Trapezium
2. Polygon
3. Kite
4. Diagonal
5. Perpendicular
6. Opposite
7. Eight
8. Triangle
9. Regular
10. Nine

Down

11. Heptagon
12. Adjacent
13. Parallelogram
14. Equal
15. Pentagon
16. Bisect
17. Rhombus



Unit 6

1. (c) 2. (a) 3. (c) 4. (a) 5. (c) 6. (d)
 7. (a) 8. (c) 9. (a) 10. (a) 11. (a) 12. (b)
 13. (b) 14. (b) 15. (c) 16. (d) 17. (d) 18. (b)
 19. (b) 20. (d) 21. (c) 22. cube 23. cuboid 24. 4
 25. $n+1$ 26. 30 27. Prism 28. Cone 29. Five 30. Six
 31. Same 32. 4 33. 1:4400000 34. 7 35. 7
 36. top 37. eight 38. 12 39. Five 40. Congruent

41. a) ☐ Front view

☐ Side view

☐ Top view

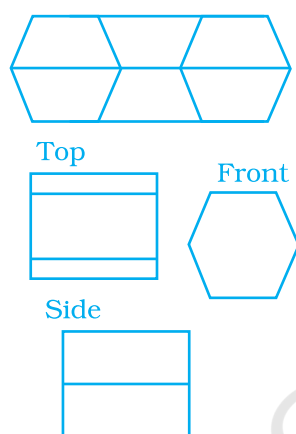
- b) i) Side view
 ii) Top view
 iii) Front view
 c) i) Side view
 ii) top view
 iii) Front view
 d) i) Side
 ii) Front
 iii) Top

42. False 43. False 44. True 45. False 46. False 47. False
 48. False 49. False 50. True 51. False 52. False 53. True
 54. False 55. True 56. True 57. False 58. True 59. True
 60. True 61. True
 62. (a) 6, 8, 12, 14, 14,
 (b) 4, 4, 6, 8, 8
 (c) 5, 5, 8, 10, 10
 (d) 5, 5, 8, 10, 10
 (e) 6, 6, 10, 12, 12
 (f) 7, 7, 12, 14, 14
 (g) 5, 6, 9, 11, 11
 (h) 6, 8, 12, 14, 14

- (i) 6, 8, 12, 14, 14
- (j) 7, 10, 15, 17, 17
- (k) 10, 16, 24, 26, 26
- (l) 9, 14, 21, 23, 23

- 63.** a) 4
b) 6
c) 9
d) 8

64.



- 65.** (a) 1 (b) none (c) none
(d) 9 (e) 4 (f) 12
- 66.** (a) 1 (b) 2 (c) none
(d) 16 (e) 18 (f) 9

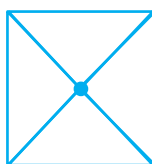
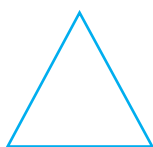
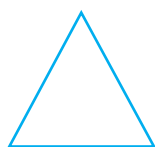
67. (c), (f), (m) and (k) are not polyhedrons

- 68.** (a) 10 (b) 10 (c) 10 (d) 9
(e) 11 (f) 9 (g) 11 (h) 110
(i) 113 (j) 66 (k) 15 (l) 14

- 69.** Front view Side view Top view
(a)



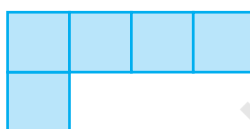
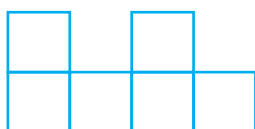
(b)



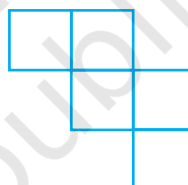
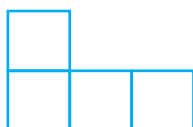
(c)



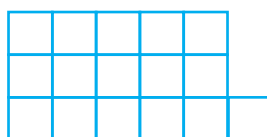
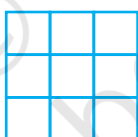
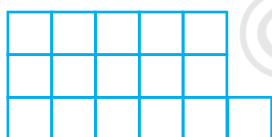
(d)



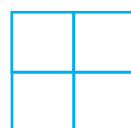
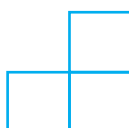
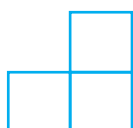
(e)



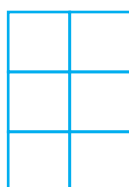
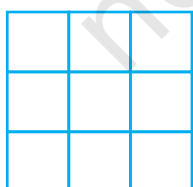
(f)

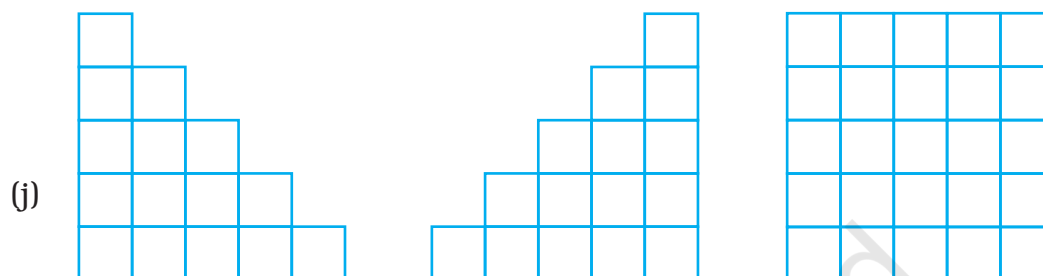
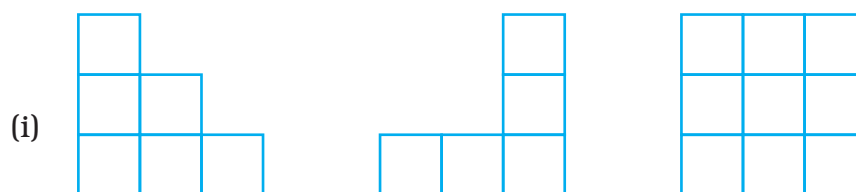


(g)



(h)





70. $x = 15$

$y = 8$

$z = 9$

$p = 8$

$q = 8$

$r = 17$

71. Yes, draw an octagonal pyramid.

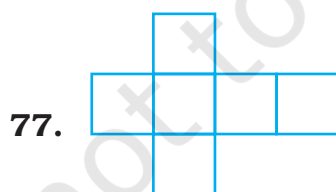
72. No.

73. 22

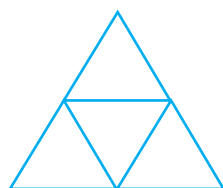
74. (a) 14 (b) 10 (c) 16

75. 30

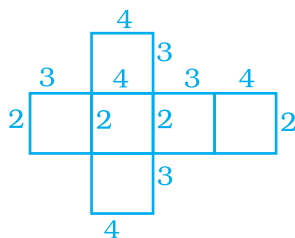
76. 22



78.



79.



80. i) b

ii) d

iii) a

iv) c

81. 1. Prism, Pyramid

3. Cone, Cylinder

5. Cylinder, Prism

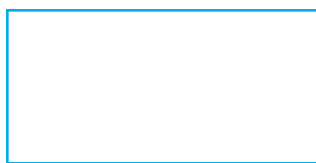
7. Cone

2. Pyramid

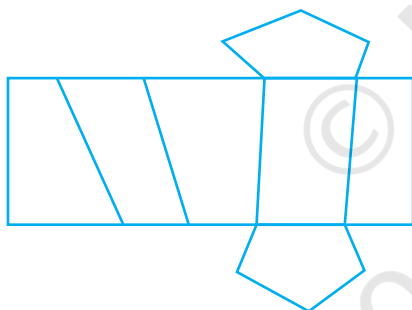
4. Prism, Pyramid

6. Pyramid

82.

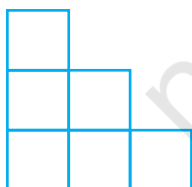


83.



84. 7

85.



86. $F = n+1$

$V = n+1$

$E = 2n$

87.



It is a cuboid. Yes.

88. a) Cylindrical mounted by hemi sphere.
b) Hexagonal prism mounted by a cone.

89. Cuboid

91. a) Cube
b) Cuboid
c) Cylinder
d) Cone
e) Square Pyramid
f) Triangular prism

92. a) 2.1 acre
b) Govt Model School I and II
c) Park A
d) B block
e) 6

93. a) AIIMS and Safdarjang Hospital
b) Sirifort Auditorium, Bhel, Asiad Tower
c) August Kranti Marg

94. a) Flower Road
Khel Marg, Mall Road
and Sneha Marg,
b) Stadium, Sector 27
B Town, B Town India
c) Sneha Marg
d) H.N.I, Nr. Bank
Sector 19, B Town India
e) Sector 27
f) Sector not mentioned
g) 3.

96. 1:2

97. 5:1

98. 25 km

99. 1) 60 km

2) 20 km

3) 35 km

100. 10 mm

101. a) 1 cm = 4 m b) 1 inch = 9 feet

102. 12 cm

Activity, Crossword Puzzle

Across

1. Prism

2. Pentagonal

5. Cuboid

7. Tetrahedron

8. Convex

Down

2. Pyramid

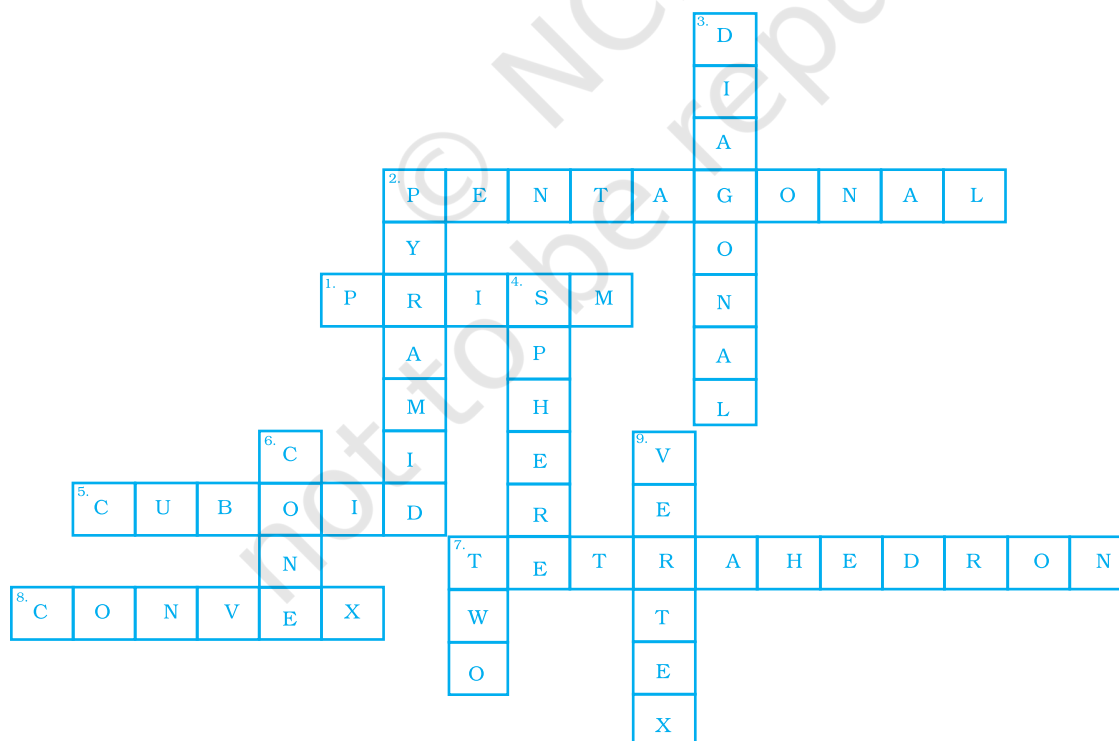
3. Diagonal

4. Sphere

6. Cone

7. Two

9. Vertex



Unit 7

1. (b) 2. (b) 3. (b) 4. (d) 5. (d) 6. (b)
7. (d) 8. (a) 9. (a) 10. (b) 11. (a) 12. (b)
13. (d) 14. (b) 15. (c) 16. (c) 17. (b) 18. (c)
19. (a) 20. (b) 21. (a) 22. (c) 23. (c) 24. (a)
25. (c) 26. (c) 27. (b) 28. (b) 29. (d) 30. (b)
31. (d) 32. (c) 33. (a) 34. positive 35. negative
36. $ab + ac$ 37. $(a - b)^2$ 38. $(a + b)(a - b)$
39. $2ab - 2b^2$ 40. $a^2 + b^2$ 41. ab 42. polynomial
43. x 44. $2m(9 + 5p)$ 45. $(2y - 3)(2y - 3)$ 46. $2x^2z$
47. $24xyz$ 48. $(67 + 37)$ 49. 205
50. $12x^2y^2$ 51. $8x^3$ 52. -37 53. 2
54. $16(a^2 + b^2)$ 55. distributive law 56. $3y$ 57. $x + 1$
58. $x + 2y$ 59. False 60. False 61. True 62. False
63. True 64. True 65. False 66. False 67. False 68. True
69. False 70. False 71. False 72. True 73. False 74. False
75. True 76. True 77. False 78. True 79. False 80. True
81. i) $10a^2bc - abc^2$ ii) $10ax - 2by + 2cz$
 iii) $4xy^2z^2 - 6x^2y^2z - 3x^2yz^2$ iv) $3x^2 + 2xy + 11y^2 + 4$
 v) $-p^4 - 10p^3 - 2p^2 - 6p - 5$ vi) $3a^2 - ab + 3ac + 2bc - 2b^2$
 vii) $6ab + 21ac + 6bc$
82. i) $-12a^2b^2c^2$ ii) $-9x^2 + 10xy + 3y^2$
 iii) $2ab^2c^2 - 14a^2b^2c + 7a^2bc^2$ iv) $-7t^4 + 12t^3 - 6t^2 + 4t + 5$
 v) $3ab - 7bc + 5ac + 10abc$ vi) $-33p^2 - 77pq$
 vii) $-3ap - 3pr - 3pq - 3px$

83. i) $91p^4q^4r^4$ ii) $51x^3y^3z^3$ iii) $255xy^3z^2$ iv) $-715a^4b^3c^3$

v) $-15x^2y^2 + 3x^3y^2$ vi) ab^2c^2

vii) $7p^2qr - 7pq^2r + 7pqr^2$ viii) $x^3y^3z^2 - x^2y^3z^3 + x^3y^2z^3$

ix) $pq - 7p + 6q - 42$ x) 0 xi) a^{12} xii) $-91S^2t^3$

xiii) $21ab^{10}$ xiv) $-\frac{25}{3}r^4s^3$ xv) $a^4 - b^4$ xvi) $a^2b^2 + 2abc + c^2$

xvii) $p^2q^2 - 4pqr + 4r^2$ xviii) $\frac{1}{2}x^2 + \frac{17}{72}xy - 2y^2$

xix) $3p^4 - \frac{19}{6}p^2q^2 - 2q^4$ xx) $2x^3 - 3x^2 - 23x + 42$

xxi) $6x^4 - 4x^3 - 23x^2 + 44x - 24$ xxii) $2x^2 + 7x - 13y - 2y^2 - 15$

84. i) $18x^2 + 8y^2$ ii) $24xy$

iii) $\frac{49}{81}a^2 + ab + \frac{81}{49}b^2$ iv) $\frac{9}{16}x^2 + \frac{16}{9}y^2$

v) $7.2pq$ vi) $2.5m^2 + 4.5q^2$ vii) x^4

viii) $a^2b^2 + c^2$ ix) $-2b^3$ x) $b^3 - 49b + 7b^2$

xi) $40.5a^2 + 27ab + 4.5b^2$ xii) $p^2q^2 + 2pq^2r + q^2r^2$

xiii) $s^4t^2 - 2s^2t^2q^2 + t^2q^4$

85. i) $x^2y^2 + 2xy^2z + y^2z^2$ ii) $x^4y^2 - 2x^3y^3 + x^2y^4$

iii) $\frac{16}{25}a^2 + 2ab + \frac{25}{16}b^2$ iv) $\frac{4}{9}x^2 - 2xy + \frac{9}{4}y^2$

v) $\frac{16}{25}p^2 + \frac{8}{3}pq + \frac{25}{9}q^2$ vi) $x^2 + 10x + 21$

vii) $4x^2 + 4x - 63$ viii) $\frac{16}{25}x^2 + \frac{4xy}{5} + \frac{3y^2}{16}$

ix) $\frac{4}{9}x^2 - \frac{4}{9}a^2$ x) $4x^2 - 20xy + 25y^2$

$$\text{xi)} \quad \frac{4}{9}a^2 - \frac{b^2}{9}$$

$$\text{xii)} \quad x^4 - y^4$$

$$\text{xiii)} \quad a^4 + 2a^2b^2 + b^4$$

$$\text{xiv)} \quad 49x^2 + 70x + 25$$

$$\text{xv)} \quad 1296a^4 + 2401b^4 - 3528a^2b^2$$

$$\text{xvi)} \quad 0.81p^2 - 0.9pq + 0.25q^2$$

$$\mathbf{86.} \quad \text{i)} \quad 2704$$

$$\text{ii)} \quad 2401$$

$$\text{iii)} \quad 10609$$

$$\text{iv)} \quad 9604$$

$$\text{v)} \quad 1010025$$

$$\text{vi)} \quad 990025$$

$$\text{vii)} \quad 2491$$

$$\text{viii)} \quad 2756$$

$$\text{ix)} \quad 9975$$

$$\text{x)} \quad 10088$$

$$\text{xi)} \quad 10403$$

$$\text{xii)} \quad 10094$$

$$\text{xiii)} \quad 98.01$$

$$\text{xiv)} \quad 99.96$$

$$\text{xv)} \quad 103.02$$

$$\text{xvi)} \quad 1050$$

$$\text{xvii)} \quad 3860$$

$$\text{xviii)} \quad 94$$

$$\text{xix)} \quad 12800$$

$$\text{xx)} \quad 89000$$

$$\text{xxi)} \quad 458000$$

$$\mathbf{87.} \quad \text{i)} \quad 18a$$

$$\text{ii)} \quad 3xy$$

$$\text{iii)} \quad y$$

$$\text{iv)} \quad lmn$$

$$\text{v)} \quad 7pqr$$

$$\text{vi)} \quad ry$$

$$\text{vii)} \quad 3xyz$$

$$\text{viii)} \quad 3prs$$

$$\text{ix)} \quad 13xy$$

$$\text{x)} \quad 1$$

$$\mathbf{88.} \quad \text{i)} \quad 6b(a + 2c)$$

$$\text{ii)} \quad -y(x + a)$$

$$\text{iii)} \quad x(ax^2 - bx + c)$$

$$\text{iv)} \quad lmn(lm + mn - ln)$$

$$\text{v)} \quad 3r(pq - 2p^2q^2r - 5r)$$

$$\text{vi)} \quad xy(x^2y + xy^2 - y^3 + 1)$$

$$\text{vii)} \quad 2xy(2y - 5x + 8xy + 1)$$

$$\text{viii)} \quad a(2a^2 - 3ab + 5b^2 - b)$$

$$\text{ix)} \quad 3pqrs(21pqr - 3qrs + 5prs - 20pqs)$$

$$\text{x)} \quad xyz(24xz^2 - 6y^2z + 15xy - 5)$$

$$\text{xi)} \quad (a + 1)(a^2 + 1)$$

$$\text{xii)} \quad (x + y)(l + m) \quad \text{xiii)} \quad x(a^2 - x^2)(a + x)$$

$$\text{xiv)} \quad (x + 2y)(2x - 1)$$

$$\text{xv)} \quad (y - 4z)(y - 2x)$$

$$\text{xvi)} \quad x(ax + by)(y - z)$$

$$\text{xvii)} \quad (a^2 + a + bc)(b + c) \quad \text{xviii)} \quad (2a + 3b)(x + y)^2$$

$$\mathbf{89.} \quad \text{(i)} \quad (x + 3)(x + 3) \quad \text{(ii)} \quad (x + 6)(x + 6) \quad \text{(iii)} \quad (x + 7)(x + 7)$$

(iv) $(x + 1)(x + 1)$

(vi) $(ax + 1)(ax + 1)$

(viii) $(ax + by)(ax + by)$

(ix) $(2x + 3)(2x + 3)$

(xi) $(3x + 4)(3x + 4)$

(xiii) $2x(x + 6)(x + 6)$

(xv) $x^2(2x + 3)(2x + 3)$

(xvii) $\left(3x + \frac{y}{3}\right)\left(3x + \frac{y}{3}\right)$

90. (i) $(x - 4)(x - 4)$

(iii) $(x - 7)(y - 7)$

(iv) $(p - 1)(p - 1)$

(vi) $(py - 1)(py - 1)$

(viii) $(3x - 2)(3x - 2)$

(x) $\left(\frac{x}{2} - 2\right)\left(\frac{x}{2} - 2\right)$

(xii) $\left(3y - \frac{2x}{3}\right)^2$

91. (i) $(x + 13)(x + 2)$

(iii) $(x + 5)(x + 13)$

(v) $(y + 7)(y - 3)$

(vii) $(9 + x)(2 + x)$

(ix) $(x - 12)(x - 5)$

(xi) $(y + 4)(y + 3)$

(xiii) $(a - 20)(a + 4)$

(v) $(2x + 1)(2x + 1)$

(vii) $(ax + b)(ax + b)$

(x) $(4x + 5)(4x + 5)$

(xii) $(3y + 5)(3y + 5)$

(xiv) $x(ax + b)(ax + b)$

(xvi) $\left(\frac{x}{2} + 2\right)\left(\frac{x}{2} + 2\right)$

(ii) $(x - 5)(x - 5)$

(v) $(2a - b)(2a - b)$

(vii) $(ay - b)(ay - b)$

(ix) $(2y - 3)(2y - 3)$

(xi) $y(ay - b)(ay - b)$

(ii) $(x + 5)(x + 4)$

(iv) $(p + 1)(p + 13)$

(vi) $(y - 5)(y + 3)$

(viii) $(x - 7)(x - 3)$

(x) $(x + 11)(x - 7)$

(xii) $(p - 15)(p + 2)$

92. (i) $(x - 3)(x + 3)$

iii) $(2x - 7y)(2x + 7y)$

v) $7a(2y - 5x)(y^2 + 5x)$

vii) $25a(x - 1)(x + 1)$

ix) $2\left(\frac{p}{5} - 4q\right)\left(\frac{p}{5} + 4q\right)$

xi) $y\left(y - \frac{1}{3}\right)\left(y + \frac{1}{3}\right)$

xiii) $\frac{1}{2}\left(\frac{x}{2} - \frac{y}{3}\right)\left(\frac{x}{2} + \frac{y}{3}\right)$

xv) $xy\left(\frac{x}{3} - \frac{y}{4}\right)\left(\frac{x}{3} + \frac{y}{4}\right)$

xvii) $b^2\left(\frac{1}{6}a - \frac{4}{7}b\right)\left(\frac{1}{6}a + \frac{4}{7}b\right)$

xiv) $2ab(2a^2 - 2ab + b^2)$

xx) $(y - 5)(y + 5)(y^2 + 25)$

xxii) $(2x - 3)(2x + 3)(4x^2 + 9)$

xxiv) $(y - 3)(y + 3)(y^2 + 9)$

xxv) $(2x - 5y)(2x + 5y)(4x^2 + 25y^2)$

xxvi) $(a - 2b + c)(a - c)$

xxviii) $(x - y)(x + y)(x^2 + y^2 + 1)$

xxx) $\left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$

ii) $(2x - 5y)(2x + 5y)$

iv) $3a^2b(b - 3a)(b + 3a)$

vi) $(3x - 1)(3x + 1)$

viii) $\left(\frac{x}{3} - \frac{y}{5}\right)\left(\frac{x}{3} + \frac{y}{5}\right)$

x) $(7x - 6y)(7x + 6y)$

xii) $\left(\frac{x}{5} - 25\right)\left(\frac{x}{5} + 25\right)$

xiv) $\left(\frac{2}{3}x - \frac{3}{4}y\right)\left(\frac{2}{3}x + \frac{3}{4}y\right)$

xvi) $11xy(11x - y)(11x + y)$

xix) $(x - 1)(x + 1)(x^2 + 1)$

xxi) $p(p - 2)(p + 2)(p^2 + 4)$

xxiii) $(x - y)(x + y)(x^2 + y^2)$

xxvii) $8xy(x^2 + y^2)$

xxix) $2a(2a - 1)(2a + 1)$

xxxi) $(3x - 3y - 3)(3x + 3y + 3)$

93. (i) $x - 2$ and $x - 4$ (ii) $x - 1$ and $x - 2$ (iii) $x - 2$ and $x - 5$

(iv) $x + 20$ and $x - 1$

(v) $x + 5$ and $x + 4$

94. (i) $3x^2y$ (ii) $4\frac{xz^3}{y}$

(iii) $-17bc$

(iv) $\frac{11p^3q^3r^3}{xy^2z^3}$

95. (i) $r - 2pqr^2$ (ii) $\frac{-a}{d}x^2 + \frac{b}{d}x - \frac{c}{d}$ (iii) $x^2y^2 + xy^2 - y^3 + 1$

(iv) $\frac{qr}{z} - \frac{pr}{x} + r$

96. (i) $x - 9$ (ii) $x + 12$ (iii) $2x$

(iv) $3x - 2$ (v) $3(x + 4)$ (vi) $x - 2$

(vii) $x^2 + 25$

97. $2x + 3y$ 98. $3x + 4y$ 99. $x + 8$

100. $y - 4$ 101. $x + 3$ 102. $\frac{1}{2}n(n+1)$

103. $(x^2 + 25)(x - 5)$ 104. $7xy(x^4 + y^4)$

105. Rs $x^2 + 8x + 16$; Rs 196

106. $4x^2 - 9$ sq. units; 391 sq. units 107. $44(ab - b(-2ac))$

108. 100 109. 200 110. 225

111. 72 112. 12

114. (i) 62 (ii) 143 (iii) 12 (iv) 8

115. $3a^2 + ab + 7ac + 2b^2 - 6bc - 4c^2$

116. $-b^3 + 2b^2 + 7b - 8$; 16 117. 51

118. $\left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 3\right)$ 119. $(p^2 + q^2 - pq)(p^2 + q^2 + pq)$

120. (i) 8 (ii) 300 121. $x(x^2 - x + 1)$

122. Side = 25 units; $x = 5$ 124. $10x(2x + 1)$ sq. units

125. (i) - (b) (ii) - (c) (iii) - (a)

Unit 8

1. (c) 2. (a) 3. (b) 4. (a) 5. (c) 6. (c)
7. (a) 8. (c) 9. (c) 10. (a) 11. (a) 12. (b)
13. (a) 14. (b) 15. (c) 16. (b) 17. (d) 18. (d)
19. (b) 20. (d) 21. (c) 22. (b) 23. (a) 24. (d)
25. (a) 26. (d) 27. (c) 28. (a) 29. (c) 30. (a)
31. (b) 32. (c) 33. (b) 34. 10^{-10} 35. a^{-7} 36. 1
37. 1 38. $\frac{1}{2^6}$ 39. 2^{-6} 40. Negative
41. Positive 42. 10^{-5} 43. $\frac{2}{13}^{-36}$ 44. $\frac{36}{22}$
45. 1 46. 1.0×10^{-8} 47. 1.234×10^7
48. 3410000 49. 2394610 50. 6^{-2} 51. 3^4 or 81
52. 3^{11} 53. 0.0000003 54. equal 55. 3.25×10^{10}
56. 8×10^{-9} 57. 0.0000000000²³ 58. 8^4
59. 2^{10} 60. 12^{-2} or $\frac{1}{144}$ 61. 6 62. 0
63. $\frac{1}{3^{-5}}$ 64. 1 65. 49 66. False
67. True 68. True 69. False 70. True
71. False 72. False 73. False 74. False
75. False 76. False 77. False 78. True
79. True 80. False 81. False 82. True
83. True 84. True 85. True 86. True
87. False 88. False 89. True 90. True
91. (i) 100^{10} (ii) 2^5 (iii) $\frac{1}{2}^{-1}$

- 92.** $\frac{1}{3^9}$ **93.** 2^{-8} **94.** $\frac{3}{4}^3$ and $\frac{-3}{4}^3$
- 95.** $\frac{4}{9}^2$ and $\frac{-4}{9}^2$ **96.** (a) $\frac{-2}{3}^{-6}$ (b) 2^{-10}
- 97.** -128
- 98.** (i) 29 (ii) $\frac{3^8}{2^7}$ (iii) $\frac{7^5}{10} z^2$ (iv) 2^{-10} or $\frac{1}{1024}$
- 99.** (i) $x = -2$ (ii) $x = -1$ (iii) $x = 0$
- 100.** 2.93×10^{-4} **101.** $(100)^9$ **102.** 1 **103.** 1
- 104.** $\frac{49}{90}$ **105.** $x = 2$ **106.** 3.9×10^8
- 107.** 5.678×10^{-6} **108.** 1.312×10^6
- 109.** 6.0×10^9 **110.** 1.5×10^7 **111.** 5.913×10^9 km
- 112.** 1.0×10^{-8} g **113.** 3.72×10^6 kg **114.** 1.25×10^{12}
- 115.** (a) 1.673×10^{-24} gm (b) 2.2×10^{-8} cm
 (c) 3.34×10^{-21} tons (d) 10^{12}
 (e) 5.6×10^4 (f) 5.0×10^5
 (g) 6.3072×10^7 sec (h) 5.0×10^8 cm²
- 116.** $x = -1$ **117.** $\frac{(-2)^7}{(3)^9}$ **118.** $n = 1$ **119.** $n = 9$
- 120.** $625x^3$ **121.** 400 **122.** $n = 6$ **123.** 16 kg
- 124.** (a) 2^{24} (b) 2^{48} **125.** B **126.** 2^8
- 127.** (a)

Number of Hops	Distance Covered	Distance Left	Distance Covered
1.	$\frac{1}{2}$	$\frac{1}{2}$	$1 - \frac{1}{2}$
2	$\frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}$	$\frac{1}{4}$	$1 - \frac{1}{4}$
3	$\frac{1}{2}\left(\frac{1}{4}\right) + \frac{3}{4}$	$\frac{1}{8}$	$1 - \frac{1}{8}$
4	$\frac{1}{2}\left(\frac{1}{8}\right) + \frac{7}{8}$	$\frac{1}{16}$	$1 - \frac{1}{16}$
5.	$\frac{1}{2}\left(\frac{1}{16}\right) + \frac{15}{16}$	$\frac{1}{32}$	$1 - \frac{1}{32}$
6.	$\frac{1}{2}\left(\frac{1}{32}\right) + \frac{31}{32}$	$\frac{1}{64}$	$1 - \frac{1}{64}$
7.	$\frac{1}{2}\left(\frac{1}{64}\right) + \frac{63}{64}$	$\frac{1}{128}$	$1 - \frac{1}{128}$
8.	$\frac{1}{2}\left(\frac{1}{128}\right) + \frac{127}{128}$	$\frac{1}{256}$	$1 - \frac{1}{256}$
9.	$\frac{1}{2}\left(\frac{1}{256}\right) + \frac{255}{256}$	$\frac{1}{512}$	$1 - \frac{1}{512}$
10.	$\frac{1}{2}\left(\frac{1}{512}\right) + \frac{511}{512}$	$\frac{1}{1024}$	$1 - \frac{1}{1024}$

127. (b) $1 - \left(\frac{1}{2}\right)^n$

(c) No, because for reaching 1, $\left(\frac{1}{2}\right)^n$ has to be zero for some finite n which is not possible.

128. (a)

x	1^x	2^x	3^x	4^x	5^x	6^x	7^x	8^x	9^x	10^x
1	1	2	3	4	5	6	7	8	9	10
2	1	4	9	16	25	36	49	64	81	100
3	1	8	27	64	125	216	343	512	729	1000
4	1	16	81	256	625	1296	2401	4096	6561	10000
5	1	32	243	1024	3125	7776	16807	32768	59049	100000
6	1	64	729	4096	15625	46656	117649	262144	531441	1000000
7	1	128	2187	16384	78125	279936	823543	2097152	4782969	10000000
8	1	256	6561	65536	390625	1679616	5764801	16777216	43046721	100000000
One digit of the Power	1	2,4,8,6	3,9,7,1	4,6	5	6	7,9,3,1	8,4,2,6	9,1	0

(b) (1) 6 (2) 1 (3) 3 (4) 5 (5) 0

(c) (1) 1 (2) 4 (3) 7 (4) 1

129. (a) Sun - 1.99×10^{30} Mercury - 3.3×10^{23} Venus - 4.87×10^{24} Earth - 5.97×10^{24} Mars - 6.42×10^{29} Jupiter - 1.9×10^{27} Saturn - 5.68×10^{26} Uranus - 8.68×10^{25} Neptune - 1.02×10^{26} Pluto - 1.27×10^{22} Moon - 7.35×10^{22}

(b) Pluto < Moon < Mercury < Venus < Earth < Uranus < Neptune < Saturn < Jupiter < Mars.

(c) Venus

130. (a) Sun - 1.496×10^8 Jupiter - 7.783×10^8 Mars - 2.279×10^8 Mercury - 5.79×10^7 Neptune - 4.497×10^9 Pluto - 5.9×10^9 Saturn - 1.427×10^9 Uranus - 2.87×10^9 Venus - 1.082×10^8

(b) Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto.

131. (a) Lead (b) Titanium

(c) Hydrogen < Lithium < Titanium < Silver < Lead

132. 2.8968192×10^{12} m 133. 2.543×10^{-2} m

- 134.** 0.000000767 **135.** $9.1093826 \times 10^{-28}$ g
- 136.** Six thousand one hundred million.
- 137.** (a) Generation Ancestor
- | | |
|----|----------|
| 1 | 2 |
| 2 | 2^2 |
| 12 | 2^{12} |
- (b) 2^n
- 138.** 1610 billion in a week or 1.61×10^{12}
83950 billion in an year or 8.395×10^{14}
- 139.** 37.5 g **140.** (a) $\frac{1}{3^7}$ (b) 5 half lines
- 141.** 1.3×10^{-15} m **142.** 5.0×10^{-2} m
- 144.** $(144) \xrightarrow{\times 2^{-3}} (18) \xrightarrow{\times 12^{-1}} \left(\frac{3}{2}\right) \xrightarrow{\times 3^{-2}} \left(\frac{1}{6}\right)$
- 145.** 1.15×10^{-5} days
- 146.** (a) Bajra, Jawar, Rice
(b) Bajra 1.3×10^3
Jawar 1.26×10^6
Rice 3.6×10^3
Wheat 7.0×10^5
(c) 3.0×10^3 hectares
- 147.** 40 cm
- 148.** (a) $(\times 2^2)$ and yes $(\times 5^2)$ hooked together
(b) $(\times 4)$ machine
- 149.** 64 cm
- 150.** (a) Two times
Total stretch is 10,000
(b) Five times
Total stretch 16,807
(c) Seven times
Total Stretch is 78,125

151. $(\times 4^3)$, $(\times 8^2)$, $(\times 2^6)$ machines **152.** It will remain same.

153. (a) They do not change its length. (b) 1 **154.** 3 cm

155. (i) 1 cm (ii) $\frac{1}{8}$ cm or 0.125 cm **156.** $\frac{1}{9}$ cm **157.** 5

158. (a) $(\times 2)$ (b) $(\times 2^2)$ (c) $(\times \frac{1}{5})$

159. (a) 2^9 (b) 100^{12} (c) 7^{61}

(d) 3^{2y} (e) 2^3 (f) $\left(\frac{1}{6}\right)^2$

160. (a) Yes, $(\times 7^5)$ (b) No (c) No

(d) Yes, $(\times (0.5)^5)$ (e) Yes, $(\times 12^5)$

161. $(\times 6^3)$ **162.** $5^2 \times 5^2$

163. (a) $(\times 2^0)$ (b) $(\times 5^{-1})$ (c) 5 cm (change in question)

(d) 3 cm

164. (a) $2^2 \times 5^2$ (b) $3^2 \times 11^1$

(c) $(\times 37)$ (d) 101×111

165. x^3 , x^9

166. $x\left(\frac{1}{2}\right)^3$

167. $a \times 25$, $a \times 125$, $a \times 625$ **168.** $\times 125$

169.

	Machine		
Input length	x^2	x^{10}	x^5
0.5	1	5	2.5
3	6	30	15
7	14	70	35

170. Give them a 8×8 grid

Now find sum of each row, e.g. 1st row

$$= 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7$$

$$= 255$$

2nd row

$$= 2^8 + 2^9 + 2^{10} + 2^{11} + 2^{12} + 2^{13} + 2^{14} + 2^{15}$$

$$= 2^8 (2^0 + 2^1 + 2^2 + 2^3 + 2^5 + 2^6 + 2^7)$$

$$= 2^8 \times 255$$

$$= 256 \times 255$$

$$= 65280$$

3rd row

$$= 2^{16} \times 255$$

$$= 16711680$$

$$2^8 = 256$$

$$2^{16} = 2^8 \times 2^8$$

$$= 256 \times 256$$

and so on

171. Diameter of sun is 100 times the diameter of earth)

172. 26.32×10^{29} kg **173.** 1492.16×10^8 m **174.** 2.7×10^8 sec

175. 3 **176.** $\frac{64}{27}$

177. (1) $x = -2$ (2) $x = -7$ (3) $x = 6$
(4) $x = 7$ (5) $x = -1$ (6) $x = 4$

178. (1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) $\frac{1}{2}$ (4) 2

179. (1) $-\left(\frac{6}{11}\right)^4$ (2) $\left(\frac{-5}{7}\right)^3$ (3) $\left(\frac{-20}{63}\right)^2$ (4) $\left(\frac{5}{10}\right)^4$ or $\left(\frac{1}{2}\right)^4$

180. (1) $\frac{8}{15}$ (2) 0 (3) $\frac{28}{169}$ (4) 0
(5) $3^7 \times t^2$ (6) $(3t)^6$

Activities

Activity 1

Number of Cuts	Number of Ballots
1	2 ($= 2^1$)
2	4 ($= 2^2$)
3	8 ($= 2^3$)
4	16 ($= 2^4$)

- (a) 2^n (b) 2^{40} (c) 9 cuts
(d)

Number of Cuts	Area (cm^2)
0	324
1	162
2	81
3	40.5
4	20.25
5	10.125
6	5.0625
7	2.53125
8	1.265625
9	0.6328125
10	0.3164062

Formula – $A \times 2^{-n}$ (changes made in question)

- (e) 8192 cm^2

Activity 2

(a)

Number of Steps	Number of Ballots
1	3
2	3^2
3	3^3
4	3^4
5	3^5

- (b) 3^{15} , 3^n (c) At least 11 steps

Unit 9

1. (a) 2. (b) 3. (b) 4. (a) 5. (c) 6. (a)
7. (c) 8. (b) 9. (d) 10. (b) 11. (a) 12. (c)
13. (c) 14. (b) 15. (a) 16. (c) 17. (c) 18. (b)
19. (b) 20. (c) 20.8% 21. Discount 22. 200 23. 1 : 10
24. Discount = M.P. – S.P. 25. Discount = Discount % of M.P.
26. Sales tax 27. $A = P \left(1 + \frac{R}{100} \right)^n$ 28. Sales tax = tax% of Bill amount
29. Conversion period 30. Overhead expenses
31. Marked Price 32. $A = P \left(1 + \frac{r}{200} \right)^{2t}$
33. equal, denominator 34. Rs 1,000
35. A = Rs 9331.20, CI = 1331.20 36. Rs 27,000
37. 10%, $1\frac{1}{2}$ years
38. $x + \frac{40}{100}x = 1,12,000$ (Let C.P. be x)
$$\frac{140x}{100} = 1,12,000$$
$$x = \frac{1,12,000 \times 100}{140} = 8000$$
39. $\frac{20}{3}\%$ or $6\frac{2}{3}\%$ 40. 100% 41. Rs 364 42. Rs 10,000
43. 400% 44. 300% 45. Rs 199.50 46. True
47. False 48. False 49. True 50. False
51. False 52. False 53. False 54. True

- 55.** False **56.** True **57.** True **58.** False
59. True **60.** False **61.** False **62.** False
63. True **64.** True **65.** False **66.** 840
67. 29.67 kg, 23.73 kg, 10.79 kg or 10.8 kg (approx.)
68. (a) Rs 5177.50 (b) Rs 1280.50 **69.** (a) Rs 500 (b) Rs 10,000
70. (a) 10% (b) 3% **71.** Rs 380 **72.** Increase 5.76 **73.** $\frac{50}{3}\%$
74. 3703 **75.** 3019.14
76. (a) 40% (b) $\frac{32}{3}\% = 10\frac{2}{3}\%$ (c) 20%
77. 55.84%, 2.23% **78.** (a) Rs 664.95 (b) Rs 1243.26
(c) Rs 2305.38 (d) Service Tax = Rs 6.29, Total = Rs 4219.88
79. (a) Rs 3,200 (b) Rs 43,200 (c) Rs 3,456 (d) Rs 46,656
80. (i) 57.55% (ii) 22.65% **81.** Rs 35 **82.** 12.5%
83. Bill amount Rs 582.01 **84.** $882.9 + 3\% = \text{Rs } 909.39$
85. (i) Rs 5,000 (ii) Rs 1,05,000 (iii) Rs 5,250 (iv) Rs 1,10,250
86. Gain 27.08% **87.** Rs 630 **88.** Rs 7,840 **89.** 7305.38
90. Rs 25,000 **91.** 7,00,000 **92.** 0% gain or no profit no loss
93. Petrol 10.96%, Diesel 6.09%, LPG 8.20%
94. A. 42.06% (increase) B. 15.94% (decrease)
C. 83.34% (decrease) D. 8.34% (decrease)
95. 18.027% or 18.03% **96.** Loss = 0.25%
97. 40% **98.** Rs 864 **99.** Rs 3561.60 **100.** 30%
101. Rs 18,400 **102.** Rs 800 **103.** Rs 1653.60, Rs 1620
104. Amount = Rs 10,75,840, Interest = Rs 51,840
105. Amount to be paid = Rs 3798.50
106. (a) (b) 690 mg (c) 120% (d) 3 : 7 **107.** Rs 90

- 108.** At store A the game is less expensive.
- 109.** (a) Rs 30.60 (b) Rs 59.40
- 110.** (a) No 2 method will give a lower price.
(b) Method 1 : Rs 202.50, Method 2: Rs 190
(c) Method 1, because in this method actual discount is less.
- 111.** Neelgiri apartments will be cheaper for the first two months by Rs 900.
- 112.** 20% increase is on original amount (if original price is Rs 100 so increased price would be Rs 120) but 20% decrease is on increased amount (i.e. 20% of 120 would be Rs 24), so decreased amount would be $120 - 24 = 96$. Hence decreased price is less than the original amount.
- 113.** 1. 93.3% 2. $\frac{3}{4}$ 3. False, as according to the claim, for $\frac{3}{100}$ affect of UV rays
1 minute = $33\frac{1}{3}$ SPF
Affect $\neq$ 30 SPF claim
- 114.** Rs 12,50,000
- 115.** Original price = Rs 3.97 per kg. Reduced Price = Rs 3.38/kg
- 116.** (1) 81.6 (2) 90.4 (3) 85 (4) 84
(5) 86.67 (6) 82.5 (7) 90 (8) 82
(9) 86.67 (10) 87 (11) 88.5
- 117.** 91.43%
- 118.** Minakshi must finish greater per cent of homework at home.
- 119.** 36% **120.** 44.4% **121.** 37.52 kg **122.** 4.431 gram
- 123.** He is finding what per cent is 5 of 32.
- 124.** Brand 1 (X) has greater sales tax rate
Brand 1 : 7.14%
Brand II (Y) : 4.84%

Unit 10

1. (c) 2. (d) 3. (a) 4. (d) 5. (a) 6. (a)
7. (a) 8. (d) 9. (d) 10. (b) 11. (c) 12. (a)
13. (a) 14. (c) 15. (d) 16. (b) 17. directly
18. inversely 19. direct, directly
20. inverse, inversely 21. inversely 22. x/y
23. directly 24. inversely
25. 16/3h or 5 h 20 mins 26. 300 27. 96
28. directly 29. constant 30. ab, constant
31. ratio 32. product 33. directly 34. 9.6 km
35. $2\frac{1}{4}$ h or 2h 15 mins 36. 90 cm 37. $y = 8$
38. = 39. $\frac{a_1}{a_2} = \frac{b_2}{b_1}$ 40. 480 cm²
41. 288 hrs 42. 0.250 km 43. False 44. False
45. False 46. False 47. False 48. False
49. True 50. False 51. False 52. False
53. False 54. True 55. True 56. False
57. False 58. True 59. True
60. (i) Inversely (ii) Direct (iii) Inverse (iv) Direct
(v) Direct
61. (i) Direct (ii) Direct (iii) Direct (iv) Direct
(v) Neither
62. (i) Direct (ii) Neither (iii) Inverse (iv) Direct
(v) Direct
63. $y = 30$ 64. $x = 128$ 65. $l = 40$ 66. $x = 20$
67. $39\frac{3}{8}$ 68. 448 person 69. 540 words 70. 96 km/h
71. (i) $\frac{l}{m}k$ (ii) $k = \frac{1}{3}$ (iii) $l = 11$ (iv) $m = 24$
72. Rs 9,000 73. 8.75 cm 74. $x = 72, y = 45$
75. 280 m 76. 60l 77. (i) No (ii) Yes (iii) Yes

- 78.** (i) $27/2 = p$, $36/13 = q$, $108/25 = r$
 (ii) $x = 45$, $y = 7.2$, $z = 9$
 (iii) $l = 12$, $m = 20/3$, $n = 12/5$
- 79.** (i) Rs 540 (ii) 60 m **80.** 12 pumps **81.** Rs 4,800
- 82.** 9 m **83.** 25 days
- 84.** (i) mixture A, (ii) mixture D, (iii) mixture F, (iv) mixture G
 Lightest blue shade in mixture D.
 30 containers of blue colours
 75 containers of white colours
- 85.** Purple (=12), Blue (=20), White (=16)
 Total = $12 + 20 + 16 = 48$
 Statement I : P : Total = $12 : 48 = 1 : 4$
 Statement II : B : Total = $20 : 48 = 5 : 12$
 Statement III : W : Total = $16 : 48 = 1 : 3$
 Statement IV : P : B : = $12 : 20 = 3 : 5$
 Statement IV : P : W : = $12 : 16 = 3 : 4$
- 86.** 5 sweets **87.** 11 cows **88.** 21 person **89.** 5 km
- 90.** 9.00 A.M.
- 91.** 1 - H, 2 - D, 3 - G, 4 - F
- 5 - C6 - A 7 - B 8 - E
- 92.** 60 g **93.** 35 km **94.** $24.9 \text{ m} \left[\because \frac{x}{21} = \frac{9.5}{8} \right]$
- 95.** Slowest elevator C (speed 13 m/sec)
 Fastest elevator D (speed 17m/sec)
 For elevator B, D distance = 2.29 km
 For elevator C, D distance = 1.820 km
- 96.** 37.5 m **97.** 5 cups **98.** Yes, $k = 1/4$
- 99.** 0.6 secs **100.** p%
- 101.** (a) $10 : 7$ (b) 98 black keys (c) $7 : 17$
- 102.** Direct proportion, 120 km.

- 103.** 1/2 cup quick cooking gas
 1/6 cup bread flour
 1/6 cup sugar syrup
 1/2 tablespoon cooking oil
 2/3 cup water
 3/2 tablespoons yeast
 1/2 tea spoon salt

104. 8 new teachers

105. 125 miles

106. (a) Rs 425, (b) 480 posts

Across

- 1 Directly
 4 Unitary
 5 Less
 7 Proportion
 9 Decrease

Down

- 2 Inverse
 3 Equivalent
 6 Constant
 7 Product
 8 Increases

Unit 11

- 1.** (c) **2.** (c) **3.** (b) **4.** (a) **5.** (b) **6.** (c)
7. (d) **8.** (a) **9.** (b) **10.** (c) **11.** (c) **12.** (d)
13. (c) **14.** (c) **15.** (a) **16.** (b) **17.** (c) **18.** (d)
19. (c) **20.** (c) **21.** (a) **22.** (a) **23.** (c) **24.** (a)
25. (c) **26.** (a) **27.** (c) **28.** (c) **29.** 24 **30.** None
31. $10a^2$ **32.** 4 times **33.** $h^3, 6h^2$ **34.** $\frac{1}{4}$
35. 50% **36.** $\frac{\pi}{4}a^3$ **37.** πb^2 **38.** $\frac{1}{2}(h_1 + h_2)d$
39. Two times **40.** 3 **41.** rectangular, different
42. equal **43.** $2\pi rh$ **44.** $2\pi rh(h + r)$
45. $\pi r^2 h$ **46.** Diagonals **47.** Twice **48.** Equal
49. Volume **50.** Lateral **51.** 3 : 1 **52.** 36 : 1
53. True **54.** False **55.** False **56.** False
57. False **58.** True **59.** False **60.** False
61. False **62.** $\frac{1}{2}$ min or 30 sec. **63.** 15 m

- 64.** $1,050 \text{ m}^2$ **65.** Rs 528
66. (1) 352.8 m^2 , 468.3 m^2 (2) 106.3 m^2 , 102.80 m^2
 (3) 13.35 m^2 , 235.6 m^2
67. 10 m **68.** 26 min 24 sec **69.** 7 : 8
70. 84 m **71.** 302 m **72.** 32.4 cm **73.** 0.636 km
74. 0.264 km/hr **75.** 13 m **76.** 53000 sq. units
77. 30100 sq. units **78.** 432 m^2 **79.** 240 m^2
80. 600 m^2 **81.** 13046 cm^2 **82.** 72 cm^2 **83.** 199.5 cm^2
84. 228.85 cm^2 **85.** 88.28 cm^2
86. (a) $\frac{x^3}{2}$ (b) $6y^3$ **87.** 1 : 5 **88.** 1 : 2π
89. 43.12 m^3 **90.** $r = 21 \text{ cm}$, $h = 14 \text{ cm}$
91. $V = 11440 \text{ cm}^3$, Weight = 91520 g
92. (a) double of the original (b) Half of the original
 (c) One fourth of the original **93.** 27 times the original
94. $h = 20 \text{ cm}$ **95.** 13280 cm^2 **96.** 22.68 m^3 , 22680 l
97. 64 cubes **98.** 6752 cm^3 **99.** $45,000 \text{ m}^3$
100. 1390.72 cm^2 **101.** 0.78 m
102. 42038.857 **103.** 1400 cm^2 **104.** B Pipe **105.** 200 m^3
106. 1 day **107.** 1440 **108.** 1848 cm^2
109. 25 dm, 20 dm, 15 dm **110.** $r = 0.07 \text{ m}$, 0.44 m^2
111. (a) 27 times (b) $\frac{1}{64}$ times **112.** $V = 3850 \text{ cm}^3$, $A = 110 \text{ cm}^2$
113. 445000 cm^2 , = 44.55 l **114.** $r = 8 \text{ cm}$, $A = 603.428 \text{ cm}^2$
115. 11180400 cm^2 , 11.180400 cm^2 **116.** 621600 l
117. 1000 **118.** $h = 8 \text{ m}$, $b = 10 \text{ m}$
119. 1 : 1 **120.** 6500 cm^3 **121.** 3 cm^2
122. 2016 cm^2 **123.** 2042 **124.** 401.2 cm^2
125. 70 cm **126.** 5082 cm^3 , 3811.5 cm^3

Unit 12

1. b 2. d 3. b 4. c 5. c 6. a
 7. d 8. c 9. c 10. d 11. line graph
 12. graph 13. pair of 14. y -axis 15. x -axis y -axis
 16. plotting 17. x 18. x -axis 19. 2 20. zero
 21. 4 22. x -coordinate/abscissa 23. (5, 4)
 24. y -coordinate/ordinate 25. origin 26. True 27. True
 28. False 29. False 30. False 31. True 32. True 33. False
 34. True 35. (1) d, (2) f, (3) e, (4) a, (5) b, (6) c
 36. (a) ii (b) iii (c) i (d) v (e) vi (f) iv
 37. (a) F (2, 0) (b) A (0, 4) (c) H (5, 1) (d) C (2, 6) (e) E (3, 3)
 38. A (0, 7.5) B (4, 5) C (7.5, 2.5) D (11, 0) E (14.5, 6.5)
 F (18, 9.5)
 40. (a) (A, f) (b) (monkeys, elephants) (c) (o, e) (d) (c, c)
 41. (a) 7, (b) 5, 90 42. (a) 5 (b) 0 (c) 7
 43. (a) Yes (b) No, square (c) No, triangle
 44. x 1 2 3 4
 y 3 6 9 12 46. (a) Rs 70, (b) 5
 47. (a) Uniform speed.
 (b) Moves with uniform speed then comes to rest.
 (c) Moves with non-uniform speed then slowly comes to rest.

48. (a)

x	0	1	2	3
y	1	4	7	10

(b)

x	0	2	4	6
y	-1	1	3	5

49. (a)

x	0	1	2	3
y	0	1	2	3

(b)

x	0	1	2	3
y	2	4	6	8

50. (a) 264 unit (b) $r = 35$ unit

51. (a) Maximum temp. in $^{\circ}\text{C}$ in the two consecutive weeks.

(b) First week (c) Wednesday (d) Friday

(e) 1st week - 37°C , 2nd week - 33°C

(f) Sunday (g) Wednesday

52. (a) April (b) March (c) April (d) 250 (e) 125 (f) $2/3$

53. (a) Subjects marks obtained (out of 10) by Sania in two terms exams in class VIII.

(b) Maths (c) English & Maths

(d) English & Hindi (e) 6 (f) Same in boths (g) Test I Maths

54. (A) (1, 1) E (5, 1) I (4, 4)
 (B) (3, 0) F (6, 3) J (4, 5)
 (C) (4, 2) G (5, 5) K (3, 6)
 (D) (2, 3) H (4, 3) L (2, 6)
 (M) (1, 5) O (2, 4) Q (0, 5)
 (N) (2, 5) P (1, 2)

55. (a) 10 am (b) 16 km (c) not travelling

(d) 40 km (e) 24 km (f) 2 pm

(g) 4 km/h, 0 km/h (h) 10 p.m.

56. (a) $p = 6$ (b) $q = 4$

57. (a) Maximum temp is 31°C in a week

(b) Sunday, 25°C (c) Wednesday

(d) Friday

58. (a) 240 km (b) 5 hours (c) 2 hours (d) 120 km

(e) Time and Distance graph

(f) P after 1 hour

R after 5 hours

Q after 3 hours

S after 6 hours

59. D (4, 4) **60.** D (3, 0) No **61.** (2, 2)

62. (a) Vendor A (b) Sunday (c) Saturday to Sunday
(d) Thursday (e) Tuesday & Wednesday

63. (a) 7°C (b) 6 a.m. (c) 3°C
(d) between 8 am to 9 am (e) between 8 am to 9 am

64. (a) 90 cm (b) 20 cm more (c) between 4 yrs to 6 yrs

65. Sneha made least progress between 25 minutes to 40 minutes

66. (a) E (0.5, 0.5) J (2, 1.5)
F (2, 2) K (8, 6)
G (4, 2) L (16, 6)
H (2.5, 0.5) M (10, 1.5)

68. (a) 0 - 20 sec. (b) 30 sec. (c) nearly 20°C
(d) It reaches 100°C at 50 sec. which is the maximum.

69. (a) line graph
(b) It represents the no. of people who visited a store at a particular time.
(c) 1 p.m. (d) less than 5 (e) 20

70. (a) 5.30 a.m. and ends at 6 p.m. (b) 12:30 hours
(c) forward (d) 3 hours

71. (a) 8:45 am for 15 minutes (b) faster (c) at 9.00 a.m.
(d) 10 km. (e) 10 km.

72. Graph 15 km. **73.** Graph

74. (a) 18 years, 17 years, (b) boys

75. (a) Time and distance
(c) 0 to 5 minutes and 5 to 10 minutes

76.

x	1	2	3	4	5
y	1.25	5	10	15	20

77. (a) highest 1990, lowest 2000 (b) 1996 (c) 4.7%

78. (a) pattern 1 2 3 4 5 6
toothpicks 4 7 10 13 16 19

(b) graph (c) pattern $y = 3x + 1$

(d)

x	7	8
y	22	25

(e) Yes

79. (a) $y = 3x - 1$

(b)

x	3	8
y	8	23

80. 1. Water, No 2. No. C (7, 5) D (5, 7) 3. (2, 7)
4. (6, 11) 5. (7, 3) (5, 5) 6. (7.5, 3) 2 km
7. (8.5, 3) 8. (6.25, 3)
9. (9, 4) (10, 4) (11, 5) 10. (7, 8) (8, 8) (9, 8) 11. (5, 3) (6, 2) (7, 2)

81. a) Makes it easy to understand the temp. change
b) Temp. increases up to 1:00 p.m. and then decreases
c) at 12 pm $19^{\circ}\text{C}.$, at 8 pm $10^{\circ}\text{C}.$

82. a) E and F b) D c) B and F, C and E
d) C, D, E e) Yes f) A g) A and C

83. (a) Height and Weight
(b) D - Ostrich B - Donkey A - Crocodile C - Dog

84. a) True b) True c) True
d) True e) False

85. Side length of purple S 1 2 3 4 5 10 100

White Tiles b 4 8 12 16 20 40 400

(c) $b = 45$

86. Rows r	4	6	8
White Tiles	9	15	21
Purple Tiles	1	6	15

Activity

1. Bar graph	2. y -axis	3. Linear graph
4. Origin	5. Coordinates	6. Right
7. Abcissa	8. Axes	9. Graph
10. Cartesia	11. Line	12. Ordinate
13. Whole	14. Histogram	15. Gaps
16. Horizontal	17. x -axis	

Unit 13

1. (c)	2. (b)	3. (c)	4. (c)	5. (c)	6. (c)
7. (d)	8. (b)	9. (a)	10. (d)	11. (b)	12. (a)
13. (c)	14. (a)	15. (c)	16. (a)	17. (b)	18. 9
19. 1, 4, 7	20. 1	21. 11	22. 9	23. 11	
24. $A = 6, B = 3$	25. $A = 2, B = 4$ (four)	26. $B = 7$	27. $x = 0$		
28. $a + c$ or 12 ($a + c$)	29. 11	30. $(a + c) - b$	31. 5		
32. values, $A = 3, B = 6$	33. t 41	34. True	35. False	36. False	
37. True	38. True	39. True	40. True	41. False	42. True
43. False	44. False	45. $a = 3$	46. $P = 6$ and $Q = 9$	47. 12	
48. 33033, 66066, 99099	49. $A = 9, Z = 8, X = 1$				
50. $A = 8, B = 1, C = 3$	51. $A = 6, B = 7, C = 1$	52. $A = 6, B = 9$			
53. $A = 5, B = 6, C = 7$	54. $A = 9, B = 1$	55. $A = 8, B = 9$			
56. $A = 7, B = 8, C = 4$	57. $A = 2, B = 5$	58. $A = 9, B = 1, C = 8$			

- 59.** $A = 7, B = 2$ **60.** $A = 7, B = 2, C = 3, D = 1$ **61.** $A = 9$
62. $X = 8$ **63.** k is either 0 or 3, 6, 9 **64.** $y = 5$ **65.** $x = 8$
66. 2 **67.** $S = 8, L = 5, M = 9, G = 1$ **68.** $S = 6, M = 9, B = 1, U = 0$
69. 96, 85, 74, 63, 52, 41, 30
70. (a) 5555555555 (b) 7777777777 (c) 72 (d) 81
71. (i) $P = 7, Q = 4$ (ii) $M = 7, L = 4$ **72.** $B = 4$ **73.** $A = 4$
74. Least value of y is 0

Cross Number Puzzle

(A) 7	(F) 4
(B) 0	(G) 8
(C) 4	(H) 9
(D) 2	(I) 1
(E) 0	(J) 0

Activity

- 3, 5, 9
- 2, 3, 6, 9
- 2, 5, 10
- 2, 3, 6, 9, 11
- 2, 4, 8